DESCHANEL'S

NATURAL PHILOSOPHY

EVERETT

PART III.—ELECTRICITY AND MAGNETISM.

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ELECTROSTATICS.

CHAPTER I.

INTRODUCTORY PHENOMENA.

1. **Fundamental Phenomena.**—If a glass tube be rubbed with a silk handkerchief, both tube and rubber being very dry, the tube will be found to have acquired the property of attracting light bodies. If the part rubbed be held near to small scraps of paper, pieces of

Fig. 1.—Attraction of Light Bodies by an Electrified Body.

cut straw, sawdust, &c., these objects will move to the tube; sometimes they remain in contact with it, sometimes they are alternately attracted and repelled, the intensity as well as the duration of these effects varying according to the amount of friction to which the tube has been subjected.

If the tube be brought near the face, the result is a sensation similar

to that produced by the contact of a cobweb. If the knuckle be held near the tube, a peculiar crackling noise is heard, and a bright *spark* passes between the tube and knuckle. The tube then has acquired peculiar properties by the application of friction. It is said to be *electrified*, and the name of *electricity* is given to the agent to which the various phenomena just described are attributed.

Glass is not the only substance which can be electrified by friction; the same property is possessed also by resin, sulphur, precious stones, amber, &c. The Greek name of this last substance (ἤλεκτρον) is the root from which the word *electricity* is derived.

At first sight it appears that this property of becoming electrified by friction is not common to all bodies; for if a bar of metal be held in the hand and rubbed with wool, it does not acquire the properties

Fig. 2.—Electrification of a Metal by Friction.

of an electrified body. But we should be wrong in concluding that metals cannot be electrified by friction; for if the bar be fitted on to a glass rod, and, while held by this handle, be struck with flannel or catskin, it may be very sensibly electrified. There is therefore no basis for the distinction formerly made between electrics and non-electrics, that is, between substances capable and incapable of being electrified by friction; for all bodies, as far as at present known, are capable of being thus excited. There is, however, an important difference of another kind between them, which was first pointed out by Stephen Grey in 1729.

2. **Conductors and Non-conductors.**—In certain bodies, such as glass and resin, electricity does not spread itself beyond the parts of the surface where it has been developed; while in other bodies, such as metals, the electricity developed at any point immediately spreads itself over the whole body. Thus, in the last-mentioned experiment, the signs of electricity are immediately manifested at the end of the metal bar which is farthest from the glass rod, if the end next the rod be submitted to friction. Bodies of the former kind, such as glass, resin, &c., are said to be *non-conductors*. Metals are said to be good *conductors*. A non-conductor is often called an *insulator*, and a conductor supported by a non-conductor is said to be *insulated*. The appropriateness of these expressions is evident. No substance is perfectly non-conducting, but the difference in conduct-

ing power between what are called non-conductors and good conductors, is enormous. The following are lists of conductors and non-conductors, arranged, at least approximately, in order of their conducting powers. In the list of conductors, the best conductors are put first; in the list of non-conductors, the worst conductors (or best insulators) are put first.

CONDUCTORS.

All metals.	Metallic ores.	Living vegetables.
Well-burned charcoal.	Animal fluids.	Flax.
Plumbago.	Sea water.	Hemp.
Concentrated acids.	Spring water	Living animals.
Dilute acids.	Rain water.	Flame.
Saline solutions.	Snow.	Moist earth and stones.

NON-CONDUCTORS.

Shellac.	Gems.	Leather.
Amber.	Ebonite.	Baked wood.
Resins.	Caoutchouc.	Porcelain.
Sulphur.	Gutta-percha.	Marble.
Wax.	Silk.	Camphor.
Jet.	Wool.	Chalk.
Glass.	Feathers.	Lime.
Mica.	Dry paper.	Oils.
Diamond.	Parchment.	Metallic oxides.

The human body is a good conductor of electricity. If a person standing on a stool with glass legs be struck with a catskin, he becomes electrified in a very perceptible degree, and sparks may be drawn from any part of his body.

When an insulated and electrified conductor is allowed to touch another conductor insulated but not electrified, it is observed that, after the contact, both bodies possess electrical properties, electricity having been communicated to the second body at the expense of the first. If the second body be much the larger of the two, the electricity of the first is greatly diminished, and may become quite insensible. This explains the disappearance of electricity when a body is put in connection with the earth, which, together with most of the objects on its surface, may be regarded as constituting one enormous conductor. On account of its practically inexhaustible capacity for furnishing or absorbing electricity, the earth is often called *the common reservoir*.

It will now be easily understood why it is not possible to electrify a metal rod by rubbing it while it is held in the hand; since the

electricity, as fast as it is generated, passes off through the body into the earth.

Air, when thoroughly dry, is an excellent insulator; and electrified conductors exposed to it, and otherwise insulated, retain their charge with very little diminution for a considerable time. Dampness in the air is, however, a great obstacle to insulation, mainly, or (as it would appear from Sir W. Thomson's experiments) entirely, by reason of the moisture which condenses on the insulating supports. Electrical experiments are accordingly very difficult to perform in damp weather. The difficulty is sometimes met by employing a stove to heat the air in the neighbourhood of the supports, and thus diminish its relative humidity. Sir W. Snow Harris employed heating-irons, which were heated in a fire, and then fixed near the insulating supports; and thus succeeded in exhibiting electrical experiments to an audience in the most unfavourable weather. Sir W. Thomson, by keeping the air in the interior of his electrometers dry by means of sulphuric acid, causes them to retain their charge with only a small percentage of loss in twenty-four hours. Dry frosty days are the best for electrical experiments, and next perhaps to these, is the season of dry cutting winds in spring.

3. Duality of Electricity.—The elementary phenomena which we have mentioned in the beginning of this chapter may be more accurately studied by means of the electric pendulum, which consists of a pith-ball suspended by a silk fibre from an insulated support. When an electrified glass rod is brought near the insulated ball, the latter is attracted; but as soon as it touches the glass tube, the attraction is changed to repulsion, which lasts as long as the ball retains the electricity which it has acquired by the contact. A similar experiment can be shown by employing, instead of the glass tube, any other body which has been electrified by friction, for example, a piece of resin which has been rubbed with flannel.

If, while the pendulum exhibits repulsion for the glass, the electrified resin is brought near, it is attracted by the latter; and conversely, when it is repelled by the resin, it is attracted by the glass. These phenomena clearly show that the electricity developed on the resin is not of the same kind as that developed on the glass. They exhibit opposite forces towards any third electrified body, each attracting what the other repels. They have accordingly received names which indicate opposition. The electricity which glass acquires when rubbed with silk, is called *positive*; and that which resin acquires by friction

with flannel, *negative*. The former is also called *vitreous*, and the latter *resinous*. On repeating the experiment with other substances,

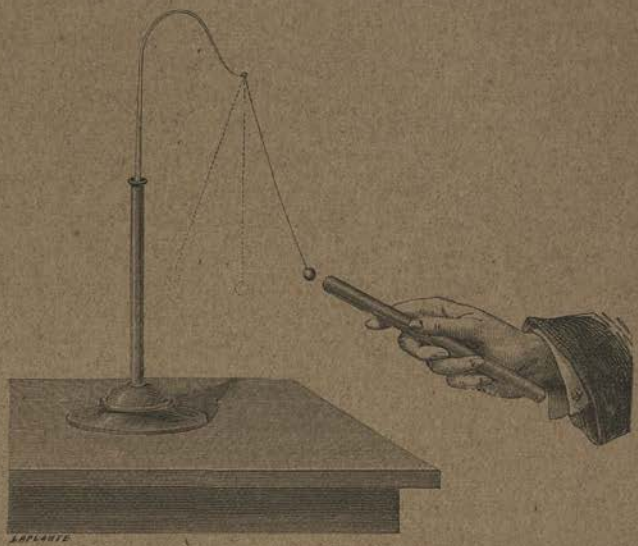

Fig. 3.—Electric Pendulum.

it is found that all electrified bodies behave either like the glass or like the resin.

4.—Without making any assumption as to what electricity is, we may speak of an electrified body as being *charged with electricity*, and we may compare quantities of electricity by means of the attractions and repulsions exerted. Bodies oppositely electrified must then be spoken of as charged with *electricities of opposite kind*, or *of opposite sign;* and experiment shows that, whenever electricity of the one kind is developed, whether by friction or by any other means, electricity of the opposite sign is always developed in exactly equal quantity. If a conductor receives two charges of electricity of equal quantity but opposite sign, it is found to exhibit no traces of electricity whatever.

Electricities of like sign repel one another and those of unlike sign attract one another.—The magnitude of the force exerted upon each other by two electrified bodies, is not altered in amount by reversing the sign of the electricity of one or both of them, provided that the quantities of electricity, and their distribution over the two

bodies, remain unchanged. If the sign of one only be changed, the mutual force is simply reversed, and if the signs of both be changed, the force is not changed at all.

5.—The simultaneous development of both kinds of electricity is illustrated by the following experiment:—Two persons stand on stools with glass legs, and one of them strikes the other with a catskin. Both of them are now found to be electrified, the striker positively, and the person struck negatively, and from both of them sparks may be drawn by presenting the knuckle.

The kind of electricity which a body obtains by friction with another body, evidently depends on the nature of their surfaces. If, for example, we take two discs, one of glass, and the other of metal, and, holding them by insulating handles, rub them briskly together, we shall find that the metal becomes negatively, and the glass positively electrified; but if the metal be covered with a catskin, and the experiment repeated, it will be the glass which will this time be negatively electrified. In the subjoined list, the substances are arranged in such order that, generally speaking, each of them becomes positively electrified by friction with those which follow it, and negatively with those which precede it.

Fur of cat.	Feathers.	Silk.
Polished glass.	Wood.	Shellac.
Woollen stuffs.	Paper.	Rough glass.

6. **Hypotheses regarding the Nature of Electricity.**—Two theories regarding the nature of electricity must be described on account of the historical interest attaching to them.

The two-fluid theory, originally propounded by Dufaye, reduced to a more exact form by Symmer, and still very extensively adopted, maintains that the opposite kinds of electricity are two fluids. Positive electricity is called the *vitreous fluid*, and negative electricity the *resinous fluid*. Fluids of like name repel, and those of unlike name attract each other. The union of equal quantities of the two fluids constitutes the neutral fluid which is supposed to exist in very large quantity in all unelectrified bodies. When a body is electrified, it gains an additional quantity of the one fluid, and loses an equal quantity of the other, so that the total amount of electric fluid in a body is never changed; and (as a consequence of this last condition) when a current of either fluid traverses a body in any direction, an equal current of the other fluid traverses it in the opposite direction.

This theory is in complete agreement with all electrical phenomena so far as at present known; but as it is conceivable that the two electricities, instead of being two kinds of matter, may be two kinds of motion, or, in some other way, may be opposite states of one and the same substance, it is more philosophical to avoid the assumption involved in speaking of *two electric fluids*, and to speak rather of *two opposite electricities*. They may be distinguished indifferently by the names *vitreous* and *resinous*, or *positive* and *negative*.

The *one-fluid theory*, as originally propounded by Franklin, maintained the existence of only one electric fluid, which unelectrified bodies possess in a certain normal amount. A positively electrified body has more, and a negatively electrified body less than its normal amount. The particles of this fluid repel one another, and attract the particles of other kinds of matter, at all distances. Æpinus, in developing this theory more accurately, found it necessary to introduce the additional hypothesis that the particles of matter repel one another. Thus, according to Æpinus, the absence of sensible force between two bodies in the neutral condition, is due to the equilibrium of four forces, two of which are attractive, and the other two repulsive. Calling the two bodies A and B, the electricity which A possesses in normal amount, is repelled by the electricity of B, and attracted by the matter of B. The matter of A is attracted by the electricity of B, and repelled by the matter of B. These four forces are all equal, and destroy one another; but, without the supplementary hypothesis of Æpinus, one of the four forces is wanting, and the equilibrium is not easily explained. To reconcile Æpinus's addition with the Newtonian theory of gravitation, it is necessary to suppose that the equality between the four forces is not exact, the attractions being greater by an infinitesimal amount than the repulsions.

The one-fluid theory in this form is, like the two-fluid theory, consistent with the explanation of all known phenomena. But it is to be remarked that there is no sufficient reason, except established usage, for deciding which of the two opposite electricities should be regarded as corresponding to an excess of the electric fluid.

Franklin was the author of the terms positive and negative to denote the two opposite kinds of electrification; but the names can legitimately be retained without accepting the one-fluid theory, understanding that opposite signs imply forces in opposite directions, and that the connection between the *positive* sign and the forces exhibited by *vitreous* electricity is merely conventional.

7. —In speaking of electric currents, the language of the one-fluid theory is almost invariably employed. Thus, if A is a conductor charged positively, and B a conductor charged negatively; when the two are put in connection by a wire, we say that the direction of the current is from A to B; whereas the language of the two-fluid theory would be, that a current of vitreous or positive electricity travels from A to B, and a current of resinous or negative from B to A.

CHAPTER II.

ELECTROSTATIC INDUCTION.

8. Induction.—In the preceding chapter we have spoken of movements of material bodies caused by electrical attractions and repulsions. We have now to treat of the movement of electricity itself in obedience to the attractions or repulsions exerted upon it by other electricity. This kind of action is called *induction*.

It may be illustrated by means of the arrangement shown in Fig. 4. The apparatus consists of a sphere C which is electrified positively, suppose, and of a conducting insulated cylinder A B placed near it. From this latter are suspended at equal distances a few pairs of pith-balls. When the cylinder is brought near the sphere, the balls are observed to diverge. The divergence of the different pairs is not the same, but goes on decreasing from the pair nearest the sphere until a point M is reached, where there is no divergence. Beyond this the divergence goes on increasing. The neutral point M does not exactly bisect the length of the cylinder, but is nearer the end A than the end B, and the former end is found to be more strongly electrified than the latter.

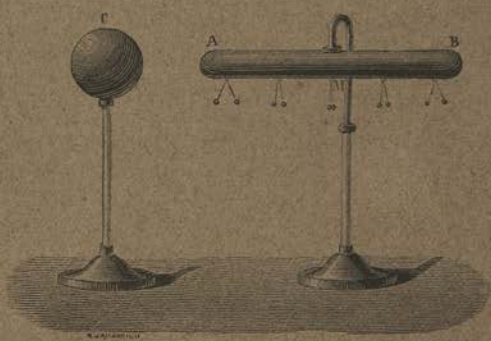

Fig. 4.—Electrification by Influence.

It is easy to show that the two ends of the cylinder are charged with opposite kinds of electricity; the end A being negatively, and

the end B positively electrified. We have only to bring an electrified stick of resin near the pith-balls at A, when these will be found to be repelled; if, on the contrary, it be held near those at B, they will be attracted.

The explanation is, that the positive electricity with which C is charged attracts the negative electricity of AB to the end A, and repels the positive to the end B. This action is more powerful at A than at B, on account of the greater proximity of the influencing body, and for the same reason the effect falls off more rapidly in the portion AM than in MB.

If the cylinder be brought closer to the sphere, the divergence of the balls increases; if it be removed farther from it, the divergence diminishes. Finally, all signs of electricity disappear if the sphere be taken away, or connected with the earth.

If, while the cylinder is under the influence of the electricity of C, the end B is connected with the earth, the pith-balls at this end

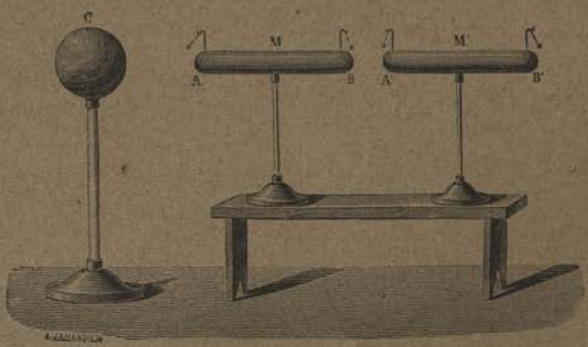

Fig. 5.—Successive Induction.

immediately collapse, while the divergence of those at A increases. The explanation is that the electricity which was repelled to the end B escapes to the earth, and thus affords an opportunity for a fresh exercise of induction on the part of the sphere, which increases the accumulation of negative electricity at A. We may also remark that the whole of the cylinder is now negatively electrified, the neutral line being pushed back to the earth. If the earth-connection be now broken, and the sphere C be then removed, the cylinder will remain negatively electrified, and will be in the same condition as if it had been touched by a negatively-electrified body. This mode

of giving a charge to a conductor is called *charging by induction*, and the charge thus given is always opposite to that of the *inducing body* C.

If a series of such conductors as A B be placed in line, without contact, and the positively-electrified body C be placed opposite to one end of the series, all the conductors will be affected in the same manner as the single conductor in the last experiment. They will all be charged with negative electricity at the end next C, and with positive electricity at the remote end, the effect, however, becoming feebler as we advance in the series. In this experiment each of the conductors acts inductively upon those next it; for example, if there be two conductors A B, A′B′, as in Fig. 5, the development of electricity at A′ and B′ is mainly due to the action of the positive electricity in M B. If the conductor A B be removed, the pith-balls at A′ and B′ will diminish their divergence.

The molecules of a body may be regarded as such a series of conductors, or rather as a number of such series. When an electrified body is brought near it, each molecule may thus become positive on one side and negative on the other. In the case of good conductors, this polarization is only instantaneous, being destroyed by the discharge of electricity from particle to particle. Good insulators are substances which are able to resist this tendency to discharge, and to maintain a high degree of polarization for a great length of time. This is Faraday's theory of "induction by contiguous particles."

9. Electrical Attraction and Repulsion.—The attraction which is observed when an electrified is brought near to an unelectrified body, is dependent upon induction. Suppose, for instance, that a body C, which is positively electrified, is brought near to an insulated and uncharged pith-ball. Negative electricity is induced on the near side of the pith-ball, and an equal quantity of positive on the further side. The former, being nearer to the body C, is more strongly attracted than the other is repelled. The ball is therefore upon the whole attracted.

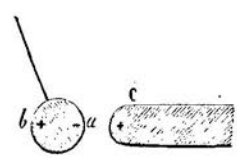

Fig. 6.—Electrical Attraction.

If the pith-ball, instead of being insulated, is suspended by a conducting thread from a support connected with the earth, it will be more strongly attracted than before, as it is now entirely charged with negative electricity.

In the case of any insulated conductor, the algebraic sum of the

electricities induced upon it by the presence of a neighbouring electrified body must be zero. If the pith-ball be insulated, and have an independent charge of either kind of electricity, the total force exerted on the pith-ball is the algebraic sum[1] of the two following quantities:—

(1) The force which the ball would experience, if it had no independent charge. This force, as we have just seen, is always attractive.

(2) The force due to the independent charge when distributed over the ball as it would be if C were removed. This second force is attractive or repulsive, according as the independent charge is of unlike or like sign to that of C. In the latter case, repulsion will generally be observed at distances exceeding a certain limit and attraction at nearer distances, the reason being that the force (1) due to the induced distribution increases more rapidly than the other as the distance is diminished.

It is important to remember this in testing, by the electric pendulum, or by any other electroscope, the kind of electricity with which a body is charged. In bringing the body towards the electroscope, the first movement produced is that which is to be observed, and repulsion is in general a more reliable test of kind of electricity than attraction.

10. Electroscopes.—An electroscope is an apparatus for detecting the presence of electricity, and determining its sign. The insulated electric pendulum is an electroscope. If the pith-ball, when itself uncharged, is attracted by a body brought near it, we know that the body is electrified. To determine the kind of electricity, the body is allowed to touch the pith-ball, which is then repelled. At this moment an excited glass tube is brought near. If it repels the ball, this latter, as well as the body which touched it, must be electrified positively. If the glass tube attracts it, or, still more decisively, if excited resin or sealing-wax repels it, the ball and the body which touched it are electrified negatively. The loss of electricity from the pith-ball is often so rapid as to render this test of sign somewhat uncertain.

The *gold-leaf electroscope* (Fig. 7) is constructed as follows:—

[1] We here suppose C to be a non-conductor, so that the distribution of its electricity is not affected by the presence of the pith-ball. If C be a conductor, the effect of induction upon it will be to favour attraction, so that an attractive force must be added to the two forces specified in the text.

Two small gold-leaves are attached to the lower end of a metallic rod, which passes through an opening in the top of a bell-glass, and terminates in a ball. The metallic rod is sometimes, for the sake of better insulation, inclosed in a glass tube secured by sealing-wax or some other non-conducting cement, and, for the same purpose, the upper part of the bell-glass is often varnished with shellac, which is less apt than glass to acquire a deposit of moisture from the air. The bell-glass is attached below to a metallic base, which excludes the external air. For the gold-leaves are sometimes substituted two straws, or two pith-balls suspended by linen threads; we have thus the *straw-electroscope* and the *pith-ball electroscope*.

Fig. 7.—Gold-leaf Electroscope

To test whether a body is electrified, it is brought near the ball at the top of the electroscope. The like electricity is repelled into the leaves, and makes them diverge, while the unlike is attracted into the ball. The sign of the body's charge may be determined in the following manner:—While the leaves are divergent under the influence of the body, the operator touches the ball with his finger. This causes the leaves to collapse, and gives to the insulated conductor composed of leaves, rod, and ball, a charge opposite to that of the influencing body. The finger must be removed while the influencing body remains in position, as the amount of the induced charge depends upon the position of the influencing body at the instant of breaking connection. On now withdrawing the influencing body, the charge of unlike electricity is no longer attracted to the ball, but spreads over the whole of the conductor, and causes the leaves to diverge. If, while this divergence continues, an excited glass tube, when gradually brought towards the ball, diminishes the divergence, we know that the body in question was electrified positively. If it increases the divergence, the body was electrified negatively.

Great caution must be used in bringing electrified bodies near the gold-leaf electroscope, as the leaves are very apt to be ruptured by

quick movements. If they diverge so widely as to touch the sides of the bell-glass, it is often difficult to detach them from the glass without tearing. To prevent this contact, two metallic columns are interposed, communicating with the ground. If the leaves diverge too widely, they touch these columns and lose their electricity.

CHAPTER III.

COULOMB'S QUANTITATIVE LAWS.

11. Coulomb's Torsion-balance.—Coulomb, who was the first to make electricity an accurate science, employed in his researches an instrument which is often called after his name, and which is still extensively employed. It depends on the principle that the torsion of a wire is simply proportional to the twisting couple. We shall first describe it, and then point out some of its applications.

It consists of a cylindrical glass case AA (Fig. 8), from the upper end B of which rises another glass cylinder DD of much smaller diameter. This small cylinder is fitted at the top with a brass cap *a*, carrying an index C. Outside of this, and capable of turning round it, is another cap *b*, the top of which is divided into 360 equal parts. In the centre of the cap *b* is an opening through which passes a small metal cylinder *d*, capable of turning in the opening with moderate friction, and having at its lower end a notch or slit. When the cap *b* is turned, the cylinder *d* turns with it; but the latter can also be turned separately, so as not to change the reading. These parts compose the *torsion-head*. A very fine metallic wire is held by the notch, and supports a small piece of metal, through which passes a light needle of shellac *f*, carrying at one end a small gilt ball *g*. A circular

Fig. 8.—Coulomb's Torsion-balance.

scale runs round the outside of the large cylinder in the plane of the needle. Finally, opposite the zero of this scale, there is a fixed ball g' of some conducting material, supported by a rod f' of shellac, which passes through a hole in the cover of the cylindrical case.

12. Laws of Electric Repulsion.—To illustrate the mode of employing this apparatus for electrical measurements, we shall explain the course followed by Coulomb in investigating the law according to which electrical repulsions and attractions vary with the distance. The index is set to the zero of the scale. The inner cylinder d, is then turned, until the movable ball just touches the fixed ball without any torsion of the wire. The fixed ball is then taken out, placed in communication with an electrified body, and replaced in the apparatus. The electricity with which it is charged is communicated to the movable ball, and causes the repulsion of this latter through a number of degrees indicated by the scale which surrounds the case. In this position the force of repulsion is in equilibrium with the force of torsion tending to bring back the ball to its original position. The graduated cap b is then turned so as to oppose the repulsion. The movable ball is thus brought nearer to the fixed ball, and at the same time the amount of torsion in the wire is increased. By repeating this process, we obtain a number of different positions in which repulsion is balanced by torsion. But we know, from the laws of elasticity, that the force (strictly the couple[1]) of torsion is proportional to the angle of torsion. Hence we have only to compare the total amounts of torsion with the distances of the two balls. By such comparisons Coulomb found that the force of electrical repulsion varies *inversely as the square of the distance.*

The following are the actual numbers obtained in one of the experiments. The original deviation of the movable ball being 36°, it was found that, in order to reduce this distance to 18°, it was necessary to turn the head through 126°, and, for a farther reduction of the deviation to 8°·5, an additional rotation through 441° was required. It will thus be perceived that at the distances of 36°, 18° and 8°·5, which may be practically considered as in the ratio of 1, $\frac{1}{2}$, and $\frac{1}{4}$, the forces of repulsion were equilibrated by torsions of 36°,

[1] The repulsive force on the movable ball is equivalent to an equal and parallel force acting at the centre of the needle (the point of attachment of the wire), and a couple whose arm is the perpendicular from this centre on the line joining the balls. This couple must be equal to the couple of torsion. The other component produces a small deviation of the suspending wire from the vertical.

$126° + 18° = 144°$, and $441 + 126 + 8{\cdot}5 = 575°{\cdot}5$ respectively. Now 144 is 36×4, and 575·5 may be considered as 576, or 36×16. Hence we perceive that, as the distance is divided by 2, or by 4, the force of repulsion is multiplied by 4 or by 16, which precisely agrees with the law enunciated above.

13. Equation of Equilibrium.—We must, however, observe that in this mode of reducing the observations two inaccurate assumptions are made. First, the distance between the balls is regarded as being equal to the arc which lies between them, whereas it is really the chord of that arc. Secondly, the force of repulsion is regarded as acting always at the same arm, whereas its arm, being the perpendicular from the centre on the chord, diminishes as the distance increases. The following investigation is more rigorous.

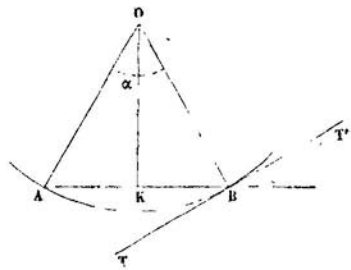

Fig. 9.

Let AOB (Fig. 9), the angular distance of the balls, be denoted by α, and let l be the length of the radius OA. Then the chord AB is $2l \sin \frac{1}{2}\alpha$, and the arm OK is $l \cos \frac{1}{2}\alpha$. Let f denote the force of repulsion at unit distance, and n the couple of torsion for 1°. Then the force of repulsion in the given position is $\frac{f}{4l^2 \sin^2 \frac{1}{2}\alpha}$ if the law of inverse squares be true, and the moment of this about the centre is $\frac{f \cos \frac{1}{2}\alpha}{4l \sin^2 \frac{1}{2}\alpha}$, which must be equal to nA, if A be the number of degrees of torsion. Hence we have

$$\frac{f}{4nl} = \text{A} \sin \tfrac{1}{2}\alpha \tan \tfrac{1}{2}\alpha,$$

and as the first member of this equation is constant, the second member must be constant also for different values of A and α, if the law of inverse squares be true. The degree of constancy is shown by the following table:—

	α	A	A $\sin \frac{1}{2}\alpha \tan \frac{1}{2}\alpha$.
1st experiment, ...	36	36	3·614
2d experiment, ...	18	144	3·568
3d experiment, ...	8·5	575·5	3·169
Supposed case, ...	9	576	3·557

The difference between the first and second numbers of the last

column is insignificant. That between the second and third is more considerable,[1] but in reality only corresponds to an error of half a degree in the measurement of the arc.

14. Case of Attraction.—The law of attractions may be investigated by a similar method. The index is set to zero, and the central piece is turned so as to place the movable ball at a known distance from the fixed ball. The two balls are then charged with electricity of different kinds. The movable ball is accordingly attracted towards the other, and settles in a position in which attraction is balanced by torsion. By altering the amount of torsion, different positions of the ball can be obtained. On comparing the distances with the corresponding torsions, it is found that the same law holds as in the case of repulsion. The experiment, however, is difficult, and is only possible when the balls are very feebly electrified. To prevent the contact of the two balls, Coulomb fixed a silk thread in the instrument, so as to stop the course of the movable ball.

15. Law of Attraction and Repulsion as depending on Amount of Charge.—We may assume as evident, that when an electrified ball is placed in contact with a precisely equal and similar ball, the charge will be divided equally between them, so that the first will retain only half the charge which it had before contact.

Suppose that an observation on repulsion has just been made with the torsion-balance, and that we touch the fixed ball with another exactly equal insulated ball, which we then remove. It will be found that the amount of torsion requisite for keeping the movable ball in its observed position is just half what it was before. The

[1] We have already seen that the mutual induction of two conductors tends to diminish their mutual repulsion, and that this inductive action becomes more important as the distance is diminished. Hence the repulsion at distance 9 should be less than a quarter of that at distance 18. The apparent error thus confirms the law.

Many persons have adduced, as tending to overthrow Coulomb's law of inverse squares, experimental results which really confirm it. Except when the dimensions of the charged bodies are very small in comparison with the distance, the observed attraction or repulsion is the resultant of an infinite number of forces acting along lines drawn from the different points of the one body to the different points of the other. The law of inverse squares applies directly to these several components, and not to the resultant which they yield. The latter can only be computed by elaborate mathematical processes.

It is incorrectly assumed in the text that the law ought to apply directly to two spheres, when by their distance we understand the distance between their nearest points. It is not obvious that the distance of the nearest points should give a better result than the distance between the centres.

The strongest evidence for the rigorous exactness of the law of inverse squares is indirect; see § 19.

same result will be obtained by touching the movable ball with a ball of its own size. We conclude that, if the charge of either body be altered, the attractive or repulsive force between the bodies at given distance will be altered in the same ratio. The law is not rigorously true for bodies of finite size, unless the distribution of the electricity on the two bodies remains unchanged. When the two bodies are very small in all their dimensions in comparison with the distance between them, their mutual force is represented by the expression

$$\frac{qq'}{D^2},$$

q and q' denoting their charges, and D the distance. If this expression has the positive sign, the force is repulsive, if negative attractive.

16. **Electricity resides on the Surface.**—Electricity (subject to the

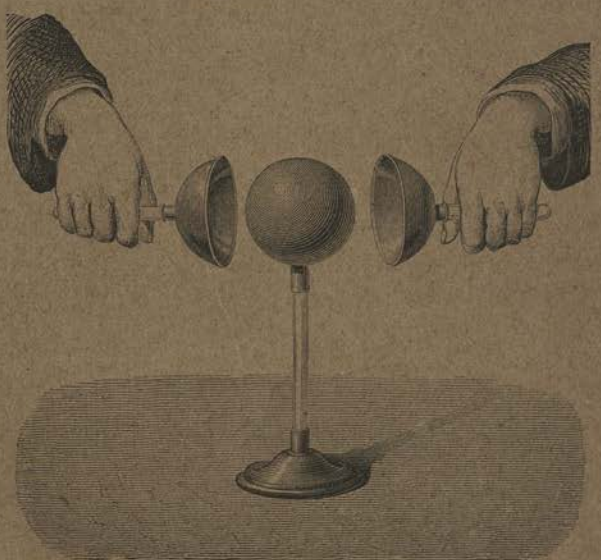

Fig. 10.—Biot's Experiment.

exceptions mentioned below) resides exclusively on the external surface of a conductor. This is perhaps implied in the experimental fact frequently observed by Coulomb, that when a solid and a hollow sphere of equal external diameter are allowed to touch each other, any charge possessed by either is divided equally between them. A

direct demonstration is afforded by the following experiment of Biot:—

We take an insulated sphere of metal, charge it with electricity, and cover it with two hemispheres furnished with insulating handles, which fit the sphere exactly (Fig. 10). If the two hemispheres be quickly removed, and presented to an electric pendulum, they will be found to be electrified, while the sphere itself will show hardly any traces of electricity. We must, however, remark that this experiment is rarely successful, and that generally the sphere remains very sensibly electrified. The reason of this is, that it is very difficult to remove the hemispheres so steadily, as not to permit their edges to touch the sphere after the first separation.

The following is a much surer form of the experiment:—

A hollow insulated sphere, with an orifice in the top, is charged with electricity (Fig. 11). A *proof-plane*, consisting of a small disc of gilt paper insulated by a thin handle of shellac, is then applied to the interior surface of the sphere, and, when tested by an electric pendulum or an electroscope, is found to exhibit no trace of electricity. But if, on the contrary, the disc be applied to the external surface of the sphere, it will be found to be electrified, and capable of attracting light bodies. Faraday varied this experiment, by substituting a cylinder of wire-gauze for the sphere. This cylinder rested on an insulated disc of metal. The disc was charged with electricity, and it was found that no trace of the electricity could be detected by applying the proof-plane to the interior surface of the cylinder.

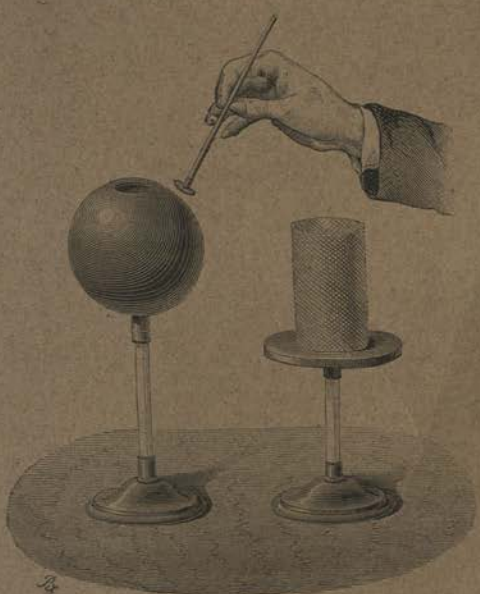

Fig. 11.—Proof-plane and Hollow Sphere.

The following experiment is also due to Faraday. A metal ring is fixed upon an insulating stand (Fig. 12). To this ring is attached a cone-shaped bag of fine linen, which is a conductor of electricity. A silk thread, attached to the apex of the cone, and extending both

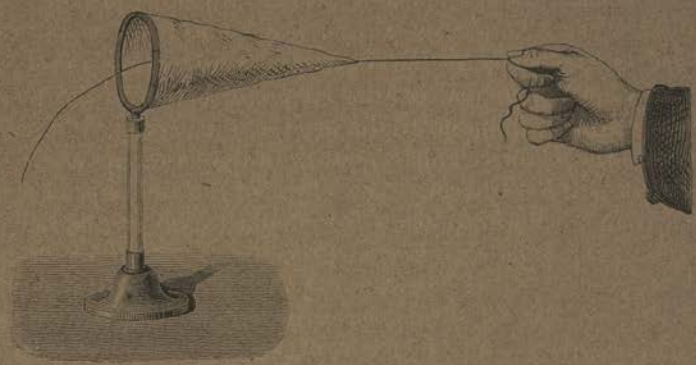

Fig. 12.—Faraday's Experiment.

ways, enables the operator to turn the bag inside out as often as required, without discharging it. When the bag is electrified, the application of the proof-plane always shows that there is electricity on the outer, but not on the inner surface. When the bag is turned inside out, the electricity therefore passes from one surface of the linen to the other.

17. **Limitations of the Rule.**—There are two exceptions to the rule that electricity is confined to the external surface of a conductor.

1. It does not hold for electric currents. We shall see hereafter in connection with galvanic electricity, that the resistance which a wire of given length opposes to the passage of electricity through it, depends not upon its circumference but upon its sectional area. A hollow wire will not conduct electricity so well as a solid wire of the same external diameter.

2. Electricity may be induced on the inner surface of a hollow conductor by the presence of an electrified body insulated from the conductor itself. If an insulated body charged with electricity be introduced into the interior of a hollow conductor, so as to be completely surrounded by it, but still insulated from it, it induces upon the inner surface a quantity equal to its own charge, but of opposite sign. If the conductor is insulated, an equal quantity, but of the same sign as the charge of the inclosed body, is repelled to the outside, and

this is true whether the conductor has an independent charge of its own or not. In this case, then, we have electricity residing on both the external and the internal surfaces of a hollow conductor, but it still resides only on the surfaces.

If a conducting body connected with the earth be introduced into the interior of a hollow charged conductor, so as to be partially surrounded by it, the body thus introduced will acquire an opposite charge by induction, and, by the reciprocal action of this charge, electricity will be induced on the inner at the expense of the outer surface of the hollow conductor, just as in the preceding case.

18. **Ice-pail Experiment.**—The effect of introducing a charged body within a hollow conductor is well illustrated by the following experiments of Faraday. Let A (Fig. 13) represent an insulated pewter ice-pail, ten and a half inches high and seven inches in diameter, connected by a wire with a delicate gold-leaf electroscope E, and let C be a round brass ball insulated by a dry thread of white silk, three or four feet in length, so as to remove the influence of the hand holding it from the ice-pail below. Let A be perfectly discharged, and let C, after being charged at a distance, be introduced into A as in the figure. If C be positive, E also will diverge positively; if C be taken away, E will collapse perfectly, the apparatus being in good order. As C enters the vessel A, the divergence of E will increase until C is about three inches below the edge of the vessel, and will remain quite steady and unchanged for any greater depression. If C be made to touch the bottom of A, all its charge is communicated to A, and C, upon being withdrawn and examined, is found perfectly discharged. Now Faraday found that at the moment of contact of C with the bottom of A, not the slightest change took place in the divergence of the gold-leaves. Hence the charge previously developed by induction on the outside of A must have been precisely equal to that acquired by the contact, that is, must have been equal to the charge of C.

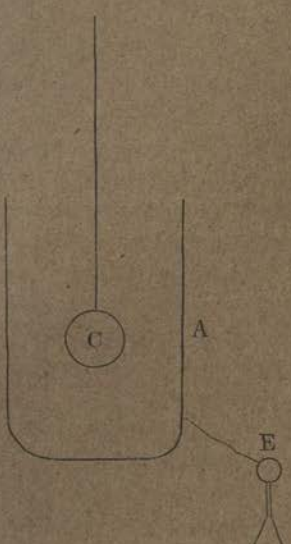

Fig. 13.—Ice-pail Experiment.

He then employed four ice-pails (Fig. 14), arranged one within the other, the smallest innermost, insulated from each other by plates of shellac at the bottom, the outermost pail being connected with the electroscope. When the charged carrier-ball C was introduced within the innermost pail, and lowered until it touched the bottom, the electrometer gave precisely the same indications as when the outermost pail was employed alone. When the innermost was lifted out by a silk thread after being touched by C, the gold-leaves collapsed perfectly. When it was introduced again, they opened out to the same extent as before. When 4 and 3 were connected by a wire let down between them by a silk thread, the leaves remained unchanged, and so they still remained when 3 and 2 were connected, and finally when all four pails were connected.

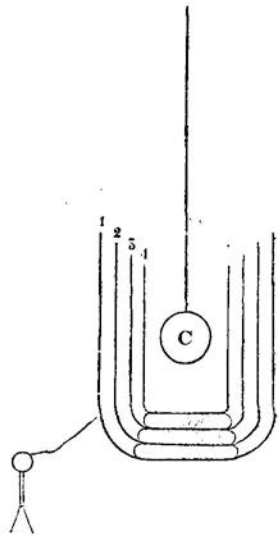

Fig. 14.—Experiment with Four Ice-pails.

19. No Force within a Conductor.—When a hollow conductor is electrified, however strongly, no effect is produced upon pith-balls, gold-leaves, or any other electroscopic apparatus in the interior, whether connected with the hollow conductor, or insulated from it, provided, in the latter case, that they have no communication with bodies external to the hollow conductor. Faraday constructed a cubical box, measuring 12 feet each way, covered externally with copper wire and tin-foil, and insulated from the earth. He charged this box very strongly by outside communication with a powerful electrical machine; but a gold-leaf electrometer within showed no effect. He says, "I went into the cube and lived in it, using lighted candles, electrometers, and all other tests of electrical states. I could not find the least influence upon them, or indication of anything particular given by them, though all the time the outside of the cube was powerfully charged, and large sparks and brushes were darting off from every part of its outer surface."

The fact that electricity resides only on the external surface of a conductor, combined with the fact that there is no electrical force in the space inclosed by this surface, affords a rigorous proof of the law

of inverse squares. For if the conductor be a sphere removed from the influence of external bodies, its charge must be distributed uniformly over its surface. Now it admits of proof, and is well known to mathematicians, that a uniform spherical shell exerts no attraction at any point of the interior space, if the law of attraction be that of inverse squares, and that the internal attraction does not vanish for any other law.

20. Electrical Density and Distribution.—When the proof-plane is applied to different parts of the surface of a conductor, the quantities of electricity which it carries off are not usually equal. But the electricity carried off by the proof-plane is simply the electricity which resided on the part of the surface covered by it, for the proof-plane during the time of its contact is virtually part of the surface of the conductor. We must therefore conclude that equal areas on different parts of the surface of a conductor have not equal amounts of electricity upon them. It is also found that if the charge of the conductor be varied, the electricity resident upon any specified portion of the surface is changed in the same ratio. The ratio of the quantities of electricity on two specified portions of the surface is in fact independent of the charge, and depends only on the form of the conductor. This is expressed by saying that *distribution* is independent of charge, and that the distribution of electricity on the surface of a conductor depends on its form.

By the *average electrical density* on the whole or any specified portion of the surface of a conductor, is meant the quantity of electricity upon it, divided by its area. By the *electrical density at a specified point* on the surface of a conductor, is meant the average electrical density on an exceedingly small area surrounding it, in other words, the *quantity of electricity per unit area* at the point. The name is appropriate, from the analogy of ordinary material density, which is mass per unit volume, and is not intended to imply any hypothesis as to the nature of electricity. The name was introduced by Coulomb, who first investigated the subject in question, and is generally employed by the best electricians in this country. The term *thickness of electrical stratum*, which was introduced by Poisson, is much used in France, but is more open to objection from the coarse assumptions which it seems to involve.

The following are some of Coulomb's results. The dotted line in each of the figures is intended to represent, by its distance from the outline of the conductor, the electric density at each point of the

latter. In all cases it is to be understood that the conductor is so far removed from external bodies as not to be influenced by them:—

1. *Sphere* (Fig. 15). The electric density is the same for all points on the surface of a spherical conductor.

2. *Ellipsoid* (Fig. 16). The density is greatest at the ends of the

Fig. 15.—Distribution on Sphere.

Fig. 16.—Distribution on Ellipsoid.

longest, and least at the ends of the shortest axis; and the densities at these points are simply proportional to the axes themselves.[1]

3. *Flat Disc* (Fig. 17). The density is almost inappreciable over the whole of both faces, except close to the edges, where it increases almost *per saltum*.

4. *Cylinder with Hemispherical Ends* (Fig. 18). The density is

Fig. 17.—Distribution on Disc.

Fig. 18.—Distribution on Cylinder with rounded ends.

a minimum, and nearly uniform, at parts remote from the ends, and attains a maximum at the ends. The ratio of the density at the ends to that at the sides increases as the radius of the cylinder diminishes, the length of the cylinder remaining the same.

5. *Spheres in Contact.*—In the case of equal spheres, the charge, which is nothing at the point of contact, and very feeble up to 30° from that point, increases very rapidly from 30° to 60°, less rapidly from 60° to 90°, and almost insensibly from 90° to 180°. When the spheres are of unequal size, the charge at any point on the smaller

[1] More generally, the density at any point on the surface of an ellipsoid is proportional to the length of a perpendicular from the centre of the ellipsoid on a tangent plane at the point.

If an ellipsoid, similar and nearly equal to the given one, be placed so that the corresponding axes of the two are coincident, we shall have a thin ellipsoidal shell, whose thickness at any point exactly represents the electric density at that point.

Such a shell, if composed of homogeneous matter attracting inversely as the square of the distance, would exercise no force at points in its interior.

or which have the sharpest convexity, and that in depressions or concavities it is small or altogether insensible. Theory shows that at a perfectly sharp edge, such, for example, as is formed by two planes meeting at any angle however obtuse, but *not rounded off*, the density must be infinite, and *a fortiori* it must be infinite at a perfectly sharp point, for example at the apex of a cone, however obtuse, *if not rounded off*. Practically, the points and edges of bodies are always rounded off; the microscope shows them merely as places of very sharp convexity (that is, of very small radius of curvature), and hence the electric density at those places is really finite; but it is exceedingly great in comparison with the density at other parts, and this is especially true of very acute points, such as the point of a fine needle. The consequence is, that if a pointed conductor is insulated and charged, the concentration of a large amount of repulsive force within an exceedingly small area produces very rapid escape of electricity at the points. Conductors intended to retain a charge of electricity must have no points or edges, and must be very smooth. If of considerable length in proportion to their breadth, they are usually made to terminate in large knobs.

24. Dissipation of Charge.—When an insulated conductor is charged and left to itself, its charge is gradually dissipated, and at length completely disappears. This loss takes place partly through the supports, and partly through the air.

As regards the supports, the loss occurs chiefly at their surface, especially if (as is usually the case) they are not perfectly dry. It is diminished by diminishing their perimeter, and by increasing their length; for example, a long fibre of glass or raw silk is an excellent insulator.

As regards the air, we must distinguish between conduction and convection. Very hot air and highly rarefied air probably act as conductors; but air in the ordinary condition acts chiefly by contact and convection. Successive layers of air become electrified by contact with the conductor, and are then repelled, carrying off the electricity which they have acquired. It is by an action of this kind that electricity escapes into the air from points, as is proved by the wind which passes off from them (§ 43). Particles of dust present in the air, in like manner, act as carriers, being attracted to the conductor, charged by contact with it, and then repelled. They also frequently adhere by one end to the conductor,

and thus constitute pointed projections through which electricity is discharged into the air.

Coulomb deduced from his observations on dissipation of charge a law precisely analogous to Newton's law of cooling, namely, that when all other circumstances remain the same, *the rate of loss is simply proportional to the charge*, so that the charges at equal intervals of time form a decreasing geometric series. Subsequent experience has confirmed this law, as approximately true for moderate charges of the same sign. Negative charges are, however, dissipated more rapidly than positive.

CHAPTER IV.

ELECTROSTATIC MACHINES

25. Electrical Machines.—The first electrical machine was invented by Otto Guericke, to whom, as we have already seen, science is indebted for the invention of the air-pump. It consisted of a ball of sulphur which was turned upon its axis by one person, while another held his hands upon the ball, thus causing the friction necessary for the production of electricity. The result was that the globe was negatively electrified, and the positive electricity escaped into the earth through the hands of the operator. This machine, however, was capable of producing only very feeble effects, and the sparks obtained from it were visible only in the dark. An English philosopher, Hawksbee, substituted a globe of glass for the globe of sulphur; the electricity thus obtained was positive, and the sparks obtained by the new machine were of considerable brightness. The machine, however, was for the time superseded by the use of glass tubes, which continued to be the favourite instruments for generating electricity until the middle of the eighteenth century, when a German philosopher, Boze, professor of physics at Wittemberg, revived and perfected Hawksbee's machine, which became universally adopted.

26. Ramsden's Machine.—The kind of friction machine most commonly employed at present is the plate-machine, invented by Ramsden about 1768, and only slightly changed and improved since.

The most usual form of this machine is shown in Fig. 19. It has a circular plate of glass, which turns on an axis supported by two wooden uprights. On each side of the plate, at the upper and lower parts of the uprights, are two cushions, which act as rubbers when the plate is turned. In front of the plate are two metallic conductors supported on glass legs, and terminating in branches

which are bent round the plate at the middle of its height, and are studded with points projecting towards it. The plate becomes charged with positive electricity by friction against the cushions, and gives off its electricity through the points to the two conductors, or, what amounts to the same thing, the conductors give off negative electricity through the points to the positively-electrified plate. In order to avoid loss of electricity from that portion of the plate which

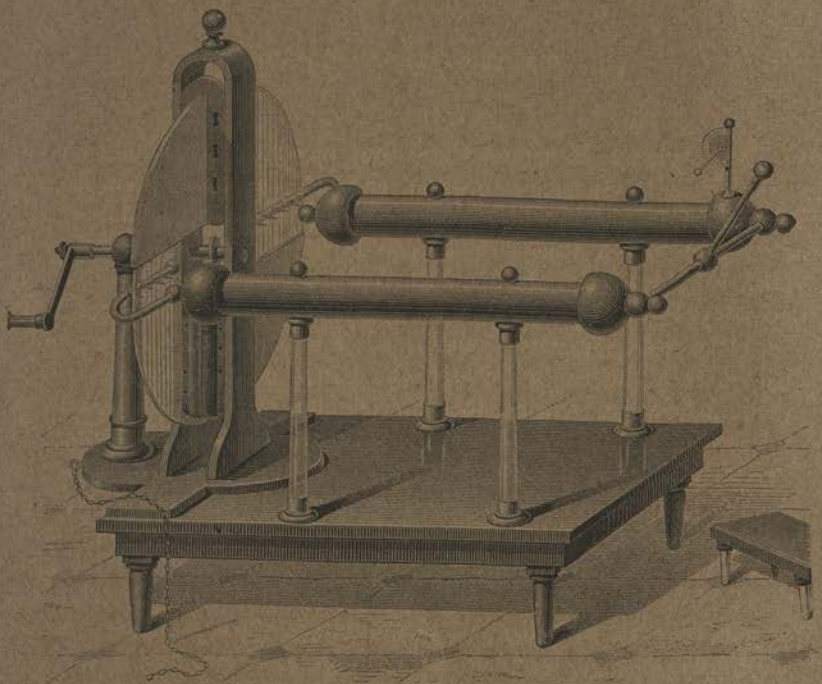

Fig. 19.—Ramsden's Electrical Machine.

is passing from the cushions to the points, sector-shaped pieces of oiled silk are placed so as to cover it on both sides. The cushions become negatively electrified by the friction; and the machine will not continue working unless this negative electricity is allowed to escape. The cushions are accordingly connected with the earth by means of metal plates let into their supports.

27. Limit of Charge.—As the conductors become more highly charged, they lose electricity to the air more rapidly, and a time soon arrives when they lose electricity as fast as they receive it from the

plate. After this, if the machine continues to be worked uniformly, their charge remains nearly constant. This limiting amount of charge depends very much upon the condition of the air; and in damp weather the machine often refuses to work unless special means are employed to keep it dry.

The rubbers are covered with a metallic preparation, of which several different kinds are employed. Sometimes it is the compound called *aurum musivum* (bisulphide of tin), but more frequently an amalgam. Kienmeier's amalgam consists of one part of zinc, one of tin, and two of mercury. The amalgam is mixed with grease to make it adhere to the leather or silk which forms the face of the cushion.

Before using the machine, the glass legs which support the conductors should be wiped with a warm dry cloth. The plate must also be cleaned from any dust or portions of amalgam which may adhere to it, and lastly, dried with a hot cloth or paper. When these precautions are taken, the machine, if standing near a fire, will always work: but the charging of Leyden jars, and especially of batteries, may be rendered impossible by bad weather.

The variations of charge are indicated by the *quadrant electroscope* (Fig. 20), which is attached to one of the conductors. It consists of an upright conducting stem, supporting a quadrant, or more commonly a semicircle of ivory, at whose centre a light needle of ivory is jointed, carrying a pith-ball at its end. When there is no charge in the conductor, this pendulum hangs vertically, and as the charge increases it is repelled further and further from the stem. In damp weather it will be observed to return to the vertical position almost immediately on ceasing to turn the machine, while in very favourable circumstances it gives a sensible indication of charge after two or three minutes.

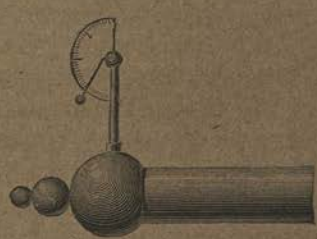

Fig. 20.—Quadrant Electroscope.

28. **Nairne's Machine.**—Ramsden's machine furnishes only positive electricity. In order to obtain negative electricity, it is necessary to insulate the cushions from the ground, and to place them in communication with an insulated conductor. An arrangement of this kind is adopted in Nairne's machine.

In this machine a large cylinder of glass revolves between two

separately insulated conductors. One of these has a row of points projecting towards the glass, and collects positive electricity. The other is connected with the rubber, and collects negative. If one

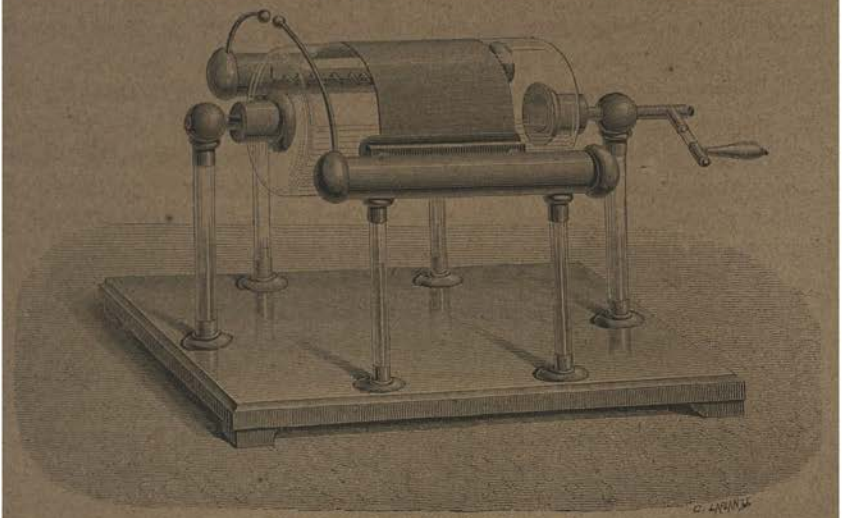

Fig. 21.—Nairne's Electrical Machine.

kind of electricity only is required, the conductor which furnishes the other must be connected with the ground.

29. Classification of Machines.—The machines above described employ friction as the means of generating electricity; and the hydro-electric machine invented in 1840 by Mr. Wm. Armstrong (now Lord Armstrong) depends on the same principle. It consists of an insulated high-pressure boiler, from which partially condensed steam rushes through a row of circuitous exits, opposite to which is an insulated metal comb, to which it gives up its electricity. A fuller description will be found in previous editions of this work.

The machines which will be described in the remainder of this chapter depend upon a very different principle.

We may remark that the name "electrical machine" is now of somewhat doubtful signification. In a general sense it must include machines which depend on magneto-electric principles and those which depend upon the rapid interruption of electric currents; but these bear so little resemblance to the friction machines which monopolized the name for a hundred years that it is not usually applied to them. It is usually confined to machines which depend

upon electrostatical principles, under which head are included both friction and electrostatic induction.

30. **Electrophorus.**—When electricity is required in comparatively small quantities, it is readily supplied by the simple apparatus called the *electrophorus*. This consists (Fig. 22) of a disc of resin, or some other material easily excited by friction, and of a polished metal disc B with an insulating handle C D. The resin disc is electrified by striking or rubbing it with catskin or flannel, and the metal plate is then laid upon it. In these circumstances the upper plate does not receive a direct charge from the lower, but, if touched with the finger (to connect it with the earth), receives an opposite charge by induction. On lifting it away by its insulating handle, it is found to be charged, and will give a spark. It may then be replaced on the lower plate (touching it at the same time with the finger), and the process repeated an indefinite number of times, without any fresh excitation, if the weather is favourable.

Fig. 22.—Electrophorus.

The resinous plate has usually a base or *sole* of metal, which is in connection with the earth while the electrophorus is being worked. When the cover receives its positive charge on being connected with the earth, the sole at the same time receives from the earth a negative charge, and as the cover is gradually lifted this negative charge gradually returns to the earth.

The most convenient form of the electrophorus is that of Professor Phillips, in which the cover, when placed upon the resinous plate, comes into metallic connection with the metal plate below. That this arrangement is allowable is evident, when we reflect that, when the upper plate is touched with the finger, it is in fact connected with the lower plate, since both are connected with the earth; and it effects a great saving of time when many sparks are required in quick succession, for the cover may be raised and lowered as fast as we please, coming alternately into contact with the resinous plate and the body which we wish to charge.

A machine which has been called a rotatory electrophorus was invented by Bertsch, and is described in previous editions of the present work.

31. **Voss' Machine.**—In Voss' machine, which is a modification of an earlier form invented by Holtz, the inducing charge may be indefinitely small at first, and is rapidly increased. It gives much more powerful effects than the friction machine, and is much easier to manage and keep in order. It is represented in Fig. 23.

There are two glass plates, a small distance asunder. The larger one is fixed, and the smaller one is made to revolve rapidly by means of a driving band passing over two grooved wheels, the larger one

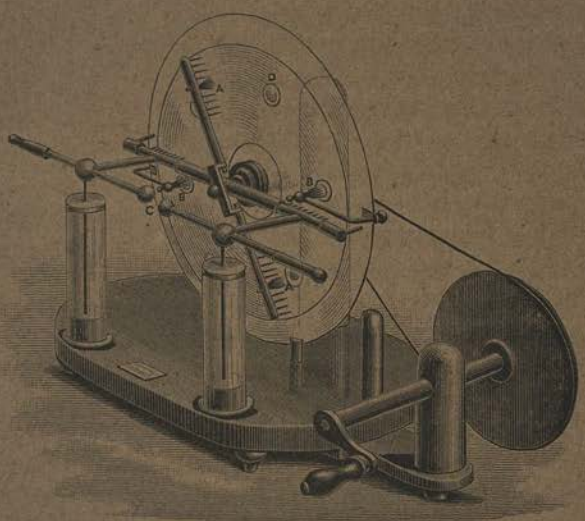

Fig. 23.—Voss' Machine.

being turned by hand. This plate has six metallic studs (like that at D) set in it at equal distances. The sloping bar which is seen in front of it is of brass, and carries two little brushes A A of thin brass wire, against which the studs rub as they pass by, and this happens at the same moment for both brushes. If we suppose that the fixed plate is charged with positive electricity in its upper part, and negative in its lower part, the upper stud will acquire a negative and the lower stud a positive charge, by induction, at the moment that the two contacts occur. When the studs have advanced about a quarter of a revolution, they come in contact with another pair of brushes B B which collect their charges.

These collecting brushes are in communication with two patches of tin-foil on the back of the fixed plate, which are not shown in

the figure. Thus the right-hand patch will be continually replenished with negative, and the left-hand patch with positive electricity. This left-hand patch extends to the top of the fixed plate, and acts as the influencing body to draw negative electricity to the upper brush and stud. The right-hand patch in like manner extends to the bottom, and attracts positive to the lower stud.

The action which we have described produces rapid increase of any slight charges that the two patches of tin-foil may possess at starting; and when the machine is dry there is generally a sufficient trace of electricity remaining in it to furnish a basis for this rapid process of multiplication. In unfavourable weather it may be necessary at the outset to employ a flat piece of vulcanite (or other suitable substance), which has been electrified by friction, and hold it at the back of the fixed plate opposite the highest or lowest brush, till the machine begins to work.

When the two patches of tin-foil have acquired their charges, a great deal more electricity is produced than is necessary for keeping them up. The surplus is collected from the revolving plate by rows of brass points, just as in the friction machine. They are ranged along the two horizontal radii of the plate, one row collecting positive and the other negative. They are in connection with the two knobs C which are seen in front of the machine, and a brilliant discharge of electricity takes place between these knobs. In the above description we have supposed the right-hand patch of tin-foil to be negative. It will accordingly attract positive electricity from the right-hand conductor to the points, through which the positive electricity will stream off on to the face of the plate, leaving the conductor with a strong negative charge. The right-hand knob will therefore be negative, and the left-hand knob positive. The knobs are at the ends of sliding rods with insulating handles, and can either be placed in contact or separated to a distance of several inches. They should be about half an inch apart at starting, and be gradually opened wider as the discharge becomes stronger.

In the original Holtz machine, in place of the brushes and studs for replenishing the charges of the *armatures* (that is, the patches of tin-foil, or paper patches answering the same purpose), these armatures are furnished with projecting points of cardboard which collect electricity from the revolving plate by discharge through the air. The plate passes these cardboard points just before it passes the brass points which supply the conductors.

32. Wimshurst's Machine.—The best of all the electrostatic machines of the present day is that of Mr. Wimshurst, Fig. 24. It has two glass plates revolving with equal speeds in opposite directions, each plate carrying on its outer face a number of equidistant

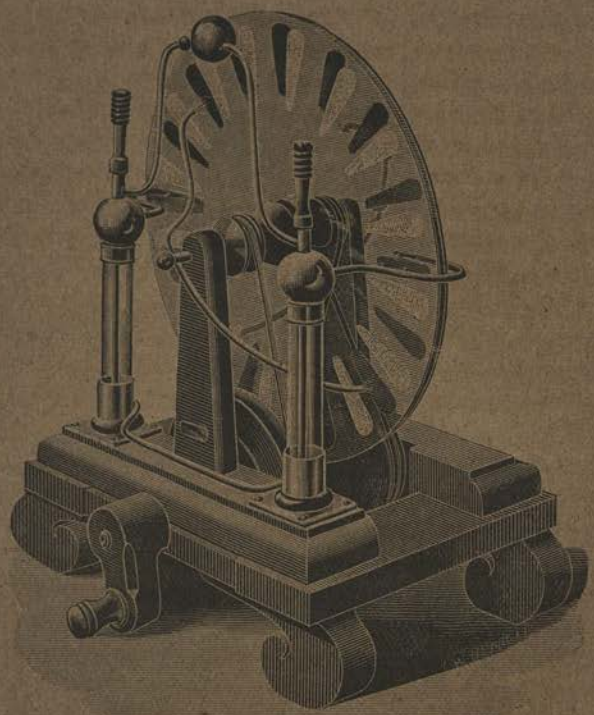

Fig. 24.—Wimshurst's Electrical Machine.

patches of tin-foil. The number should not be less than sixteen, and must be the same for both plates. Sometimes each patch is furnished with a projecting stud to make contact with the brushes, but this is of doubtful benefit. The curved bar of metal seen in front of the machine carries brass brushes at its two ends which make simultaneous contact with a pair of diametrically opposite patches. This bar is adjustable and is usually set to a slope of about 45°. At the back of the machine, just visible in the figure, is a similar bar sloping the opposite way, so that the two bars are nearly at right angles to one another.

The electricity furnished by the machine is collected by brass combs embracing the plates at the ends of a horizontal diameter and connected with the two conductors, one positive and the other negative. In the figure (which has been kindly supplied by Mr. Wimshurst expressly for this work) each of the two conductors consists of a fixed sphere of brass, in which turns, by means of an insulating handle, a curved arm of brass terminating in a ball. The two balls are of unequal sizes, and can be interchanged so as to make either of them positive at pleasure. Their distance apart is regulated by rotating the insulating handles.

As shown in the figure the conductors are connected with the inner coatings of two Leyden jars, whose outer coatings are connected with each other. To disconnect the conductors from the jars, the vertical rods seen in the centres of the jars are lifted out.

33. The working of the machine will be understood from the accompanying diagram, Fig. 25, which indicates the directions of rotation of the two plates, and the signs of the charges on their tin-foil patches when the right-hand conductor is the positive one. At the moment when the brushes at A and B make contact with a pair of tin-foil patches on the front plate, these two patches, together with the connecting rod A B, form one conductor, which is influenced by the negative electricity on the portion of the back plate opposite to A, and the positive on the portion opposite to B. Thus the patch at A acquires a positive and that at B a negative charge, and these charges are carried on till they are partly delivered up to the combs. Similar reasoning applies to the patches on the back plate. In passing C they become negatively and in passing D positively charged by the inductive action of the charges on the front plate. Thus the right-hand combs are supplied with positive electricity by both plates and the left-hand combs with negative.

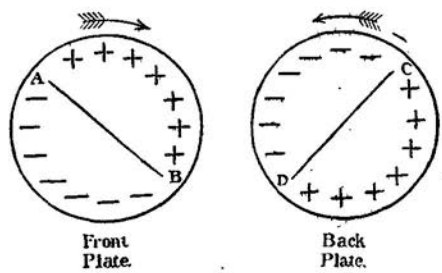

Fig. 25.

Quite recently (1892) Mr. Wimshurst has exhibited a modification of his machine, in which each of the two conductors becomes alternately positive and negative, the reversals succeeding each other with extreme rapidity.

CHAPTER V.

EXPERIMENTS ON ELECTRIC DISCHARGE.

34. Spark, Brush, and Glow.—Luminous discharges of electricity are usually classed as of three kinds: the spark, the brush, and the glow. The spark, as furnished by ordinary electrical apparatus, may be of any length, from a small fraction of an inch to several inches. When short it is usually straight, and accompanied by an instantaneous sound which may be described as a crack. The crack becomes louder in proportion as the length of the spark is increased, and still more as the capacity of the discharging conductor is increased, the discharge of a battery of Leyden jars even across a distance of an inch being of deafening loudness. When the spark is long and loud it is usually crooked, resembling on a small scale a flash of forked lightning. Such sparks are easily obtained from an induction machine when its Leyden jars are in connection with the terminals. When the jars are out of connection there is no loud crack, but the discharges succeed each other so rapidly as to produce a sizzling sound, and a continuous stream of fire (often consisting of several nearly parallel threads) plays across the interval. As the knobs are drawn further apart, a dark interval is left near the negative knob. Lines of fire extend from the positive knob to a certain distance, where they terminate in mid air; the negative knob exhibits a luminous glow, and the intervening region is dark. When the room is darkened, the direct discharge is seen to be surrounded by a nebulous luminosity of purple tint, shaped like an egg, with its ends at the two knobs, and an interruption or defect of brightness at the dark space above-mentioned.

35. When a conductor is giving off electricity from points or sharp edges into the air, it exhibits in the dark an appearance of

lines of fire issuing from the points and spreading out into broader streams of fainter luminosity. This is called the *brush* discharge. It is best seen when the electricity which is escaping from the conductor is positive.

The glow discharge as seen in the dark renders the surface of the conductor luminous, but the luminosity does not extend into the surrounding air.

When an induction machine is working in the dark, the discharges which take place between the combs and the revolving glass-plates appear very different at the two combs. The positive conductor is receiving positive or giving off negative electricity through its comb. The points of its comb are accordingly covered with a glow, and some luminosity is seen on the surface of the plate. The negative conductor is receiving negative or giving off positive electricity through its comb. Lines of fire proceed from its points and spread out into a large sheet of light on the surface of the plate.

Very interesting variations in the appearance of the discharge between the terminal knobs can be obtained by making the knobs very unequal in size and interchangeable, as in the pattern of Wimshurst machine, shown in Fig. 24. The description given in § 34 ceases to apply when the small knob is negative.

It is probable that the passage of a spark is always preceded by a state of stress in the intervening air, the stress being most intense along the path of least resistance, which at last gives way, the spark passing along the line of rupture.

36. Duration of the Spark.—We can form no judgment of the duration of the electric spark from what we see with the unaided eye; for impressions made upon the retina remain uneffaced for something like $\frac{1}{10}$ of a second, and the duration of the spark is incomparably less than this. The discharges of a Winshurst or Voss machine with Leyden jars cause the revolving plates with their tin-foil sectors to be seen as if stationary. As this appearance can be seen even in full daylight, it shows that the spark, though very brief, is an extremely powerful illuminant while it lasts.

Wheatstone, in a classical experiment, measured the duration of the spark of a Leyden jar by means of a revolving mirror; an expedient which has since been employed with great advantage in many other researches, especially in determining the velocity of light.

Let $m n$ (Fig. 26) be a mirror revolving with great velocity about an axis passing through c, and suppose that, during the rotation, an electric spark is produced at a. An eye stationed at o will see an image in the symmetrical position a'. If the spark is strictly instantaneous, its image will be seen as a luminous point at a', notwithstanding the rotation of the mirror; but if it has a finite duration, the image will move from a' to a'', while the mirror moves from $e e'$ to $t t'$, the latter being its position when the spark ceases. What is actually seen in the mirror will therefore not be a point, but a luminous track $a' a''$.

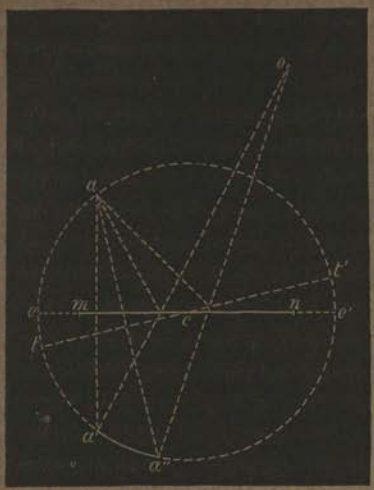

Fig. 26.—Duration of Spark.

The length of this image will be double of the arc $e t$; for the angle $e c t$ at the centre is equal to the angle $a' a a''$ at the circumference, the sides of the one being perpendicular to those of the other. In Wheatstone's experiment, the mirror made 800 turns in a second, and the image $a' a''$ was an arc of 24°; the mirror therefore turned through 12°, or $\frac{1}{30}$ of a revolution, while the spark lasted. The duration of the spark was therefore $\frac{1}{30}$ of $\frac{1}{800}$, that is, $\frac{1}{24000}$ of a second.

By examining the brush in the same way, Wheatstone found it to consist of a succession of sparks.

37. Oscillatory Discharge.—Feddersen, by taking instantaneous photographs of the spark of a Leyden jar as seen in a revolving mirror, has shown that it consists not of one discharge only, but of a number of successive discharges, a result which was predicted by Sir Wm. Thomson from theory many years before. The successive discharges are in opposite directions, and are analogous to the successive swings of a pendulum, first to one side and then to the other, in a medium whose resistance produces a rapid decrease of amplitude.

The appearance of the spark is greatly modified by rarefying the air in which it is taken. We shall return to this subject in a later chapter.

38. Velocity of Electricity.—Soon after the invention of the Leyden jar, various attempts were made to determine the velocity with which the discharge travels through a conductor connecting the two coatings. Watson, about 1748, took two iron wires, each more than a mile long, which he arranged on insulating supports in such a way that all four ends were near together. He held one end of each wire in his hands, while the other ends were connected with the two coatings of a charged jar. Although the electricity had more than a mile to travel along each wire before it could reach his hands, he could never detect any interval of time between the passage of the spark from the knob of the jar and the shock which he felt. The velocity was in fact far too great to be thus measured.

Wheatstone, about 1836, investigated the subject with the aid of the revolving mirror of which we have spoken above (§ 36). He connected the two coatings of a Leyden jar by means of a conductor which had breaks in three places, thus giving rise to three sparks. When the sparks were taken in front of the revolving mirror, the positions of the images indicated a retardation of the middle spark, as compared with the other two, which were taken near the two coatings of the jar, and were strictly simultaneous. The middle break was separated from each of the other two by a quarter of a mile of copper wire. He calculated that the retardation of the middle spark was $\frac{1}{1,152,000}$ of a second, which was therefore the time occupied in travelling through a quarter of mile of copper wire. This is at the rate of 288,000 miles per second, a greater velocity than that of light, which is only about 186,000 miles per second.

Since the introduction of electric telegraphs, several observations have been taken on the time required for the transmission of a signal. For instance, trials in Queenstown harbour, in July, 1856, when the two portions of the first Atlantic cable, on board the *Agamemnon* and *Niagara*, were for the first time joined into one conductor, 2500 miles long, gave about $1\frac{3}{4}$ seconds as the time of transmission of a signal from induction coils, corresponding to a velocity of only 1400 miles per second. In 1858, before again pro-

ceeding to sea, a quicker and more sensitive receiving instrument—Thomson's mirror galvanometer—gave a sensible indication of rising current at one end of 3000 miles of cable about a second after the application of a Daniell's battery at the other.

It seems to be fully established by experiment that electricity has no definite velocity, and that its apparent velocity depends upon various circumstances, being greater through a short than through a long line, greater (in a long line) with the greater intensity and suddenness of the source, greater with a copper than with an iron wire, and much greater in a wire suspended in air on poles than in one surrounded by gutta-percha and iron sheathing, and buried under ground or under water. In a long submarine line, a short sharp signal sent in at one end, comes out at the other as a signal gradually increasing from nothing to a maximum, and then gradually dying away.

Fig. 27.
Spangled Tube.

39. Multiplication of the Electric Spark.—The old electricians contrived several pieces of apparatus for multiplying the electric spark. The principle of all is the same. Small squares of tin-foil are arranged in series at a small distance from each other on an insulating surface. The first of the series is connected with a metallic knob which can be brought near the electrical machine; and the last of them is connected with another knob which is in communication with the earth. By allowing a discharge to pass through the series, sparks can be simultaneously obtained at all the intervals between the successive squares.

In the *spangled tube* (Fig. 27) the squares of tin-foil are arranged spirally along a cylindrical glass tube which has a brass cap at each end. One cap is put in communication with the machine, and the other with the earth.

Sometimes a glass globe is substituted for the cylinder. We have thus the *spangled globe* (Fig. 28).

In the *sparkling pane* a long strip of tin-foil is disposed in one continuous crooked line (consisting of parallel strips connected at alternate ends) from a knob at the top to another knob at the bottom of the pane. A pattern is then traced by scratching away the tin-foil in numerous places with a point, and when the spark

passes, it is seen at all these places, so as to render the pattern luminous (Fig. 29).

40. Physiological Effects of the Spark: Electric Shock.—When the finger is held between the knobs of an induction machine, with the jars disconnected, the sparks produce a sharp pricking sensation.

Fig. 28.—Spangled Globe.

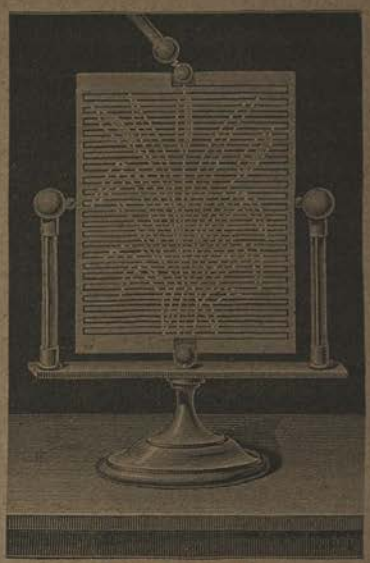

Fig. 29.—Sparkling Pane.

In general a pricking sensation more or less strong is felt at the place where a spark enters or leaves the body.

When the spark is taken from a Leyden jar, a shock is felt; especially if one hand is kept in contact with the outer coating of the jar while the other hand is brought to the knob. The sensation experienced is altogether unique, and is accompanied by spasmodic contraction of the muscles.

At the distance of a few feet from a machine in powerful action, a tickling sensation is felt on the exposed parts of the body, due to the movement of the hairs in obedience to electrical force. These phenomena are exhibited in a still more marked manner when a person stands on a stool with glass legs, and keeps his hand upon the conductor. He thus becomes highly charged with electricity. His hair stands on end, and is luminous if seen in the dark. If a

conductor connected with the earth is presented to him, a spark passes, and his hair falls again.

Electricity has frequently been resorted to for medical purposes. The electrical machine was first employed, and afterwards the Leyden jar, but both have now been abandoned in favour of magneto-electric machines and other apparatus for obtaining induced currents, which we shall describe in a later chapter.

41. Heating and Mechanical Effects.—The heating power of the electric spark may be conveniently shown by taking the discharge of an induction machine (with or without jars attached) across gas escaping from a burner. The gas is instantly ignited.

If a person standing on a stool with glass legs keeps one hand in permanent contact with one conductor of the machine, while the other conductor is connected to the earth, he can light gas by giving a spark from his other hand to the burner. The luminosity of the spark is doubtless due, either wholly or in part, to the very high temperature which is produced in the particles of air traversed by the discharge.

The commotion produced in the air by the spark can be exhibited

Fig. 30.—Kinnersley's Thermometer.

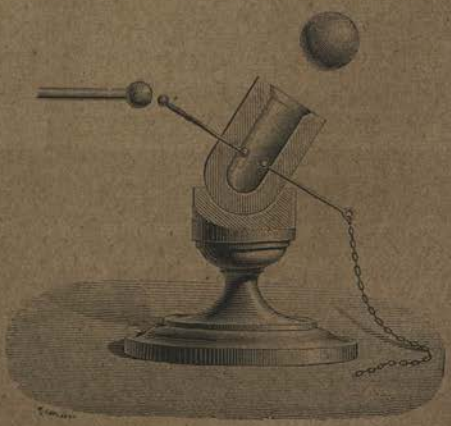

Fig. 31.—Electric Mortar.

by means of Kinnersley's (so called) thermometer (Fig. 30), which consists of two glass tubes of unequal diameters, the smaller being open at the top, while the larger is completely closed, with the exception of a side passage, by which it communicates with the smaller. The caps which close the ends of the large tube are traversed by rods terminating in knobs, and the upper one can be

raised and lowered to vary the distance between the knobs. Both tubes are filled, to a height a little below the lower knob, with a very mobile liquid such as alcohol. When the spark passes between the knobs, the liquid is projected with great violence, and may rise to a height of several yards if the spark is very strong. The same property of the spark is exhibited in the experiment of the electric mortar, which is sufficiently explained by the figure (Fig. 31).

The spark may be obtained in the interior of a non-conducting liquid, which it agitates in a similar manner. If the liquid is contained in a closed vessel, this is often broken.

The spark of an induction machine with jars attached will puncture card, the hole being larger or smaller according to the strength of the spark. Thin glass can be punctured by the discharge of a battery of jars.

42. Chemical Properties of the Spark.—Discharges of electricity, whether in the form of a single strong spark, or a succession of sparks, or a continuous discharge like that obtained between the conductors of an induction machine without the jars, are capable of producing on a very small scale the effects of a voltaic battery. Electrolytes can be very slowly decomposed and metallic thread can be heated. The former result can be easily obtained by moistening a piece of blotting-paper with solution of iodide of potassium and placing it close to the two knobs of the machine. Iodine is set free at the positive knob and produces discoloration of the paper.

43. Wind from Points.—If a metallic rod terminating in a point be attached to the conductor of the electrical machine, electricity escapes in large quantity from the point, which, accordingly, when viewed in the dark, is seen to be crowned with a tuft of light. A layer of air in front of the point is electrified by contact, and then repelled, to make way for other portions of air, which are in their turn repelled. A continuous current of air is thus kept up, which is quite perceptible to the hand, and produces a very visible effect on the flame of a taper.

Fig. 32.—Electric Whirl.

The *electric whirl* (Fig. 32) consists of a set of metallic arms, radiating horizontally from a common centre about which they can turn freely, and bent, all in the same direction,

at the ends, which are pointed. When the central support is mounted on the conductor of the machine, the arms revolve in a direction opposite to that in which their ends point. This effect is due to the mutual repulsion between the pointed ends and the electrified air which flows off from them.

It is instructive to remark that if, by a special arrangement, the rotating part be inclosed in a well-insulating glass case, the rotation soon ceases, because, in these circumstances, the inclosed air quickly attains a state of permanent electrification.

44. Electric Watering-pot.—Let a vessel containing a liquid, and furnished with very fine discharge tubes, be suspended from the conductor of the machine. When the vessel is not electrified, the liquid comes out drop by drop; but when the machine is turned, it issues in continuous fine streams. It has, however, been observed that the quantity discharged in a given time is sensibly the same in both cases. This must be owing to the equality of action and reaction between different parts of the issuing stream.

It is found that the drops of a small fountain exhibit a marked tendency to coalesce when the jet from which they issue is electrified even to a very feeble degree, or when an electrified body is held near it so as to give an induced charge to the issuing water.

CHAPTER VI.

ELECTRICAL POTENTIAL, AND LINES OF ELECTRICAL FORCE.

45. The object of the present chapter is to give a brief outline of the methods by which mathematicians have succeeded in bringing numerous electrical problems within the range of accurate reasoning.

The fundamental conception in the mathematical theory of electricity is that of attraction and repulsion, acting according to the law of inverse squares; and the unit quantity of electricity is defined to be that quantity which would attract or repel an equal quantity at unit distance with unit force.

The influence which an electrified body exercises in the region around it can be specified by stating the force of attraction or repulsion which the body would exert upon a small charged body placed in various parts of this region or field (which is accordingly called a field of electrical force), the force being stated not only in magnitude but also in direction. In this sense we speak of *the electrical force at a point*, meaning the force which would be exerted upon a unit of electricity placed at the point; and in any such specification the unit of electricity is supposed not to disturb by its presence the previously existing distribution of electricity.

There can be electrical force at points in the air, or in the substance of any non-conductor, without disturbance of equilibrium; but electrical force in a conductor instantly produces a current of electricity in the direction of the force. At the *surface* of a conductor electrical force can exist, but it must always be normal to the surface; for if there were any tangential component, a current would be produced along the surface.

46. **Definition of Difference of Potential.**—We know, by the principle of the conservation of energy, that the work done upon a unit

of electricity in its passage from one point to another, must be independent of the path pursued; and we agree to call this work the difference of potential of the two points.

47. **Relation between Potential and Force.**—If V denote the potential at a point, and $V - \delta V$ the potential at a neighbouring point. δV is the work which electrical attractions and repulsions do upon a unit of positive electricity in its passage from the first point to the second; and since work is equal to force multiplied by distance, the average force along the joining line can be computed by dividing δV by the distance, which we will call δs. Hence the limiting value of $\frac{\delta V}{\delta s}$ as the two points are taken nearer together, is the component force in the direction δs; that is, *the rate of variation of potential in any direction is equal to the component force in that direction.*

The direction in which the variation is most rapid will be the direction of the resultant force; and when δs is measured in this direction $\frac{\delta V}{\delta s}$ will be equal to the resultant force.

48. **Lines of Force.**—If a line be traced such that every small portion of it (small enough to be regarded as straight) is the direction of resultant force at the points which lie upon it, it is called a line of force; in other words, a line of force is a line whose tangent at any point is the direction of the force at that point. We may express this briefly by saying that lines of force are the lines along which resultant force acts.

49. **Equipotential Surfaces.**—An equipotential surface is a surface over the whole of which there is the same value of potential. When δs lies in such a surface, the value of $\frac{\delta V}{\delta s}$ is zero; and therefore there is no component force along any line lying in the surface. The resultant force must therefore be normal; that is, *lines of force cut equipotential surfaces* at right angles.

When we are dealing with gravitational forces instead of with electrical attractions and repulsions, equipotential surfaces are called *level* surfaces, and lines of force are called *verticals*.

If two equipotential surfaces are given, their potentials being V_1 and V_2, the work done in carrying a unit of electricity from any point of the one to any point of the other, is constant, and equal to the difference of V_1 and V_2.

If we consider two equipotential surfaces very near one another, so that the portions which they intercept on the lines of force may

be regarded as straight, the intensity of force at different points of the intermediate space will vary inversely as the distance between the two equipotential surfaces; for, when equal amounts of work are done in travelling unequal distances, the forces must be inversely as the distances.

50. Potential of a Conductor.—When electrical potential is constant throughout a given space, there is no electrical force in that space; and conversely, if there be an absence of electrical force in a given space, the potential throughout that space must be uniform. These propositions apply to the space within a hollow conductor. They also apply to the whole substance of a solid conductor, and to the whole space inclosed within the outer surface of a hollow conductor. Whenever a conductor is in electrical equilibrium, it has the same potential throughout the whole of its substance, and also through any completely inclosed hollows which it may contain.

When a conductor is not in electrical equilibrium, currents set in, tending to restore equilibrium; and the direction of such currents is always from places of higher to places of lower potential.

It is usual to assume as the zero of potential the potential of the earth; but this assumption is not consistent with itself, since the existence of earth currents proves that there are differences of potential between different parts of the earth. The absolute zero of potential is the potential of places infinitely distant from all electricity.

51. Energy of a Charged Conductor.—When positive electricity is allowed to run down from a conductor of higher to one of lower potential, there is a loss of potential energy, just as there is a loss of potential energy in the running down of a heavy body from a higher to a lower level; and on the other hand, to make positive electricity pass from a conductor of lower to one of higher potential, work must be expended from some external source, just as work must be expended to raise a heavy body. In the case of the heavy body, the work expended in the latter case, or the potential energy which runs down in the former, is equal to its weight multiplied by the difference of levels; and in the analogous case of the electrical operation, the work or the energy is the product of the quantity of electricity which passes from one conductor to the other by the difference of potentials of the two conductors,[1] provided that these potentials remain sensibly constant during the operation.

[1] The closeness of the analogy will be better understood when it is remembered that if

When a conductor is charged in the ordinary way, its charge is drawn from the earth, the potential of which is unaffected. If we suppose the charge to be communicated in a numerous succession of small equal parts, the potential of the conductor, which is originally zero, is increased by a succession of equal steps, till it attains its final value. Hence it is only the last part that is raised through the full difference of potential, and the mean value of the difference of potential through which the successive parts are raised is the half of this. Hence the work done in charging a conductor, or the energy which runs down in discharging it into the earth, is half the product of its potential and its charge.

52. **Tubes of Force.**—If we conceive a narrow tube bounded on all sides by lines of force, and call it a *tube of force*, we can lay down the following remarkable rules[1] for the comparison of the forces which exist at different points in its length. (1) *In any portion of a tube of force not containing electricity, the intensity of force varies inversely as the cross-section of the tube,* or the product of *intensity of force by section of tube* is constant.[2] (2) *When a tube of force cuts through electricity, this product changes, from one side of this electricity to the other, by the amount* $4\pi q$, where q denotes the quantity of the electricity inclosed by the tube.

The following are particular cases of (1):—

When the electricity to which the force is due is collected in a point, the lines of force are straight, the tubes of force are cones (in the most general sense), and the law of force becomes the law of inverse squares, since the section of a cone varies as the square of the

a series of level surfaces are described completely surrounding the earth, and one foot apart at the equator, they will be less than a foot apart at the poles, for the distance between them will, by the reasoning in the text, be inversely as the intensity of gravity. The work done in lifting a body from any one of these surfaces to any other, will be proportional to the product of its mass (not its weight) by the number of intervals crossed.

If one of the surfaces passes through the top of Mount Everest, and another through a point on the Indian coast, the distance between them will be greater at the coast than at the mountain. Hence the height of the mountain above the coast is an ambiguous quantity.

[1] For the proof of these rules, as mathematical deductions from the law of inverse squares, the student may refer to Everett's edition of Todhunter's *Analytical Statics*, articles 228, 235.

[2] This is obviously analogous to the law which applies to the comparison of the velocities of a liquid in different parts of a tube through which it flows, since the product of area by velocity is the volume of liquid which flows past any section in unit time. The tube may be an imaginary one, bounded by lines of flow in a large body of liquid flowing steadily. Lines of flow are thus the analogues of lines of force.

distance from the vertex. These results also apply to the case of electricity uniformly distributed over the surface of a sphere, the common vertex in which the cones would meet if produced being now at the centre of the sphere.

When the electricity consists of the charges of two oppositely electrified parallel plates, whose length and breadth exceed the distance between them (the plates being conductors, and placed opposite to each other), the lines of force between their central portions are sensibly straight and parallel, the tubes of force are therefore cylinders (in the most general sense), and the force is constant, being equal to the difference of the potentials of the plates divided by the distance between them. The same thing holds if, instead of being oppositely electrified, the plates are similarly electrified, but not to the same potential.

53. Force Proportional to Number of Tubes which cut Unit Area.—The cross-sections of tubes of force are portions of equipotential surfaces. If one equipotential surface be divided into portions, such that the product of *area by force-intensity* shall be the same for all, then, if all neighbouring space not containing electricity be cut up into tubes, springing from these portions as their respective bases, the product of any cross-section of any one of these tubes by the force-intensity over it will be constant. The force-intensities at any points in this space are therefore inversely as the cross-sections of the tubes at these points, or are directly as the number of tubes per unit area of equipotential surfaces at the points.

54. Force just Outside an Electrified Conductor.—Since there is no force in the interior of a conductor, the lines and tubes of force become indeterminate; but proposition (2) of § 52 can be shown to hold when we give them any shape not discontinuous. Let ρ denote the electric density at a point on the surface, and a a small area around this point, which area we shall regard as a section of a tube of force cutting through the surface. Let F denote the intensity of force just outside the surface opposite this point, then, since the intensity inside is zero, we have

$$\mathrm{F}a = 4\pi q = 4\pi\rho a \quad , \quad \mathrm{F} = 4\pi\rho;$$

that is, *the intensity of force just outside any part of the surface of a charged conductor, is equal to the product of* 4π *into the density at the nearest part of the surface.*

55. Relation of Induction to Lines and Tubes of Force.—Lines of

force are also the lines along which induction takes place. On Faraday's theory of induction by contiguous particles, the line of poles, for any particle, is coincident with the line of force which passes through the particle. Apart from all theory, it is matter of fact that *a tube of force extending from an influencing to an influenced conductor, and not containing any electricity in the interval between, has equal quantities of electricity on its two ends, these quantities being of opposite sign.* This equality follows at once from § 52 (2), if we consider the tube as penetrating the two conductors; for the product of force by section, which is constant for the portion of the tube in air, is zero in both conductors; and the quantity of electricity on *either* end of the tube must be the quotient of this constant product by 4π. In connection with this reasoning, it is to be remarked that the surface of a conductor is an equipotential surface, and is cut at right angles by lines of force.

In Faraday's ice-pail experiment, a tube of force springing from the upper side of the charged ball, and of such small section at its origin as to inclose only an insensible fraction of the charge of the ball, opens out so fast, as it advances, that it fills the whole opening at the top of the pail.

In every case of induction, therefore, *the total quantities of inducing and induced electricity are equal, and of opposite sign.*

When the inducing electricity resides in or upon a non-conductor, for example on the surface of a glass rod, or in the substance of a mass of air, the quantity of electricity induced on the base of a tube of force is equal and opposite to the quantity contained within the tube. In the simplest case, all the tubes will have a common apex, which will be a point of maximum or minimum potential.

56. **Potential defined as $\Sigma \frac{q}{r}$.**

Let a quantity q of electricity be collected at a point O, and let A, B be any two points very near together. The forces at A and B due to q will be $\frac{q}{OA^2}$ and $\frac{q}{OB^2}$, and these will be nearly equal to each other or to $\frac{q}{OA \cdot OB}$. When a unit of electricity is carried from A to B its motion can be resolved into two components, one of them in the direction of the force and equal to $OB - OA$, and the other perpendicular to the direction of the force. Hence the work done will be $q\frac{(OB-OA)}{OA \cdot OB}$, that is, $\frac{q}{OA} - \frac{q}{OB}$; or if r denote the distance of any point from O, the *work done in a* small *movement is equal to*

the change in the value of $\frac{q}{r}$. Since any movement can be resolved into a succession of small movements, we may omit the word *small*, and the proposition will still be true. As r increases to infinity, $\frac{q}{r}$ will diminish to zero. Hence, $\frac{q}{r}$ denotes the work from distance r to infinite distance.

As regards sign, $\frac{q}{r}$ is the work done by electrical force when a unit of positive electricity is carried from distance r to infinite distance.

Next suppose several quantities, q_1, q_2, &c., to be collected at different points, O_1, O_2, &c. Let P be any other point, and let $O_1P = r_1$, $O_2P = r_2$, &c. Then in the passage of a unit of electricity from P to infinite distance, the electrical work is, by the preceding section,

$$\frac{q_1}{r_1} + \frac{q_2}{r_2} + \quad \&c.,$$

which we will denote by $\Sigma\frac{q}{r}$, the symbol Σ being read "the sum of such terms as."

$\Sigma\frac{q}{r}$ is therefore the general expression for the potential at a point due to any quantity of electricity distributed in any manner; in other words, the potential is equal to the sum of the quotients obtained by dividing each element of electricity by its distance from the point. The distances are essentially positive. If the electricity is not all of one sign, some of the quotients, $\frac{q}{r}$, will be positive and others negative, and their algebraical sum is to be taken.

57. Application to Sphere.—In the case of a charged conducting sphere, all the elements q are equally distant from the centre of the sphere, and the sum of the quotients $\frac{q}{r}$, when we are computing the potential at the centre, will be $\frac{Q}{R}$, Q denoting the charge, and R the radius of the sphere. But the potential is the same at all points in a conductor. $\frac{Q}{R}$ is therefore the potential of a sphere of radius R, with charge Q, when uninfluenced by any other electricity than its own.

58. Capacity.—The electrical capacity of a conductor is *the quantity of electricity required to charge it to unit potential*, when it is not influenced by any other electricity besides its own charge and the electricity which this induces in neighbouring conductors. Or,

since, in these circumstances, potential varies directly as charge, capacity may be defined as the *quotient of charge by potential.* Let C denote capacity, V potential, and Q charge, then we have

$$C = \frac{Q}{V} \quad ; \quad V = \frac{Q}{C} \quad ; \quad Q = VC.$$

But we have seen that, for a sphere of radius R, at a distance from other conductors or charged bodies, $V = \frac{Q}{R}$. Hence $C = R$; that is, the *capacity of a sphere is numerically equal to its radius.*

This is a particular instance of the general proposition that the capacities of similar conductors are as their linear dimensions; which may be proved as follows:—

Let the linear dimensions of two similar conductors be as $1 : n$. Divide their surfaces *similarly* into very small elements, which will of course be equal in number. Then the areas of corresponding elements will be as $1 : n^2$, and, if the electrical densities at corresponding points be as $1 : x$, the charges on corresponding elements are as $1 : n^2x$. The potential at any selected point of either conductor is the sum of such terms as $\frac{q}{r}$ (§ 56). Selecting the corresponding point in the other conductor, and comparing potentials, the values of q are as $1 : n^2x$, and the values of r are as $1 : n$; therefore the values of $\frac{q}{r}$ are as $1 : nx$. Hence the potentials of the two conductors are as $1 : nx$. If they are equal, we have $nx = 1$, and therefore $n^2x = n$; that is, the charges on corresponding elements, and therefore also on the whole surfaces, are as $1 : n$.

We shall see, in the next chapter, that the capacity of a conductor may be greatly increased by bringing it near to another conductor connected with the earth.

59. **Connection between Potential and Induced Distribution.**—In the circumstances represented in Fig. 4 (§ 8), if we suppose the influencing body C to be positively charged, the potential due to this charge will be algebraically greater at the near end A of the influence conductor than at the remote end B. The induced electricity on A B must be so distributed as to balance this difference, in fact the potential due to this induced electricity is negative at A and positive at B. All cases of induced electricity upon conductors fall under the rule that *the potential at all parts of a conductor must be the same, and hence, wherever the potential due to the influencing*

electricity is algebraically greatest, the potential due to the electricity on the influenced conductor must be algebraically least.

As there can be no force in the interior of a conductor, the force at any point in the interior, due to the influencing electricity, must be equal and opposite to the force due to the electricity on the surface of the conductor. This holds, whether the conductor be solid or hollow. A hollow conductor thus completely screens from external electrical forces all bodies placed in its interior.

60. Electrical Images.—If a very large plane sheet of conducting material be connected with the earth, and an electrified body be placed in front of it near its middle, the plate will completely screen all bodies behind it from the force due to the electrified body. The induced electricity on the plate therefore exerts, at all points behind the plate, a force equal and opposite to that of the electrified body, or, what is the same thing, a force identical with that which the electrified body would exert if its electricity were reversed in sign. But electricity distributed over a plane surface must act symmetrically towards both sides. Hence the force which the induced electricity exerts in front, is identical with that which would be exerted by a body precisely similar to the given electrical body, symmetrically placed behind the plane, and charged with the opposite electricity. The total force at any point in front of the plane is the resultant of the force due to the given electrified body, and the force due to this imaginary image. The name and the idea of *electrical images*, of which this is one of the simplest examples, are due to Sir W. Thomson.

The mathematical student will have no difficulty in proving that the joint effect of a small electrified body in front of the plane and its image behind the plane is to produce at any point close in front of the plane a force which varies inversely as the cube of the distance of this point from the electrified body; whence, by § 52, the distribution of induced electricity on the plane is inversely as the cube of the distance.

CHAPTER VII.

ELECTRICAL CONDENSERS.

61. Condensers.—The process called *condensation of electricity* consists in increasing the capacity of a conductor by bringing near it another conductor connected with the earth, or with some other source from which electricity can be supplied. The two conductors are usually thin plates or sheets of metal, placed parallel to one another, with a larger plate of non-conducting material between them.

Let A and B (Fig. 33) be the two conducting plates, of which A, called the *collecting plate*, is connected with the conductor of the machine, and B, called the *condensing plate*, with the earth; and let C be the non-conducting plate (or dielectric) which separates them. Then, if the machine has been turned until the limit of charge is attained, the surface of B which faces towards A is covered with negative electricity, drawn from the earth, and held by the attraction of the positive electricity of A; and, conversely, the surface of A which faces towards B, is covered with positive electricity, held there by the attraction of the negative of B, in addition to the charge which would reside upon it if the conductor were at the existing potential, and B and C were absent. In fact, the electrical density on the face of A, as well as the whole charge of A, would, in this latter case, be almost inappreciable, in comparison with those which exist in the actual circumstances. By condensation of electricity, then,

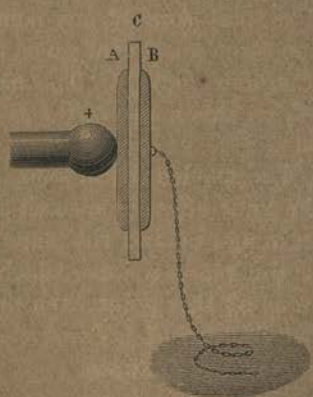

Fig. 33.—Theoretical Condenser.

we are to understand *increase*—usually enormous increase—*of electrical density on a given surface, attained without increase of potential.* If two conducting plates, in other respects alike, but one with, and the other without a condensing plate, be connected by a wire, and the whole system be electrified, the two plates will have the same potential, but nearly the whole of the charge will reside upon the face of that which is accompanied by a condensing plate.

62. Calculation of Capacity of Condenser.—The lines of force between the two plates A and B are everywhere sensibly straight and perpendicular to the plates, with the exception of a very small space round the edge, which may be neglected. The tubes of force (§ 52) are therefore cylinders, and the intensity of force is constant at all parts of their length. Also, since the potential of the plate B is zero, if we take V to denote the potential of the plate A, which is the same as the potential of the conductor, and t to denote the thickness of the intervening plate C, the rate at which potential varies along a line of force is $\frac{V}{t}$, which is therefore (§ 47) the expression for the force at any point between the plates A, B. The whole space between the plates may be regarded as one cylindrical tube of force of cross-section S equal to the area of either plate, the two ends of the tube being the inner faces of the plates. The quantities of electricity $\pm Q$ residing on these faces are therefore equal, but of opposite sign (§ 55); and as the force changes from nothing to $\frac{V}{t}$ in passing from one side to the other of the electricity which resides on either of these surfaces, we have (§ 52)

$$\frac{V}{t} \cdot S = 4\pi Q.$$

Hence the capacity of the plate A, being, by definition, equal to $\frac{Q}{V}$, is equal to

$$\frac{S}{4\pi t}$$

We should, however, explain that, if the intervening plate C is a solid or liquid, we are to understand by t not the simple thickness, but the thickness reduced to an equivalent of air, in a sense which will be explained further on (§ 69). This reduced thickness is, in the case of glass, about half the actual thickness.

If s denote an element of area of A, and q the charge residing

upon it, it is evident, from considering the tube of force which has s for one of its ends, that

$$\frac{V}{t} \cdot s = 4\pi q;$$

and the electric density $\frac{q}{s}$ on the element is equal to $\frac{V}{4\pi t}$, which is constant over the whole face of the plate.

To give a rough idea of the increase of capacity obtained by the employment of a condensing plate, let us compare the capacity of a circular disc of 10 inches diameter, accompanied by a condensing plate at a reduced distance of $\frac{1}{20}$ of an inch, with the capacity of a globe of the same diameter as the disc. The capacity of the globe is equal to its radius, and may therefore be denoted by 5. The capacity of the disc is $\frac{25\pi}{4\pi \times \frac{1}{20}} = 125$, or 25 times the capacity of the globe. It is, in fact, the same as the capacity of a globe 250 inches (or 20 ft. 10 in.) in diameter.

63. **Discharge of Condenser.**—If, by means of a jointed brass discharger (Fig. 34) with knobs M N at the ends, and with glass

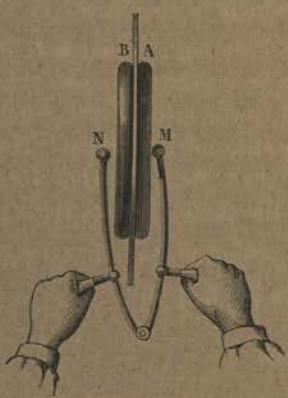

Fig. 34.—Discharge of Condenser.

Fig. 35.—Discharger without Handles.

handles, we put the two plates A and B in communication, a brilliant spark is obtained, resulting from the combination of the positive charge of A with the negative of B, and the condenser is discharged. When the quantity of electricity is small, the glass handles are unnecessary, and the simpler apparatus represented in Fig. 35 may be employed, consisting simply of two brass rods jointed together, and with knobs at their ends, care being taken to touch the plate B, which is in communication with the earth, before the other. The

operator will then experience no shock, as the electricity passes in preference through the brass rods, which are much better conductors than the human body. If, however, the operator discharges the condenser with his hands by touching first the plate B, and then also the plate A, the whole discharge takes place through his arms and chest, and he experiences a severe shock. If he simply touches the plate A, while B remains connected with the earth by a chain, as in Fig. 33, he receives a shock, but less violent than before, because the discharge has now to pass through external bodies which consume a portion of its energy. If, instead of a chain, B is connected with the earth by the hand of an assistant touching it, he too will receive a shock when the operator touches A.

64. Discovery of Cuneus.—The invention of the Leyden jar was brought about by a shock accidentally obtained. Some time in the

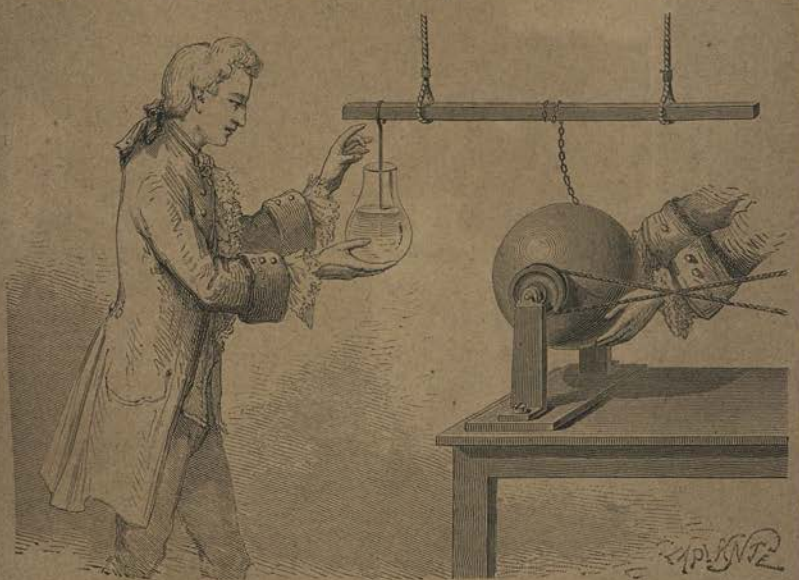

Fig. 36.—Experiment of Cuneus.

year 1746, Cuneus, a pupil of Muschenbroeck, an eminent philosopher of Leyden, wishing to electrify water, employing for this purpose a wide-mouthed flask, which he held in his hand, while a chain from the conductor of the machine dipped in the water (Fig. 36). When the experiment had been going on for some time, he wished to disconnect the water from the machine, and for this purpose was about

to lift out the chain; but, on touching the chain, he experienced a shock, which gave him the utmost consternation, and made him let fall the flask. He took two days to recover himself, and wrote to Réaumur that he would not expose himself to a second shock for the crown of France. The news of this extraordinary experiment spread over Europe with the rapidity of lightning, and it was eagerly repeated everywhere. Improvements were soon introduced in the arrangement of the flask and its contents, until it took the present form of the *Leyden Phial* or *Leyden Jar*. It is easy to see that the effect obtained by Cuneus depended on condensation of electricity, the water in the vessel serving as the collecting plate, the hand as condensing plate, and the vessel itself as the dielectric. When he

Fig. 37.—Leyden Jar.

Fig. 38.—Discharge of Leyden Jar.

touched the chain, the two oppositely charged conductors were put in communication through the operator's body, and he received a shock.

65. **Leyden Jar.**—The Leyden Jar, as now usually constructed, consists of a glass jar coated, both inside and out, with tin-foil, for

about four-fifths of its height. The mouth is closed by a cork, through which passes a metallic rod, terminating above in a knob, and connected below with the inner coating, either by a chain depending from it, or by pieces of metallic foil with which the jar is filled. The interior of the jar must be thoroughly dry before it is closed, and the cork and neck are usually covered with sealing-wax, and shellac varnish, which is less hygroscopic than glass. The Leyden jar is obviously a condenser, its two coatings of tin-foil performing the parts of a collecting plate and a condensing plate. If the inner coating is connected with the electrical machine, and the outer coating with the earth, the former acquires a positive, and the latter a negative charge. On connecting them by a discharger, as in Fig. 38, a spark is obtained, whose power depends on the potential of the inner coating, and on its electrical capacity. If these be denoted respectively by V and C, and if Q denote the quantity of electricity residing on either coating, the amount of electrical energy which runs down and undergoes transformation when the jar is discharged, is $\frac{1}{2} Q V = \frac{1}{2} C V^2 = \frac{1}{2} \frac{Q^2}{C}$. (§ 51).

The quantities Q, V, C, which are, properly speaking, the charge, potential, and capacity of the *inner coating*, are usually called the charge, potential, and capacity of the jar.

66. Residual Charge.—When a Leyden jar has been discharged by connecting its two coatings, if we wait a short time we can obtain another but much smaller spark by again connecting them, and other sparks may sometimes be obtained after further intervals. These are called secondary discharges, and the electricity which thus remains after the first discharge is called the *residual charge*. It appears to arise from a state of strain into which the glass is thrown by the charge, and from which it takes some time to recover.

The whole charge of the outer coating, and all except an insignificant portion of the charge of the inner coating, resides on the side of the foil which is in contact with the glass, or, more probably, on the surfaces of the glass itself, the mutual attraction of the two opposite electricities causing them to approach as near to each other as the glass will permit. This is illustrated by Franklin's experiment of the *jar with movable coatings* (Fig. 39). The jar is charged in the ordinary way, and placed on an insulating stand. The inner coating is then lifted out by a glass hook, and touched with the hand to discharge it of any electricity which it may retain. The

glass is then lifted out, and the outer coating also discharged. The jar is then put together again, and is found to give nearly as strong a spark as it would have given originally.

Fig. 39.—Jar with Movable Coatings.

67. Discharge by Alternate Contacts.—Instead of discharging a Leyden jar at once by connecting its two coatings, we may gradually discharge it by alternate contacts. To do this we must set it on an insulating stand (or otherwise insulate both coatings from the earth), and then touch the two coatings alternately. At every contact a small spark will be drawn. The coating last touched has always rather less electricity upon it than the other, but the difference is an exceedingly small fraction of the whole charge, and, after a great number of sparks have been drawn by these alternate contacts, we may still obtain a powerful discharge by connecting the two coatings.

The quantities of electricity thus alternately discharged from the two coatings form two decreasing geometric series, one for each coating. In fact, if m and m' be two proper fractions such that, when the outer coating is connected with the earth, the ratio of its charge to that of the inner is $-m$; and, when the inner coating is connected with the earth, the ratio of its charge to that of the outer is $-m'$, we have the following series of values:—

	On inner coating.		On outer coating.
Original charges, ...	$+Q$	...	$-mQ$
After 1st contact, ...	$+m'mQ$	...	$-mQ$
2d „	$+m'mQ$	...	$-m'm^2Q$
3d „	$+m'^2m^2Q$	...	$-m'm^2Q$
	&c.		&c.

The quantities discharged from the inner coating are, successively $(1-m'm)Q$, $m'm(1-m'm)Q$, $m'^2m^2(1-m'm)Q$, &c.; and the quantities successively discharged from the outer, neglecting sign, are $m(1-m'm)Q$, $m'm^2(1-m'm)Q$, $m'^2m^3(1-m'm)Q$, &c.

The quantity $(1-m'm)Q$ discharged at the first contact represents that portion of the charge[1] which is not due to condensation; so

[1] This portion of the original charge is said to be *free*, and the remaining portion to be *bound*, *dissimulated*, or *latent*. These terms are applicable to all cases of condensation.

that the actual capacity of the Leyden jar is to the capacity of the inner coating if left to itself as $1 : 1 - m'm$.

The discharge by alternate contacts can be effected by means of a carrier suspended between two bells, as in Fig. 40. The rod from the inner coating terminates in a bell, and the outer coating is connected, by means of an arm of tin, with another bell supported on a metallic column. An insulated metallic ball is suspended between the two. This is first attracted by the positive bell. Then, being repelled by this and attracted by the other, it carries its charge of positive electricity to the negative bell, and receives a charge of negative, which it carries to the positive bell, and so on alternately. The whole apparatus stands upon an insulating support. It is not, however, necessary that the carrier should be insulated from the earth, but it must be insulated from both coatings.

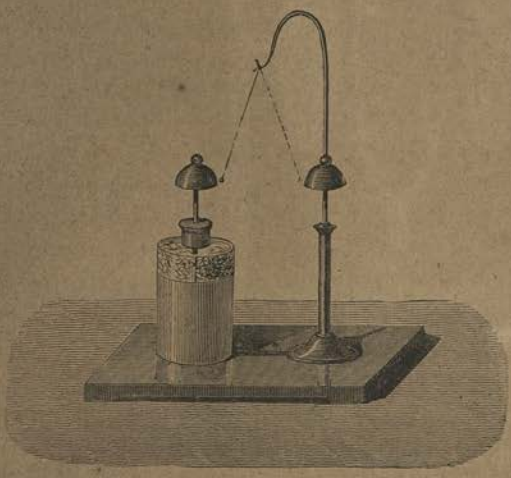

Fig. 40.—Alternate Discharge.

68. Condensing Power.—By the condensing power of a given arrangement is meant the ratio in which the capacity of the collecting plate is increased by the presence of the condensing plate, which ratio, as we have seen in last section, is equal to the fraction $\frac{1}{1-m'm}$. Riess has investigated its amount experimentally under varying conditions, by means of the apparatus represented in Fig. 41, which is a modification of the condenser of Æpinus. It consists of two metallic plates A and B, supported on glass pillars, and travelling on a rail, so that they can be adjusted at different distances. Between them is a large glass plate C. A is charged from the machine, B being at the same time touched to connect it with the ground. The electrical density on the anterior face of A was observed by means of Coulomb's proof-plane and torsion-balance.

Riess' experiments are completely in agreement with the theory laid down in the preceding sections of this chapter; for example, he found, among other results, that the condensing power was independent of

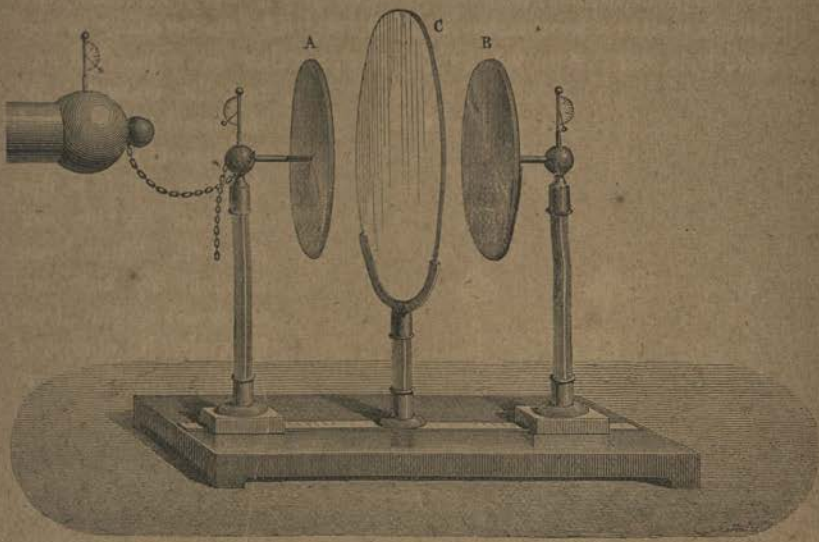

Fig. 41.—Condenser of Æpinus.

the absolute charge, and that it varied nearly in the inverse ratio of the distance.

69. Influence of the Dielectric.—Faraday discovered that the amount of condensation obtained in given positions of the two conducting plates depended upon the material of the *intervening non-conductor* or *dielectric*. Fig. 42 represents a modification of one of Faraday's experiments. A is an insulated metallic disc, with a charge, which we will suppose to be positive. B and C are two other insulated metallic discs at equal distances from A, each having a small electric pendulum suspended at its back. Let B and C be touched with the hand; they will become negatively electrified by induction, but their negative electricity will reside only on their sides which face towards A, and the pendulums will hang vertically. If, while matters are in this condition, we move B nearer to A, we shall see both the pendulums diverge, and on testing, we shall find that the pendulum B diverges with positive, and C with negative electricity. The reason is obvious. The approach of B to

A causes increased induction between them, so that more negative is drawn to the face of B, and positive is driven to its back; at the same time the symmetrical distribution of electricity on A is disturbed, a portion being accumulated on the side next B at the expense of the side next C. The inductive action of A upon C is thus diminished, and a portion of the negative charge of C is left free to spread itself over the back, and affect the pith-ball.

Fig. 42.—Change of Distal

If, while the discs are in their initial position, B and C being equidistant from A, and the pendulums vertical, we interpose between B and A a plate of sulphur, shellac, or any other good insulator, the same effect will be produced as if B had been brought nearer to A. We see, then, that the insulating plate of a condensing arrangement serves not only to prevent discharge, but also to increase the inductive action and consequent condensation, as compared with a layer of air of the same thickness; inductive action through a plate of sulphur or shellac of given thickness, is the same as through a thinner plate of air. The numbers in the subjoined table (which contains Faraday's results) denote the thickness of each material which is equivalent to unit thickness of air. For example, the mutual induction through 2·24 inches of sulphur is the same as through 1 inch of air. These numbers are called

SPECIFIC INDUCTIVE CAPACITIES.

Air or any gas,	1·00	Pitch,	1·80
Spermaceti,	1·45	Wax,	1·86
Glass,	1·76	Shellac,	2·00
Resin,	1·77	Sulphur,	2·24

The quotient of the actual thickness of the plate by the specific inductive capacity of its material may appropriately be called the *thickness reduced to its equivalent of air*, or simply the *reduced thickness*.

When the comparisons are made by very rapid charges and dis-

charges (so as to minimize the residual charge), larger values are found; for example,—

Glass,	7 to 9	India-rubber,	2
Shellac,	3·4	„ vulcanized,	2·8
Sulphur,	3	Ebonite,	2·6
Solid paraffin,	2	Turpentine,	2·2

70. Faraday's Determinations.—Faraday, to whom the name and discovery of specific inductive capacity are due, operated by comparing the capacities of condensers, alike in all other respects, but differing in the materials employed as dielectrics. One of his condensers is represented in Fig. 43. It is a kind of Leyden jar, containing a metallic sphere A, attached to the rod M, and forming with it the inner conductor. The outer conductor consists of the hollow sphere B divided into two hemispheres which can be detached from each other. The interval between the outer and inner conductor can be filled, either with a cake of solid non-conducting material, or with gas, which can be introduced by means of the cock R. The method of observation and reduction will be best understood from an example.

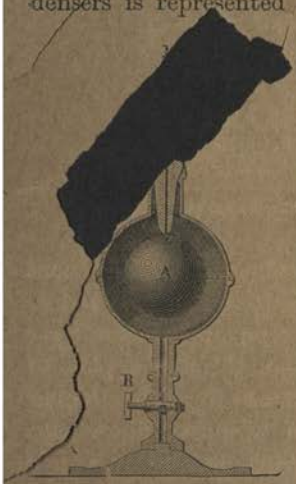

Fig. 43.—Apparatus for Specific Inductive Capacity.

The interval being occupied by air, the apparatus was charged, and a carrier-ball, having been made to touch the summit of the knob M, was introduced into a Coulomb's torsion-balance, and found to be charged with a quantity of electricity represented by 250° of torsion. When the second apparatus was precisely similar to the first, it was found that, on contact of the two knobs, the charge divided itself equally, and the carrier-ball, if applied to either knob, took a charge represented very nearly by 125°.

The conditions were then changed in the following way. The first jar still containing air, the interval between the two conductors in the second was filled with shellac. It was then found that the air-jar, being charged to 290°, was reduced, by contact of its knob with that of the shellac-jar, to 114°, thus losing 176°. If no allowance be made for dissipation, the capacities of the air-jar and shellac-jar would

therefore be as 114 : 176, or as 1 : 1·54, and the specific inductive capacity of shellac would be 1·54.

71. **Polarization of the Dielectric.**—As the interposed non-conductor, or dielectric, modifies the mutual action of the two electricities which it separates, and does not play the mere passive part which was attributed to it before Faraday's experiments, it is natural to conclude that the dielectric must itself experience a peculiar modification. According to Faraday, this modification consists in a polarization of its particles, which act inductively upon each other along the lines of force, and have each a positive and a negative side, the positive side of each facing the negative side of the next. This polarization is capable of being sustained for a great length of time in good non-conductors; but in good conductors it instantly leads to discharge between successive particles, and the opposite electricities appear only at the two surfaces.

The polarization of dielectrics is clearly shown in the following experiment. In a glass vessel (Fig. 44) is placed oil of turpentine,

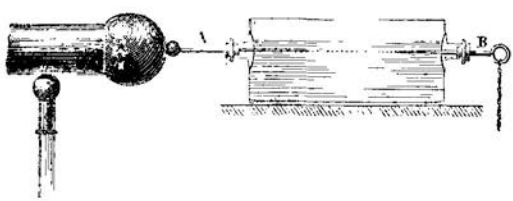

Fig. 44.—Polarization of Dielectric.

containing filaments of silk 2 or 3 millimetres long. Two metallic rods, A, B, each terminating within in a point, are connected, one with the ground, and the other with an electric machine. On working the machine, the little filaments are seen to arrange themselves in a line between the points, and, on endeavouring to break the line with a glass rod, it will be found that they return to this position with considerable pertinacity. On stopping the machine, they immediately fall to the bottom.

An experiment of Matteucci's demonstrates this polarization still more directly. A number of thin plates of mica are pressed strongly together between two metallic plates. One of the metallic plates is charged, while the other is connected with the ground; and, on removing the metallic plates by insulating handles, it is found that all the mica plates are polarized, the face turned towards the positive

metal plate being covered with positive electricity, and the other face with negative.

Dr. Kerr has recently shown that glass and other transparent insulators, when subjected to strong dielectric action, become for the time doubly-refracting, a property which is also producible in such substances as glass by longitudinal extension or compression. In some substances, including glass itself, the dielectric effect is identical with the effect of *compression* along the lines of force. In others it is identical with the effect of *extension* along these lines. Liquid as well as solid dielectrics are thus affected, the only difference being that in solids the effect takes about half a minute to attain its maximum, and dies away gradually when the electrical forces are removed; whereas in liquids, the full effect is attained instantaneously, and the disappearance is also instantaneous. The direction of vision, in the experiments, was at right angles to the lines of force; and the optical effect, per unit of thickness in this direction, was found to vary, in any given liquid, directly as the square of the electric force.

72. Limit to Thinness of Interposed Plate.—We have seen (§ 62) that the capacity of a condenser varies inversely as the distance between the collecting and the condensing plate. But if this distance is very small, the resistance of the interposed dielectric (which varies directly as its thickness) may be insufficient to prevent discharge, and it will not be practicable to establish a great difference of potential between the two plates. We may practically distinguish two sorts of condensers, one sort having a very thin dielectric and very great condensing power, but only capable of being charged to feeble[1] potential; the other having a dielectric thick enough to resist the highest tensions attainable by the electrical machine. The Leyden jar comes under the second category. The first includes the electrophorus (except in so far as its action is aided by the metallic sole), and the condenser of Volta's electroscope.

73. Condensers for Galvanic Electricity.—Condensers of very large surface are used for certain applications of galvanic electricity, especially in connection with telegraphy. They are constructed by arranging a number of sheets of tin-foil in a pile, with either thin

[1] *Strong* potential is potential differing very much from zero either positively or negatively. *Feeble* potential is potential not differing much from zero. *Tension* is measured by difference of potential; and when the earth is one of the terms of the comparison, tension becomes identical with potential.

mica or paper saturated with paraffin wax between them. The 1st, 3d, 5th, &c., sheets of tin-foil are connected together and correspond to one coating of a Leyden jar, and the 2d, 4th, 6th, &c., sheets are connected and correspond to the other coating. They are charged by connecting one of these coatings to one pole of a battery and the other coating to the other pole.

74. **Volta's Condensing Electroscope.**—This instrument, which has rendered very important services to the science of electricity, differs from the simple gold-leaf electroscope previously described (§ 10), in having at its top two metal plates, of which the lower one is connected with the gold-leaves, and is covered on its upper face with insulating varnish, while the upper is varnished on its lower face, and furnished with a glass handle. These two plates constitute the condenser. In using the instrument, one of the two plates (it matters not which) is charged by means of the body to be tested, while the other is connected with the earth. They thus receive opposite and sensibly equal charges. The upper plate is then lifted off, and the higher it is raised the wider do the gold-leaves diverge. The separation of the plates diminishes the capacity, and strengthens the potential of both, one becoming more strongly positive, and the other more strongly negative. This involves increase of potential energy, which is represented by the amount of work done against electrical attraction in separating the plates. No increase in quantity of electricity is produced by the separation; hence the instrument is chiefly serviceable in detecting the presence of electricity which is available in large quantity but at weak potential. The glass handle of the upper plate is by no means essential, as it is only necessary that the lower plate should be insulated. The charge may be given by induction; in which case one plate must be connected with the earth while the inducing body is held near it, and the other plate must be kept connected with the earth while the influencing body is withdrawn. The plates will then be left charged with opposite electricities, that

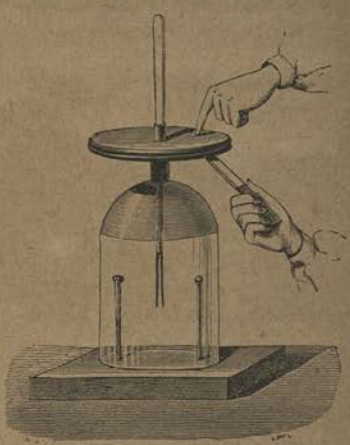

Fig. 45.—Condensing Electroscope.

which was more remote from the influencing body having acquired a charge similar to that of the body. For inductive charges, however, the condensing arrangement serves no useful purpose, beyond enabling the electroscope to retain its charge for a longer time, the effect finally obtained on separating the plates being no greater than would have been obtained by employing only the lower plate.

75. Leyden Battery.—The Leyden battery consists of a number of Leyden jars, placed in compartments of a box lined with tin-foil, which serves to establish good connection between their outer coat-

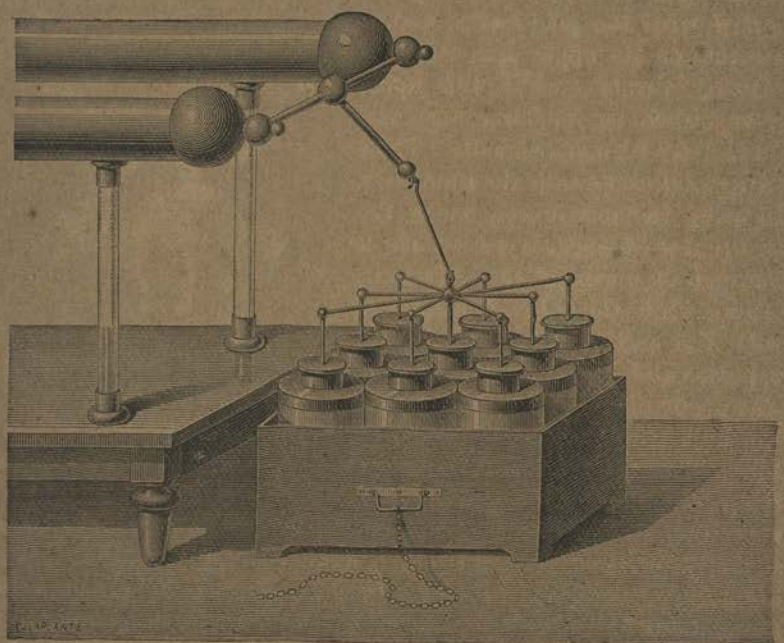

Fig. 46.—Battery of Leyden Jars.

ings, while their inner coatings are connected by brass rods. It is advisable that the outer coatings should have very free communication with the earth. For this purpose a metallic handle, which is in metallic communication with the lining of the box, should be connected, by means of a chain, with the gas or water pipes of the building.

The capacity of a Leyden battery is the sum of the capacities of the jars which compose it. The charge is given in the ordinary way,

by connecting the inner coatings with the conductor of the machine. In bad weather this is usually a very difficult operation, on account of the large quantity of electricity required for a full charge, and the large surface from which dissipation goes on.

Holtz's machine can be very advantageously employed for charging a battery, one of its poles being connected with the inner, and the other with the outer coatings. In dry weather it gives the charge with surprising quickness.

76. Lichtenberg's Figures.—An interesting experiment devised by

Fig. 47.—Lichtenberg's Figures.

Lichtenberg serves to illustrate the difference between the physical properties of positive and negative electricity.

A Leyden jar is charged, and the operator, holding it by the outer

coating, traces a design with the knob on a plate of shellac or vulcanite. He then places the jar on an insulating stand, to enable him to transfer his hold to the knob, and traces another pattern on the cake with the outer coating. A mixture of flowers of sulphur and red-lead, which has previously been well dried and shaken together, is then sprinkled over the cake. The sulphur, having become negatively electrified by friction with the red-lead, adheres to the pattern which was traced with positive electricity, while the red-lead adheres to the other. The yellow and red colours render the patterns very conspicuous. The particles of sulphur (represented by the inner pattern in Fig. 47) arrange themselves in branching lines, while the red-lead (shown in the outer pattern) forms circular spots; whence it would appear that positive electricity travels along the surface more easily than negative. A similar difference has already been pointed out between positive and negative brushes.

77. **Charge by Cascade.**—Instead of connecting all the inner coatings together, and all the outer coatings together, as in the Leyden battery, we may connect a number of jars in series. The inner coating of the first jar is to be connected with the prime conductor of the machine; and its outer coating, which must be insulated from the earth, is to be connected with the inner coating of the second jar. The outer coating of this is in like manner to be connected with the inner coating of the next, and so on to the last jar, the outer coating of which must be connected with the earth. When the machine is worked, a positive charge is given to the inner coating of the first, and a sensibly equal negative charge is induced upon its outer coating, this negative charge being drawn from the inner coating of the second, which accordingly acquires a positive charge sensibly equal to that given from the machine to the first jar. This reasoning can be extended through the whole chain. Hence if we denote by Q the charge given to the inner coating of the first jar, the inner coating of each jar in the series has a charge $+Q$, and the outer coating a charge $-Q$. If we further suppose all the jars to have the same capacity C, and if we denote the potentials of the inner coatings by $V_1, V_2, V_3, \ldots V_n$, we shall have

$$\frac{Q}{C} = V_1 - V_2 = V_2 - V_3 =, \text{ \&c., } = V^n - 0,$$

since the quotient of the charge of a jar by its capacity is equal to the difference of potential of the two coatings, and the potentials of the outer coatings are $V_2, V_3, \ldots V_n, 0$. By adding the n differ-

ences $V_1 - V_2$, $V_2 - V_3 \ldots V_n - 0$, we obtain V_1, which is accordingly equal to n times $\frac{Q}{C}$, and we have

$$\frac{Q}{C} = V_1 - V_2 = V_2 - V_3 =, \text{ \&c., } = \frac{V_1}{n}.$$

If we compare the charges of these jars with the charge which the first jar would have received if its outer coating had been connected to earth in the ordinary way, the prime conductor being supposed to attain the same potential V_1 in both cases, we have

$$Q = \frac{1}{n} CV_1$$

for each jar in the series, whereas we should have had $Q' = CV_1$ for the single jar. The charge of each jar in the series is therefore $\frac{1}{n}$ of its ordinary charge.

As regards energy; for the single jar the energy would be $\frac{1}{2}Q'V_1 = \frac{1}{2}CV_1^2$, while for any one jar in the series the energy would be

$$\tfrac{1}{2}Q(V_1 - V_2) = \tfrac{1}{2}\frac{1}{n}CV_1\frac{V_1}{n} = \tfrac{1}{2}\frac{1}{n^2}CV_1^2,$$

which is $\frac{1}{n^2}$ of the energy of the single jar.

Jars thus arranged are said to be charged *by cascade*, the name being suggested by the successive falls of potential from jar to jar. They can either be discharged in succession by connecting the two coatings of each, or all together by connecting the inner coating of the first with the outer coating of the last. In the former case the energy of each spark is $\frac{1}{2}\frac{1}{n^2}CV_1^2$, as appears from the above calculation. In the latter case the energy of the single spark is $\frac{1}{2}\frac{1}{n}CV_1^2$.

78. Force modified by value of K.—The following modifications must be made in the foregoing statements when specific inductive capacity is taken into account.

The mutual force between two charges q_1 and q_2 collected at points at distance r in a medium of specific capacity K is $q_1 q_2 / \mathrm{K} r^2$.

The intensity of electrical force at a point due to a charge q at distance r is $q / \mathrm{K} r^2$.

The work done on unit charge which moves from distance r_1 to distance r_2 by the repulsion of a charge q is $q / \mathrm{K} r_1 - q / \mathrm{K} r_2$.

When there are several repelling or attracting charges, the work done by their forces in a given displacement of a unit charge is $\Sigma \frac{q}{\mathrm{K} r_1} - \Sigma \frac{q}{\mathrm{K} r_2}$. Hence the potential at a point is $\Sigma \frac{q}{\mathrm{K} r}$.

When a tube of force incloses a quantity q of electricity, the product

$$\text{intensity of force} \times \text{section} \times \text{K}$$

changes by the amount $4\pi q$ in passing from one side of the inclosed electricity to the other. This product is called the *quantity of induction* within the tube. It is independent of the medium, and does not change its value when the tube passes out of one medium into another; but the force does change, being inversely as K. Maxwell accordingly preferred the name "tubes of induction" to "tubes of force."

The force just outside a conductor with a surface-density ρ is $4\pi\rho/\text{K}$.

The force at a point between two parallel plates with potentials V_1 and V_2, at a small distance x apart, is $\frac{V_2 - V_1}{x}$, and is also $\frac{4\pi\rho}{\text{K}}$ or $\frac{4\pi Q}{\text{KS}}$, S denoting the area of either plate, and Q the charge upon it. Hence the capacity of the condenser formed by the plates is $C = \frac{Q}{V_2 - V_1} = \frac{KS}{4\pi x}$.

79. Analogy between Specific Inductive Capacity and Thermal Conductivity.—Suppose steady flow of heat to be taking place uniformly in all directions from a small source in a medium of conductivity k. Let the quantity of heat emitted by the source in unit time be denoted by $4\pi H$. Then, since the flow is steady, this quantity crosses in unit time every spherical surface $4\pi r^2$ that can be described about the source, and the flow across unit area of such a surface is H/r^2, r denoting the radius of the sphere. This flow must be equal to k multiplied by the gradient, that is, to $-k\,d\theta/dr$, θ denoting the temperature at distance r from the source. Thus we have

$$d\theta = -\frac{H}{k r^2} dr = d\frac{H}{kr};$$

whence it can be shown that, when there are any number of sources, the difference of the values of $\Sigma \frac{H}{kr}$ at any two points in the medium is the difference of the temperatures at these points.

Thus in the analogy between Electrostatics and Heat, potential corresponds to temperature, q to H, K to k, and intensity of electrical force at a point to intensity of flow of heat at a point.

[At this stage the student should turn to the Appendix and read to the end of the account of Electrostatic Units.]

CHAPTER VIII.

ELECTROMETERS.

80. Object of Electrometers.—Electrometers are instruments for the measurement of differences of electrical potential. The gold-leaf electroscope, the straw-electroscope, and other instruments of the same type, afford rough indications of the difference of potential between the diverging bodies and the air of the apartment, and more measurable indications are given by the electrometers of Peltier and Dellmann; but none of these instruments are at all comparable in precision to the various electrometers which have been invented from time to time by Sir Wm. Thomson.

81. Attracted-disc Electrometers, or Trap-door Electrometers.[1]—We shall first describe what Sir Wm. Thomson calls "Attracted-disc Electrometers." These instruments, one of which is represented in Figs. 48, 49, contain two parallel discs of brass, *g*, *h*, with an aperture in the centre of one of them, nearly filled up by a light trap-door of aluminium *f*, which is supported in such a way as to admit of its electrical attraction towards the other disc being resisted by a mechanical force which can be varied at pleasure. The trap-door and the perforated plate surrounding it must have their faces as nearly as possible in one plane when the observation is taken, and, as they are electrically connected, they may then be regarded as forming *one disc of which a small central area is movable.* There is always attraction between the two parallel discs, except when they are at the same potential.

Let their potentials be denoted by V and V′, the electrical densities on their faces by ρ and ρ', and their mutual distance by D. We have seen (§ 62) that, in such circumstances, ρ and ρ' are constant

[1] Sometimes called "guard-ring" or "guard-plate" electrometers, the trap-door being "guarded" by the fixed plate which surrounds it.

(except near the edges of the discs), opposite in sign, and equal, and that the intensity of force in the space between them is everywhere the same, and equal at once to $\frac{V-V'}{D}$ and to $4\pi\rho$. This force is jointly due to attraction by one plate and repulsion by the other, each of these having the intensity $2\pi\rho$, or half the total intensity.

Let A denote the area of the trap-door. The quantity of electricity upon it will be ρ A, and the force of attraction which this experiences will be $\rho A \times 2\pi\rho = 2\pi\rho^2 A$, which we shall denote by F. Then from the equations

$$F = 2\pi\rho^2 A \qquad \frac{V-V'}{D} = 4\pi\rho, \tag{1}$$

we find, by eliminating ρ,

$$F = \frac{A}{8\pi}\left(\frac{V-V'}{D}\right)^2, \text{ or } V - V' = \pm D\sqrt{\frac{8\pi F}{A}}. \tag{2}$$

82. **Absolute Electrometer.**—In the *absolute electrometer*, which somewhat resembles Fig. 49 turned upside down, the force of electrical attraction on the trap-door is measured by direct comparison with the gravitating force of known weights. This is done by first observing what weights must be placed on the trap-door to bring it into position when no electrical force acts (the plates being electrically connected), and by then removing the weights, allowing electrical force to act, and adjusting the plates at such a distance from one another, by the aid of a micrometer screw, that the trap-door shall again be brought into position. Then, in equation (2), F, A, and D are known, and the difference of potentials $V - V'$ can be determined. In the absolute electrometer, the perforated disc h is uppermost, so that the direction of electrical attraction on the trap-door is similar to the direction of the gravitating force of the weights. The reverse arrangement is usually adopted in the portable electrometer, which we shall next describe. In both instruments, the trap-door constitutes one end of a very light lever fil of aluminium, balanced on a horizontal axis.

83. **Portable Electrometer.**—In the *portable electrometer* (Figs. 48, 49) this axis passes very accurately through the centre of gravity of the lever, the suspension being effected by means of a fine platinum wire ww tightly stretched, which is secured at its centre to the lever in such a manner that, when the trap-door comes into position, the wire is under torsion tending to draw back the disc from the attracting plate g. This torsion (except in so far as it is

affected by causes of error such as temperature and gradual loss of elasticity), is always the same when the disc is in position, and as it is to be balanced in every observation by electrical attraction, the latter must also be always the same; that is to say, the quantity F in equations (2) is constant for all observations with the same instrument; whence it is obvious that $V - V'$ is directly proportional to D, the distance between the plates. The observation for difference of

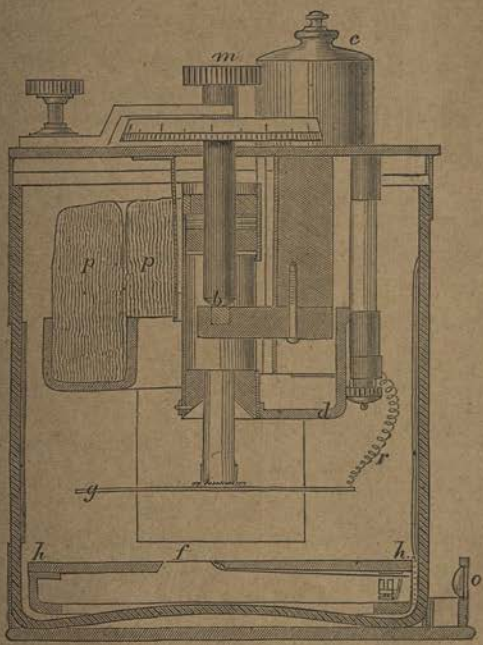

Fig. 48.—Portable Electrometer.

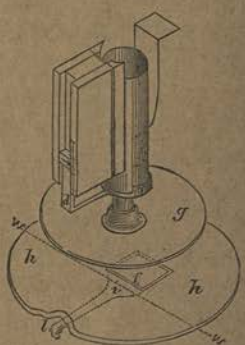

Fig. 49.—Parallel Discs.

potential therefore consists in altering this distance until the trap-door comes into position. This is done by turning the micrometer screw, by means of the milled head *m*. The divided circle of the micrometer indicates the amount of turning for small distances, and whole revolutions are read off on the vertical scale traversed by the index carried by the arm *d*. The correct position is very accurately identified by means of two sights, one of them being attached to a fixed portion of the instrument, and the other to one end *l* of the lever. One of these sights moves up and down close in front of the other, and they are viewed through a lens *o* in

front of both. This arrangement is also adopted in the absolute electrometer.

One of the two parallel plates h is connected with the inner coating of a Leyden jar,[1] which, being kept dry within by means of pumice p wetted with sulphuric acid, retains a sufficient charge for some weeks. The other plate g is in communication, by means of

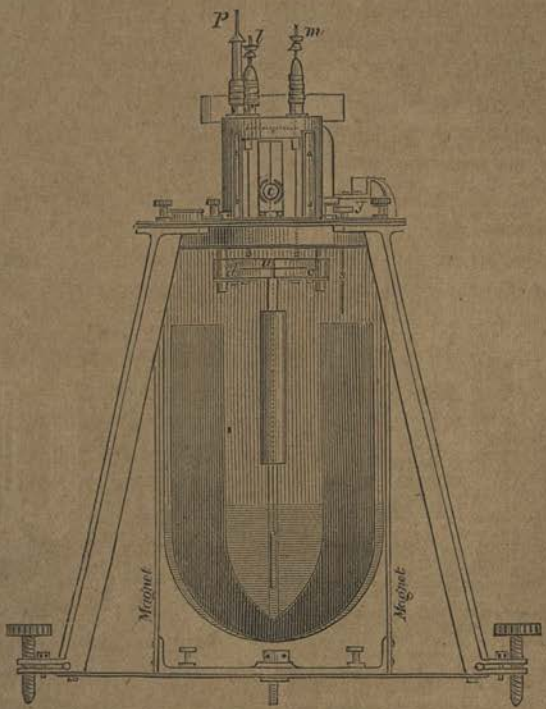

Fig. 50.—Quadrant Electrometer.

the spiral wire r, with the insulated umbrella c, which can be connected with any external conductor; and, in order to determine the potential of any conductor which we wish to examine, two observations are taken, one of them giving the difference of potential

[1] The use of the Leyden jar is to give constancy of potential. Its capacity is so much greater than that of the disc with which it is connected that the electricity which enters or leaves the latter in consequence of the inductive action of the other disc is no sensible fraction of the whole charge of the jar, and produces no sensible change in its potential. Its great capacity in comparison with the extent of surface exposed likewise tends to prevent rapid loss of potential by dissipation of charge.

between this conductor and the Leyden jar, and the other the difference between the earth and the jar. We thus obtain, by subtraction, the difference of potential between the conductor in question and the earth.

84. **Quadrant Electrometer.**—The most sensitive instrument yet invented for the measurement of electrical potential is the *quadrant electrometer*, which is represented in front view in Fig. 50, some of its principal parts being shown on a larger scale in Figs. 51, 52.

In this instrument, the part whose movements give the indications is a thin flat piece of aluminium u, narrow in the middle and broader towards the ends, but with all corners rounded off. This piece, which is called the *needle*, and is represented by the dotted line in Fig. 51, is inclosed almost completely in what may be described as a shallow cylindrical box of brass, cut into four quadrants, c, d, c', d'. These parts are shown in plan in Fig. 51, and in front view in Fig. 50. The needle u is attached to a stiff platinum wire, which is supported by a silk fibre hanging vertically. The same wire carries a small concave mirror t (Fig. 50) for reflecting the light from an illuminated vertical slit. An image of the slit is thus formed at the distance of about a yard, and is received upon a paper scale of equal parts, by reference to which the movements of the image can be measured. The movements of the image depend upon the movements of the mirror, which are precisely the same as those of the needle. We have now to explain how the movements of the needle are produced.

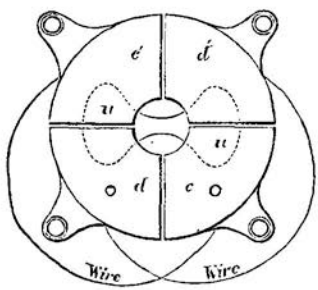

Fig. 51.—Needle and Quadrants.

One pair of opposite quadrants $c\,c'$ are connected with each other, and with a stiff wire l projecting above the case of the instrument. The other quadrants $d\,d'$ are in like manner connected with the other projecting wire m. The projecting parts $l\,m$ are called the *chief electrodes*, and are to be connected respectively with the two conductors whose difference of potential is required, one of which is usually the earth. Suppose the needle to have a positive charge of its own, then if the potential of c and c' be higher (algebraically) than that of d and d', one end of the needle will experience a force urging it from c to d, and the other end will experience a force urging it

from c' to d'. These two forces constitute a couple tending to turn the needle about a vertical axis. If the potential of c and c' be lower than that of d and d', the couple will be in the opposite direction. To prevent the needle from deviating too far under the action of this couple, and to give it a definite position when there is no electrical couple acting upon it, a small light magnet is attached to the back of the mirror, and by means of controlling magnets outside the case the earth's magnetism is overpowered, so that, whatever position be chosen for the instrument, the needle can be made to assume the proper zero position. In more recent instruments the magnets are dispensed with, and a bifilar suspension is substituted for the single silk fibre. The permanent electrification of the needle is attained by connecting it, by means of a descending platinum wire, with a quantity of strong sulphuric acid, which occupies the lower part of the containing glass jar. The acid, being an excellent conductor, serves as the inner coating of a Leyden jar, the outside of the glass opposite to it being coated with tin-foil, and connected with the earth. The acid at the same time serves the purpose of keeping the interior of the apparatus very dry. The charge is given to the jar through the *charging electrode* p, which can be thrown into or out of connection at pleasure. As the sensibility of the instrument increases with the potential of the jar, a *gauge* and *replenisher* are provided for keeping this potential constant. The *gauge* is simply an "attracted-disc electrometer," in which the distance between the parallel discs is never altered, so that the aluminium square only comes into position when the potential of one of the discs, which is connected with the acid in the jar, differs by a certain definite amount from the potential of the other, which is connected with the earth. A glance at the gauge shows, at any moment, whether the potential of the jar has the normal strength. If it has fallen below this point, the *replenisher* is employed to increase the charge.

This apparatus, which is separately represented, dissected, in Fig. 52, and is for simplicity omitted in Fig. 50, consists of a vertical stem of ebonite s, which can be rapidly twirled with the finger by means of a milled head y, and which carries two metal wings or *carriers*, b, b, insulated from each other. In one part of their revolution, these come in contact with two light steel springs $f f$, which simply serve to connect them for the instant with each other. In another part of their revolution, they come in contact with two other springs $e\,e$, connected respectively with the acid of the jar and

with the earth. The first of these contacts takes place just before the wings emerge from the shelter of the larger metallic sectors or *inductors* $a\,a$, of which one is connected with the acid, and the other with the earth. Suppose the acid to have a positive charge. Then, at the instant of contact, an inductive movement of electricity takes place, producing an accumulation of negative electricity in the carrier which is next the positive inductor, and an accumulation of positive in the other. The next contacts are effected when the carrier which has thus acquired a positive charge is well under cover of the positive inductor, to which accordingly it gives up its electricity; for, being in great part surrounded by this inductor, and being connected with it by the spring, the carrier may be regarded as forming a portion of the interior of a concave conductor, and the electricity accordingly passes from it to the external surface, that is to the inductor, and to the acid connected with it, which forms the lining of the jar. The negative electricity on the other carrier is, in like manner, given off to the other inductor, and so to the earth.

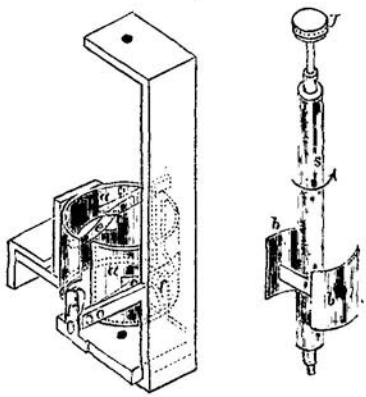

Fig. 52.—Replenisher.

The jar thus receives an addition to its charge once in every half-revolution of the replenisher; and, as these increments are very small, it is easy to regulate the charge so that the gauge shall indicate exactly the normal potential. If the charge is too strong, it can be diminished by turning the replenisher in the reverse direction.

NOTE ON THE ENERGY OF A SYSTEM OF CHARGED CONDUCTORS, WITH APPLICATION TO THOMSON'S QUADRANT ELECTROMETER.

(1.) By the energy of a system of charged conductors is meant the work which must have been spent in charging them, or, what is the same thing, the energy which will run down when the conductors are connected with the earth. We shall investigate its amount in terms of the charges Q_1, Q_2, &c., of the conductors, and their potentials V_1, V_2, &c., these latter being supposed to depend only on the charges of the system itself.

Let the conductors be charged gradually all at the same time, and let

their charges at any time be xQ_1, xQ_2, &c., the value of x being the same for all the conductors. The potentials at the same time will be xV_1, xV_2, &c. By § 51, the work required to bring the small quantity of electricity $Q_1\,dx$ from the earth to the conductor of potential xV_1 is $xV_1\,Q_1\,dx$; thus, when x receives the small increase dx, the whole addition of energy to the system is

$$(V_1Q_1 + V_2Q_2 + \&c.)\,x\,dx.$$

If this operation is repeated time after time, beginning with $x = 0$, and ending with $x = 1$, the conductors will begin with being uncharged, and will end by having the given charges. Since the integral of $x\,dx$ between these limits is $\frac{1}{2}$, *the whole energy acquired by the system is* $\frac{1}{2}(V_1Q_1 + V_2Q_2 + \&c.)$, which may be written $\frac{1}{2}\Sigma VQ$.

(2.) Hence when any small changes dQ_1, dQ_2, &c., are made in the charges, and any small changes dV_1, dV_2, &c., in the potentials, either with or without displacement of the conductors, the increase of energy is $\frac{1}{2}\Sigma(V\,dQ + Q\,dV)$.

(3.) If the conductors are *stationary*, another simple expression can be found for the increase of energy; for the work required to bring the electricity dQ_1 to the conductor of potential V_1 is $V_1\,dQ_1$; thus the whole increase of energy is $\Sigma V\,dQ$; and by comparing this with the expression for the same thing in (2.) we see that a third expression for the increase of energy will be $\Sigma Q\,dV$.

(4.) If the conductors are insulated, so that their *charges remain constant*, the increase of energy when they are displaced will be the difference between the initial energy $\frac{1}{2}\Sigma QV$ and the final energy $\frac{1}{2}\Sigma QV'$, that is, will be $\frac{1}{2}\Sigma Q(V' - V)$, where V' denotes the final potential of the conductor whose initial potential is V. If the charges are small the increase of energy will be $\frac{1}{2}\Sigma Q\,dV$. This, it will be noticed, is exactly half the increase in (3.), the changes of potential being supposed the same in both cases.

When insulated charged conductors are allowed to move under the influence of their own mutual forces, these forces will do positive work, and the system will lose electrical energy of the same amount. On the other hand, if external forces move the conductors in opposition to their mutual forces, there will be a gain of electrical energy equal to the work done by external forces against the forces of the system. These consequences follow immediately from the principle of the conservation of energy.

(5.) If the conductors while displaced are kept *at constant potentials*, their charges must change, and we cannot make the same direct application of the principle of conservation of energy which we have made above, unless we include in our reasonings the external sources from which electricity

increase of loudness. When any considerable portion of the path of discharge is at a uniform distance from the observer, the simultaneous arrival of the disturbances propagated from all this portion will produce a specially loud burst of sound.

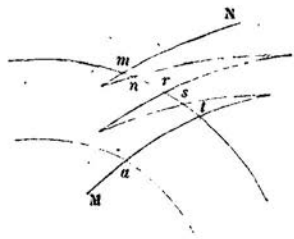

Fig. 54.—Simultaneous Explosions.

88. Shock by Influence.—Persons near whom a flash of lightning passes, frequently experience a severe shock by induction. This is analogous to the phenomenon, first observed by Galvani, that a skinned frog in the neighbourhood of an electrical machine, although dead, exhibits convulsive movements every time a spark is drawn from the conductor.

89. Lightning Conductors.—Experience having shown that electricity travels in preference through the best conductors, it is easy to understand that, if a building be fitted with metallic rods terminating in the earth, lightning will travel through these instead of striking the building. But further, if these rods terminate above in a point, they may exercise a preventive influence by enabling the earth and clouds to exchange their opposite electricities in a gradual way, just as the conductor of a machine is prevented from giving powerful sparks by presenting to it a sharp point connected with the earth. The following conditions should always be complied with:—

1. The connection with the ground should be continuous.

2. The conductor must be everywhere of so large a section that it will not be melted by lightning passing through it.

3. The earth contact must be good. The conductor may be connected at its base with the iron pipes which supply the neighbourhood with water; or it may terminate in the water of a well or pond. Failing these, it should be provided with branches traversing the soil in different directions and surrounded by coke, which is a good conductor.

4. At no part of its course above ground should it come near to the metal pipes which supply the house with water or gas, nor to any large masses of metal in the house. All large masses of metal on the outside of the house, such as lead roofing, should be well connected with the conductor.

5. It should terminate above in a sharp point or points, which should project above the highest parts of the building.

90. Ordinary Electricity of the Atmosphere.—The presence of electricity in the upper regions of the air is not confined to thunder-clouds, but can be detected at all times. In fine weather this electricity is almost invariably positive, but in showery or stormy weather negative electricity is as frequently met with as positive; and it is in such weather that the indications of electricity, whether positive or negative, are usually the strongest.

91. Methods of obtaining Indications.—One of the early methods of observing atmospheric electricity consisted in shooting up an arrow, attached to a conducting thread, having at its lower end a ring, which was laid upon the top of a gold-leaf electroscope. As the arrow ascends higher, the leaves diverge more and more with electricity of the same sign as that overhead; and they remain divergent after the ring has been lifted off by the movement of the arrow.

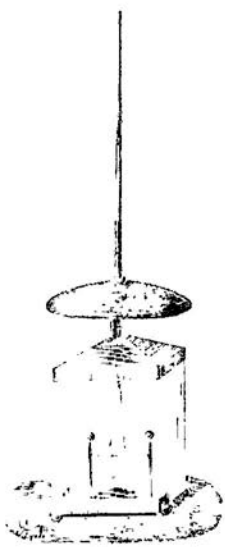

Fig. 55.—Early form of Electrometer.

Sometimes, instead of the arrow, a point on the top of the electroscope is employed to collect electricity from the air, as in Fig. 55. Both these methods are very uncertain in their action.

A better method of collecting electricity from the air was long ago devised by Volta, who employed for this purpose a burning match attached to the top of a rod connected with the gold-leaves or straws of his electroscope. If there is positive electricity overhead, its influence causes negative electricity to collect at the upper end of the rod, whence it passes off by convection in the products of combustion of the match, leaving the whole conducting system positively electrified. In like manner, if the electricity overhead be negative, the system will be left negatively electrified.

Another method which, in the hands of Peltier, Quetelet, and Dellmann, has yielded good results, consists in first exposing, in an elevated position such as the top of a house, a conducting ball supported on an insulating stand, and, while exposed, connecting it with the earth; then insulating it, and examining the charge which it has

acquired. This charge, being acquired from the earth by the inductive action of the electricity overhead, is opposite in sign to the inducing electricity.

Another method, which in principle resembles that of Volta, but is speedier in its action, has been introduced by Sir W. Thomson. It consists in allowing a fine stream of water to flow from an insulated metallic vessel, through a pipe, which projects through an open window or other aperture in the wall of a house, so that the nozzle from which the water flows is in the open air. The apparatus for this purpose, called the water-dropping collector, is represented in Fig. 56. *a* is a copper can, containing water, which can be discharged through the brass pipe *b* by turning a tap. The mode of insulation is worthy of notice. The can is supported on a glass stem *c*, which is surrounded, without contact, by a ring or rings of pumice *d d*, moistened with sulphuric acid. These are protected by an outer case of brass *e e*, having a hole in its top rather larger than the glass stem, the brass being separated from the moist pumice by an inner case of gutta percha. The acid needs renewal about once in two months.

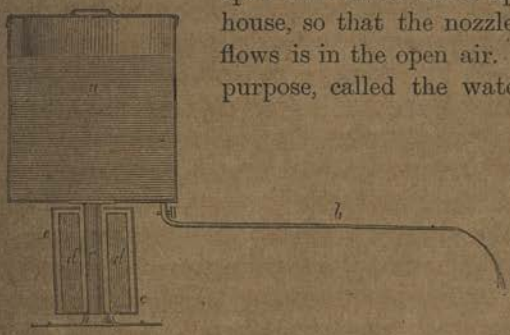

Fig. 56.—Water-dropping Collector.

In severe frost, burning matches can be used instead of water, and are found to give identical indications. Whether water or match be used, the principle of action[1] is that, as long as any differ-

[1] The following quotation from an article by Sir W. Thomson puts the matter very clearly:—"If, now, we conceive an elevated conductor, first belonging to the earth, to become insulated, and to be made to throw off, and to continue throwing off, portions from an exposed part of its surface, this part of its surface will quickly be reduced to a state of no electrification, and the whole conductor will be brought to such a potential as will allow it to remain in electrical equilibrium in the air, with that portion of its surface neutral. In other words, the potential throughout the insulated conductor is brought to be the same as that of the particular equi-potential surface in the air, which passes through the point of it from which matter breaks away. A flame, or the heated gas passing from a burning match, does precisely this: the flame itself, or the highly heated gas close to the match, being a conductor which is constantly extending out, and gradually becoming a non-conductor. The drops [into which the jet from the water-dropper breaks] produce the same effects, with more pointed decision, and with more of dynamical energy to remove the rejected matter, with the electricity which it carries, from the neighbourhood of the fixed conductor."—*Nichol's Cyclopædia*, second edition, art. "Electricity, Atmospheric."

ence of potential exists between the insulated conductor and the point of the air where the issuing stream (whether of water or smoke) ceases to be one continuous conductor, and begins to be a non-conductor or a succession of detached drops, so long will each drop or portion that detaches itself carry off either positive or negative electricity, and thus diminish the difference of potential. The time required to reduce the system to the potential which exists at the point above specified, is practically about half a minute with the water-jet, and from half a minute to a minute or more, according to the strength of the wind, with a match.

The water-dropper is the most convenient collecting apparatus when the observations are taken always in the same place. For portable service, Sir Wm. Thomson employs blotting-paper, steeped in solution of nitrate of lead, dried, and rolled into matches. The portable electrometer carries a light brass rod or wire projecting upwards, to the top of which the matches can be fixed.

92. **Interpretation of Indications.**—We have seen that the collecting apparatus, whether armed with water-jet or burning match, is merely an arrangement for reducing an insulated conductor to the potential which exists at a particular point in the air. An electrometer will then show us the difference between this potential and that of any other given conductor, for example the earth. The earth offers so little resistance to the passage of electricity, that any temporary difference of potential which may exist between different parts of its surface, must be very slight in comparison with the differences of potential which exist between different points in the non-conducting atmosphere above it. As there is no possible method of determining absolute potential, since all electric phenomena would remain unchanged by an equal addition to the potentials of all points, it is convenient to assume, as the zero of potential, that of the most constant body to which we have access, namely the earth; and under the name earth we include trees, buildings, animals, and all other conductors in electrical communication with the soil.

Now we find that, as we proceed further from the earth's surface, whether upwards from a level part of it, or horizontally from a vertical part of it, such as an outer wall of a house, the potential of points in the air becomes more and more different from that of the earth, the difference being, in a broad sense, simply proportional to the distance. Hence we can infer[1] that there is electricity residing on the

[1] By § 54, if ρ denote the quantity of electricity per unit area on an even part of the

surface of the earth, the density of this electricity, at any moment, in the locality of observation, being measured by the difference of potential which we find to exist between the earth and a given point in the air near it. Observations of so-called atmospheric electricity[1] made in the manner we have described, are in fact simply determinations of the quantity of electricity residing on the earth's surface at the place of observation. The results of observations so made are however amply sufficient to show that electricity residing in the atmosphere is really the main cause of the variations observed. A charged cloud or body of air induces electricity of the opposite kind to its own on the parts of the earth's surface over which it passes; and the variations which we find to occur in the electrical density at the parts of the surface where we observe, are so rapid and considerable, that no other cause but this seems at all adequate to account for them. We may therefore safely assume that the difference of potential which we find, in increasing our distance from the earth, is mainly due to electricity induced on the surface of the earth by opposite electricity in the air overhead.

As electrical density is greater on projecting parts of a surface than on those which are plane or concave, we shall obtain stronger indications on hills than in valleys, if our collecting apparatus be at the same distance from the ground in both cases. Under a tree, or in any position excluded from view of the sky, we shall obtain little or no effect.

93. Results of Observation.—The first regular series of observations taken with Sir Wm. Thomson's instruments which were published[2] consisted of two years' continuous observations with self-recording apparatus at Kew Observatory, and two years' observations, at three stated times daily, and at other irregular times, at Windsor in Nova Scotia (lat. 45° N.). The electrometer used at Kew was an earlier

earth's surface, the force in the neighbouring air is $4\pi\rho$. This must be equal to the change of potential in going unit distance (§ 47). If potential increases positively, ρ is negative.

[1] No good electrical observations have yet been made in balloons, and very little is known regarding the distribution of electricity at different heights in the air. A method of gauging this distribution by balloon observations is suggested by the principles of § 52, which show that, when the lines of force are vertical, and the tubes of force consequently cylindrical, the difference of electrical force at different heights is proportional to the quantity of electricity which lies between them.

[2] The observations at Windsor, N.S., and at Kew, are described in three papers by the editor of this work, *Proc. R. S.*, June 1863, January 1865, and *Trans. R. S.*, December 1867. Dellmann's observations at Kreuznach, which were taken with apparatus devised by himself, are described in *Phil. Mag.*, June 1858. Quetelet's observations (taken with Peltier's apparatus) are described in his volume *Sur le Climat de la Belgique* (Brussels, 1849).

form of the quadrant electrometer already described; and the autographic registration was effected by throwing the image of a bright point (a small hole with a lamp behind it) upon a sheet of photographic paper drawn upwards by clock-work, whereas the movements of the image, formed by means of the mirror attached to the needle, were horizontal. The curves thus obtained give very accurate information respecting the potential of the air at the point of observation, when of moderate strength; but fail to record it when of excessive strength, as the image on these occasions passed out of range. The Windsor observations were taken with the cage-electrometer, of which two forms were employed, one being much more sensitive than the other. The more sensitive form was usually employed. When the potential became inconveniently strong, the first step was to shorten the discharging pipe by screwing off some of its joints. This reduced the strength of potential in about the ratio of 3:1; but even this reduction was often not enough for the more sensitive instrument, and on such occasions the other (which was intended as a portable electrometer) was employed instead. As the ratio of the indications of the two instruments was known, a complete comparison of potentials in all weathers was thus obtained. The results are as follows:

Employing a unit in terms of which the average fine-weather potential for the year was +4, the potential was seldom so weak as 1, though on rare occasions it was for a few minutes as low as 0·1. In wet weather, especially with sudden heavy showers, the potential was often as strong as ±20 to ±30, and it was fully as strong during hail. With snow, the average strength was about the same as with heavy rain, but it was less variable, and the sign was almost always positive. Occasionally, with high wind accompanying snow, during very severe frost, it was from +80 to +100, or even higher. With fog, it was always positive, averaging about +10. In thunderstorms it frequently exceeded ±100, and on a few occasions exceeded −200. There was usually a great predominance of negative potential in thunder-storms. Change of sign was a frequent accompaniment of a flash of lightning or a sudden downpour of rain. At all times, there was a remarkable absence of steadiness as compared with most meteorological phenomena, wind-pressure being the only element whose fluctuations are at all comparable, in magnitude and suddenness, with those of electrical potential. Even in fine weather, its variations during two or three minutes usually amount to as much

as 20 per cent. In changeable and stormy weather they are much greater; and on some rare occasions it changes so much from second to second that, notwithstanding the mitigating effect of the collecting process, which eases off all sudden changes, the needle of the electrometer is kept in a continual state of agitation.

94. Annual and Diurnal Variations.—Observations everywhere[1] concur in showing that the average strength of potential is greater in winter than in summer; but the months of maxima and minima appear to differ considerably at different places. The chief maximum occurs in one of the winter months, varying at different places from the beginning to the end of winter; and the chief minimum occurs everywhere in May or June. Both Kew and Windsor show distinctly two maxima in the year, but Brussels, and apparently Kreuznach, show only one. The ratio of the highest monthly average to the lowest is at Kew about 2·5, at Windsor 1·9, and at Kreuznach 2·0.

The Kew observations, being continuous, are specially adapted to throw light on the subject of diurnal variation. They distinctly indicate for each month two maxima, which in July occur at about 8 A.M. and 10 P.M., in January about 10 A.M. and 7 P.M., and in spring and autumn about 9 and 9. The result of the Brussels observations is about the same.

95 Causes of Atmospheric Electricity.—Various conjectures have been hazarded regarding the sources of atmospheric electricity; but little or no certain knowledge has yet been obtained on this subject. Evaporation has been put forward as a cause, but, as far as laboratory experiments show, whenever electricity has been generated in connection with evaporation, the real source has been friction, as in Armstrong's hydro-electric machine. The chemical processes involved in vegetation have also been adduced as causes, but without any sufficient evidence. It is perhaps not too much to say that the only natural agent which we know to be capable of electrifying the air is the friction of solid and liquid particles against the earth and against each other by wind. The excessively strong indications of electricity observed during snow accompanied by high wind, favour the idea that this may be an important source.

Without knowing the origin of atmospheric electricity, we may, however, give some explanation of the electrical phenomena which

[1] The remarks in this section express the results of observation at places all of which are in the north temperate zone.

occur both in showers and in thunder-storms. Very dry air is an excellent non-conductor; very moist air has, on the other hand, considerable conducting power. When condensation of vapour takes place at several centres, a number of masses of non-conducting matter are transformed into conductors, and the electricity which was diffused through their substance passes to their surfaces. These separate conductors influence one another. If one of them is torn asunder while under influence, its two portions may be oppositely charged; and if rain falls from the under surface of a cloud which is under the influence of electricity above it, the rain which falls may have an opposite charge to the portion which is left suspended.

Lord Rayleigh has found that drops of water which are slightly electrified have a tendency to coalesce, whereas unelectrified drops usually rebound after collision.[1] It is probable that electricity may in this way give rise to showers.

The coalescence of small drops to form large ones, though it increases the electrical density on the surfaces of the drops, does not increase the total quantity of electricity, and therefore (§ 56) cannot directly influence the observed potential.[2]

Fig. 57.—Electric Hail.

Thunder-storms and other powerful manifestations of atmospheric electricity seem to be accompaniments of very sudden and complete condensation which gives unusually free scope to the causes of irregular distribution just indicated.

96. Hail.—Hail has sometimes been ascribed to an electrical origin, and a singular theory was devised by Volta to account for the supposed fact that hailstones are sustained in the air. He imagined

[1] The observation was made upon jets discharged from a nozzle directed upwards, and it was found that connecting the discharging vessel with one pole of a single Grove's cell (the other pole being to earth) was sufficient to produce coalescence of the drops. *Proc. Royal Society*, Feb. 27, 1879.

[2] In several modern works there is an attempt to prove that the coalescence of small

that two layers of cloud, one above the other, charged with opposite electricities, kept the hailstones continually moving up and down by alternate attraction and repulsion. An experiment called *electric hail* is sometimes employed to illustrate this idea. Two metallic plates are employed (Fig. 57), the lower one connected with the earth, and the upper one with the conductor of the electrical machine; and pith-balls are placed between them. As the machine is turned, the balls fly rapidly backwards and forwards from one plate to the other.

97. Waterspouts. — Waterspouts, being often accompanied by

Fig. 58.—Waterspouts.

strong manifestations of electricity, have been ascribed by Peltier and others to an electrical origin; but the account of them given in the subjoined note appears more probable.[1]

drops to form large ones must increase their potential. The reasoning employed regards the potential of a conductor as depending only on its own charge. But it is easy to show that, in computing the potential of one drop in a group of millions, the part depending on its own charge is negligible; and as regards the terms depending on the charges of the other drops, they involve volume-densities, not surface-densities, and are unchanged by coalescence.

[1] "On account of the centrifugal force arising from the rapid gyrations near the centre of a tornado, it must frequently be nearly a vacuum. Hence when a tornado passes over a building, the external pressure, in a great measure, is suddenly removed, when the atmosphere within, not being able to escape at once, exerts a pressure upon the interior, of perhaps nearly fifteen pounds to the square inch, which causes the parts to be thrown in

MAGNETISM.

CHAPTER X.

GENERAL STATEMENT OF FACTS AND LAWS.

98. Magnets, Natural and Artificial.—Natural magnets, or *lodestones*, are exceedingly rare, although a closely allied ore of iron, capable of being strongly acted on by magnetic forces, and hence called *magnetic iron-ore*, is found in large quantity in Sweden and elsewhere. Artificial magnets are usually pieces of steel, which have been permanently endowed with magnetism by methods which we shall hereafter describe. Magnets are chiefly characterized by the property of attracting iron, and by the tendency to assume a particular orientation when freely suspended.

99. Force Greatest at the Ends.—The property of attracting iron is very unequally manifested at different points of the surface of a magnet. If, for example, an ordinary bar-magnet be plunged in iron-filings, these cling in large quantities to the terminal portions, and leave the middle bare, as in the lower diagram of Fig. 59. If

every direction to a great distance. For the same reason, also, the corks fly from empty bottles, and everything with air confined within explodes. When a tornado happens at sea, it generally produces a waterspout. This is generally first formed above, in the form of a cloud shaped like a funnel or inverted cone. As there is less resistance to the motions in the upper strata than near the earth's surface, the rapid gyratory motion commences there first. . . . This draws down the strata of cold air above, which, coming in contact with the warm and moist atmosphere ascending in the middle of the tornado, condenses the vapour and forms the funnel-shaped cloud. As the gyratory motion becomes more violent, it gradually overcomes the resistances nearer the surface of the sea, and the vertex of the funnel-shaped cloud gradually descends lower, and the imperfect vacuum of the centre of the tornado reaches the sea, up which the water has a tendency to ascend to a certain height, and thence the rapidly ascending spiral motion of the atmosphere carries the spray upward, until it joins the cloud above, when the waterspout is complete. The upper part of a waterspout is frequently formed in tornadoes on land. When tornadoes happen on sandy plains, instead of waterspouts they produce the moving pillars of sand which are often seen on sandy deserts."—W. Ferrel, in *Mathematical Monthly*.

the magnet is very thick in proportion to its length, we may have filings adhering to all parts of it, but the quantity diminishes rapidly towards the middle. The name *poles* is used, in a somewhat loose sense, to denote the two terminal portions of a magnet, or to denote two points, not very accurately defined, situated in these portions. The middle portion, to which the filings refuse to adhere, is called *neutral*.

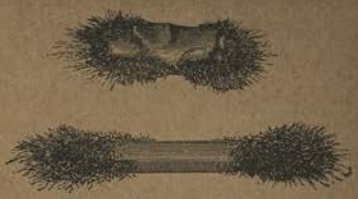

Fig. 59.—Magnets dipped in Filings.

100. Lines Formed by Filings.—If a sheet of card is laid horizontally upon a magnet, and wrought-iron filings are sifted over it, these will, with the assistance of a few taps given to the card, arrange

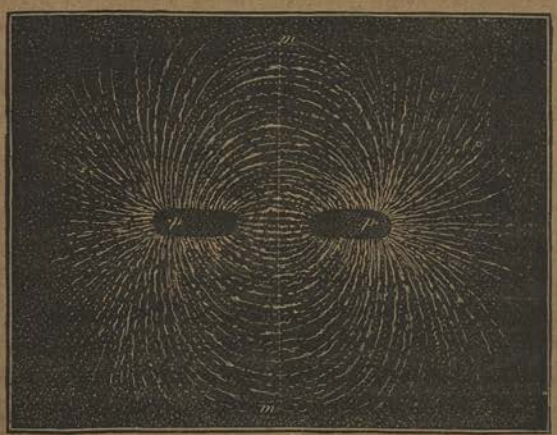

Fig. 60.—Magnetic Curves.

themselves in a system of curved lines, as shown in Fig. 60. These lines give very important indications both of the direction and intensity of the force produced by the magnet at different points of the space around it.[1] They cluster very closely about the two poles $p\ p$, and thus indicate the places where the force is most intense.

101. Curve of Intensities.—Some idea may be obtained of the relative intensities of magnetic force at different points in the length of

[1] The lines formed by the filings may be called the lines of *effective force for particles only free to move in the plane of the card*. The lines of total force cut the card at various angles, and are at some places perpendicular to it, as shown by the filings standing on end. For the definition of lines of magnetic force, see § 109.

a magnet, by measuring the force required to detach an iron ball at various points. Better determinations can be obtained by counting the number of vibrations made by a small magnetized needle when suspended opposite different parts of the bar, the bar being in a vertical position, and the vibrations of the needle being horizontal. The intensity of the force is nearly as the square of the number of vibrations; on the same principle that the force of gravity at different places is proportional to the square of the number of vibrations of a pendulum. Both these methods of determination were employed by Coulomb, who was the first to make magnetism an accurate science; and the results which he obtained are represented by the curve of intensities A M B (Fig. 61). M is the middle of the bar, O one end of it, and the ordinates of the curve (that is, the distance of its points from the line O X) represent the intensities of force at the different points in its length. The curve was constructed from observations of the force at several points in the length; but in dealing with the observation made opposite the very end, the force actually observed was multiplied by 2. Perfect symmetry was found between the intensities over the two halves of the length. In the figure we have inverted the curve for one-half, in order to indicate an opposition of properties, which we shall shortly have to describe. The curves of intensities for two magnets of different sizes but of the same form are usually similar.

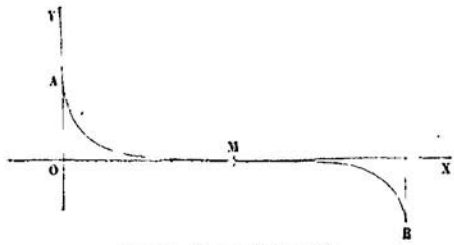

Fig. 61.—Curve of Intensities.

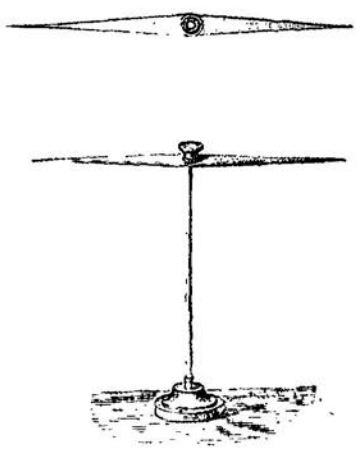
Fig. 62.—Magnetized Needle.

102. **Magnetic Needle.**—Any magnet freely suspended near its centre is usually called a *magnetic needle*, or more properly a *magnetized needle*. One of its most usual forms is that of a very elon-

gated rhombus of thin steel, having, very near its centre, a concavity or *cup* by means of which it can be balanced on a point. When it is thus balanced horizontally, it does not, like a piece of ordinary matter, remain in equilibrium in all azimuths[1] but assumes one particular direction, to which it always comes back after displacement. In this position of stable equilibrium, one of its ends points to magnetic north, and the other to magnetic south, which differ in general by several degrees from geographical (or true) north and south. This is the principle on which compasses are constructed.

103. Declination.—The difference between magnetic and true north, or the angle between the magnetic meridian and the geographical meridian, is called *magnetic declination.*[2] It is very different at different places, and at a given place undergoes a gradual change from year to year, besides smaller changes, backwards and forwards, which are continually taking place. At Greenwich, at the present time, its value is about 17° W., that is, magnetic north is west of true north by this amount.

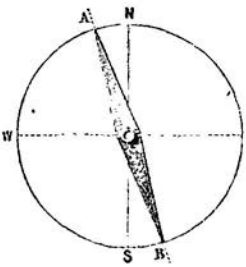

Fig. 63.—Declination.

104. Inclination or Dip.—If, before magnetizing a needle, we mount it on an axis passing through its centre of gravity, and support the ends of the axis, as in Fig. 64, by a thread without torsion, the needle will remain in equilibrium in any position in which it may be placed. If it be then magnetized, it will no longer be indifferent, but will place itself in a particular vertical plane called the magnetic meridian, and will take a particular direction in this plane. This direction is not horizontal, but inclined, generally at a considerable angle, to the horizon; and this angle is called *dip* or

[1] All lines in the same vertical plane are said to have the same *azimuth*. Azimuthal angles are angles between vertical planes, or between horizontal lines. The azimuth of a line when stated numerically, is the angle which the vertical plane containing it makes with a vertical plane of reference, and this latter is usually the plane of the meridian. Some readers may be glad to be reminded that by the plane of the *meridian* is meant a vertical plane passing through the place of observation, and through or parallel to the earth's axis. A horizontal line in this plane is a meridian line. The *magnetic meridian* is the vertical plane in which a magnetized needle, when freely suspended, tends to place itself.

[2] The nautical name for magnetic declination is *variation;* but it is most inconvenient and confusing to denote the element itself by the same name as the variations of the element.

inclination. Its value at Greenwich is about 67½°, the end which points to the north pointing at the same time downwards. In the northern hemisphere generally, it is the north end of the needle which dips, and in the southern hemisphere it is the end which points south.

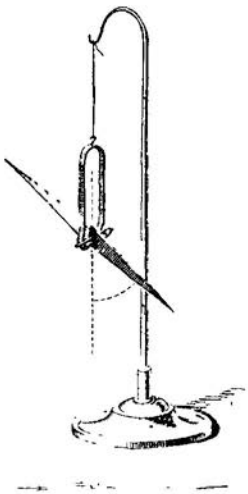

Fig. 64.—Dip.

It follows that, if a magnetized needle is to be balanced in a horizontal position, the point or axis of support must not be in the same vertical with the centre of gravity, but must be between the centre of gravity and the end which tends to dip. Needles thus balanced, as in the ordinary mariner's compass, are called *declination needles.*

105. Mutual Action of Poles.—On presenting one end of a magnet to one end of a needle thus balanced, we obtain either repulsion or attraction, according as the pole which is presented is similar or dissimilar to that to which it is presented. *Poles of contrary name attract each other; poles of the same name repel each other.*

This property furnishes the means of distinguishing a body which is merely magnetic (that is, capable of temporary magnetization) from a permanent magnet. The former, a piece of soft iron for example, is attracted by either pole of a magnet; while a body which has received permanent magnetization has, in ordinary cases, two poles, of which one is attracted where the other is repelled. Magnetic attractions and repulsions are exerted without modification through any body which may be interposed, provided it be not magnetic.

106. Names of Poles.—The phenomena of declination and inclination above described, evidently require us to regard the earth, in a broad sense, as a magnet, having one pole in the northern and the other in the southern hemisphere. Now since poles which attract one another are dissimilar, it follows that the magnetic pole of the earth which is situated in the northern hemisphere is *dissimilar* to that end of a magnetized needle which points to the north. Hence

there is some doubt as to which end of a needle or magnet is to be called the *north* pole. We shall avoid this ambiguity by calling that end of it which seeks the north, the *north-seeking* end or pole, and the other the *south-seeking* end or pole. Sir Wm. Thomson calls the north-seeking pole the *south* pole, and the other the *north* pole, because the former is similar to the south, and the latter to the north pole of the earth. In like manner most French writers call the north-seeking pole of a needle the *austral*, and the other the *boreal* pole. Popular usage in this country calls the north-seeking end the *north*, and the other the *south* pole, a nomenclature which introduces great confusion whenever we have to reason respecting the earth regarded as a magnet. Faraday, to avoid the ambiguity which has attached itself to the names north and south pole, calls the north-seeking end the *marked*, and the other the *unmarked* pole. Airy, for a similar reason, employed, in his *Treatise on Magnetism*, the distinctive names *red* and *blue* to denote respectively the north-seeking and south-seeking ends, these names, as well as those employed by Faraday, being purely conventional, and founded on the custom of marking the north-seeking end of a magnet with a transverse notch or a spot of red paint.

107. Magnetic Induction.—When a piece of iron is in contact with a magnet, or even when a magnet is simply brought near it, it becomes

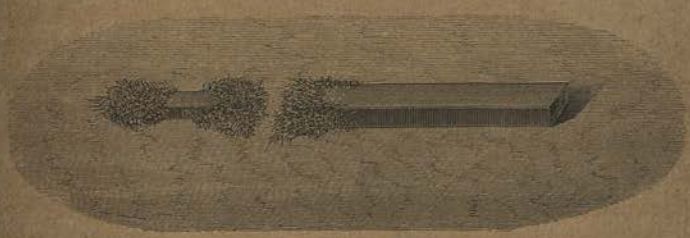

Fig. 65.—Induced Magnetism.

itself for the time a magnet, with two poles and a neutral portion between them. If we scatter filings over the iron, they will adhere to its ends, as shown in Fig. 65. If we take away the influencing magnet, the filings will fall off, and the iron will retain either no traces at all or only very faint ones of its magnetization. If we apply similar treatment to a piece of steel, we obtain a result similar in some respects, but with very important differences in degree. The steel, while under the influence of the magnet, exhibits much weaker

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effects than the iron; it is much more difficult to magnetize than iron, and does not admit of being so powerfully magnetized; but, on the other hand, it retains its magnetization after the influencing magnet has been withdrawn. This property of retaining magnetism when once imparted has been (somewhat awkwardly) named *coercive force*. Steel, especially when very hard, possesses great coercive force; iron, especially when very pure and soft, scarcely any.

In magnetization by influence, which is also called *magnetic induction*, it will be found, on examination, that the pole which is next the inducing pole is of contrary name to it; and it is on account of the mutual attraction of dissimilar poles that the iron is attracted by the magnet. The iron can, in its turn, support a second piece of iron; this again can support a third, and so on through many steps. A magnetic chain can thus be formed, each of the component pieces having two poles. An action of this kind takes place in the clusters of filings which attach themselves to one end of a magnetized bar, these clusters being composed of numerous chains of filings.

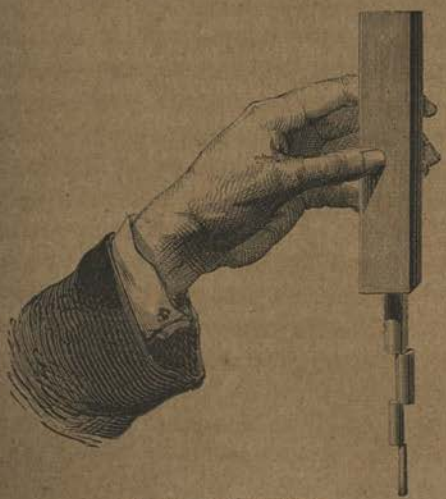


Fig. 66.—Magnetic Chain.

In comparing the phenomena of magnetic induction with those of electrical induction, we find both points of resemblance and points of difference. In the case of electricity, if the influencing and influenced body are allowed to come in contact, the former loses some of its own charge to the latter. In the case of magnetism there is no such loss, a magnet after touching soft iron is found to be as strongly magnetized as it was before.

108. Effect of Rupture on a Magnet.—If a magnet is broken into any number of pieces, every piece will be a complete magnet with poles of its own. In the case of an ordinary bar-magnet or needle, the similar poles of the pieces will all be turned the same way, as

in Fig. 67, which represents a magnet A B broken into four pieces. The ends a, a, a, a are of one name, and the ends b, b, b, b of the opposite name.

109. Imaginary Magnetic Fluids: Magnetic Potential.—All mutual forces between magnets can be reduced to attractions and repulsions

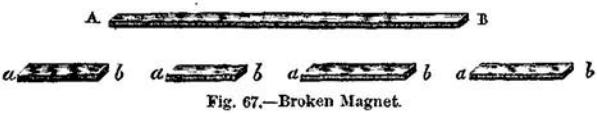


Fig. 67.—Broken Magnet.

between different portions of two imaginary fluids, called *magnetisms*,[1] one of which may be called *positive* and the other *negative*. Neither magnetism can exist apart from the other; every magnet possesses equal quantities of both; quantity being measured by force of attraction or repulsion at given distance, just as in the case of electricity, like portions repelling, and unlike portions attracting each other inversely as the square of the distance. Equal quantities of the two magnetisms, when coexisting at the same place, produce no resultant effect, and may be regarded as destroying each other. Any magnetism which is not thus cancelled is called *free*.

With reference to these imaginary fluids, *magnetic potential* can be defined in the same way as electrical potential (§§ 46, 47, 56), and magnetic *lines of force* possess the same properties as electrical lines of force (§§ 48, 49, 52, 53). The direction of magnetic force at a point can either be defined as the direction in which a pole of a magnet would be urged if brought to the point, or as the direction in which a small magnetized needle, if brought to the point and balanced at its centre of gravity, would place its line of poles; and lines of magnetic force are lines to which this direction is everywhere tangential. It is important to remark that a linear piece of soft iron, though it sets its length along a line of force, does not travel along a line of force, but deviates towards the concave side. On tapping the card represented in Fig. 60, it will be found that filings placed on the line $m\ m$ move along that line, and therefore at right angles to the lines of force.

The force which is specified by magnetic "lines of force" is the force which *one pole* of a permanent magnet would experience; and

[1] Poisson, following Coulomb, spoke of *two magnetic fluids*, and laid down a theory of their action. Sir W. Thomson, avoiding the hypothetical parts of Poisson's theory, speaks of *imaginary magnetic matter* of two dissimilar kinds.

it is the same in intensity, but opposite in direction, for dissimilar poles. The two poles of a small magnet (temporary or permanent) in the position of stable equilibrium as regards rotation, are pulled in nearly opposite directions; and the force which tends to produce movement of translation is the resultant of these two nearly opposite pulls. The direction of this resultant for a small sphere is the direction in which the intensity of the field increases most rapidly.[1]

110. Specification of Magnetization.—A piece of steel is said to be *uniformly magnetized*, if equal and similar portions, cut in parallel directions from all parts of it, are precisely alike in their magnetic properties.

If a piece of magnetized steel be suspended at its centre of gravity, so as to be free to turn all ways about it, the effect of the earth's magnetism upon it consists in a tendency for a particular line through this centre of gravity to take a determinate direction, which is the direction of terrestrial magnetic force. When the line is placed in any other position, the couple tending to bring it back is proportional to the sine of the angle between the two positions, and is the same for all directions of deviation. The line which possesses this property is the *magnetic axis* of the body, and the name is sometimes given to all lines parallel to it. If the piece of steel be uniformly magnetized, this axis is the direction of magnetization; or the *direction of magnetization is the common direction of all those lines which tend to place themselves along lines of force* in a field[2] where the lines of force are parallel.

111. Ideal Simple Magnet: Thin Bar, uniformly and longitudinally Magnetized.—The mutual actions of magnets admit of very accurate expression when the magnets are very thin in comparison with their length, uniform in section, and uniformly magnetized in the direction of their length. Such bars, which may be called *simple magnets*, behave as if their forces resided solely in their ends, which may

[1] The following considerations will help to explain the direction of motion:—

Suppose a small piece of iron wire bent to coincide with a portion of a line of force. It will be pulled at both ends, and will therefore move towards the concave side.

Also if the force is stronger at one end of the piece than at the other, the stronger pull will prevail.

Both these actions combine to move the iron from weaker to stronger parts of the field.

[2] A *field of force* is any region of space traversed by lines of force; or, in other words, any region pervaded by force of attraction or repulsion. A *magnetic field* is any region pervaded by magnetic force. All space in the neighbourhood of the earth is a magnetic field, and within moderate distances the lines of force in it may be regarded as parallel, unless artificial magnets, pieces of iron, or other sources of disturbance are present.

therefore in the strictest sense be called their poles. The two poles of any one such bar are equal in strength; that is to say, one of them attracts a pole of another simple magnet with the same force with which the other repels it at the same distance. They have no free magnetism except at the two ends, and the quantities at the two ends are equal but of opposite sign. The same number which denotes the quantity of free magnetism at either pole, denotes the *strength of the pole*, or, as it is often called, the *strength of the magnet*. Its definition is best expressed by saying that the force between a pole of one simple magnet and a pole of another, is the product of their strengths divided by the square of the distance between them.[1]

The force which a pole of a simple magnet experiences in a magnetic field, is the *product of the strength of the pole and the intensity of the field*. This rule applies to the force which a pole experiences from the earth's magnetism, the intensity of the field being in this case the intensity of terrestrial magnetic force; and, from the uniformity of the field, the forces on the two poles are in this case equal, constituting a couple, whose arm is the line joining the poles multiplied by the sine of the angle which this line makes with the lines of force.

The product of the line joining the two poles by the strength of either pole is called the *moment of the magnet*, and it is evident, from what has just been said, that the continued product of *the moment of the magnet, the intensity of terrestrial magnetic force, and the sine of the angle between the length of the magnet and the lines of force*, is equal to the moment of the couple which the earth's magnetism exerts upon the magnet.

112. Compound Magnet of Uniform Magnetization.—Any magnet which is not a simple magnet in the sense defined in § 111 may be called a *compound magnet*. It is convenient to define the moment of a compound magnet by the condition stated in the concluding words of that section, so that the moments of different magnets, whether simple or compound, may be compared by comparing the couples exerted on them by terrestrial magnetism when their axes are equally inclined to the lines of force.

If a number of simple magnets of equal strength be joined end to

[1] We here, and throughout the remainder of this chapter, ignore the existence of induction, which, however, is not altogether absent even in the hardest steel. The effect of induction is always to favour attraction. The attractions will therefore be somewhat stronger, and the repulsions somewhat weaker, than our theory supposes.

end, with their similar poles pointing the same way, there will be mutual destruction of the two magnetisms at every junction, and the system will constitute one simple magnet of the same strength as any one of its components; but its moment will evidently be the sum of their moments.

If any number of simple magnets be united, either end to end or side to side, provided only that they are parallel, and have their similar poles turned the same way, the resultant couple exerted upon the whole system by terrestrial magnetism will be the sum of the separate couples exerted by it on each magnet, and the moment of the system will be the sum of the moments of its parts. But any piece of uniformly magnetized material may be regarded as being thus built up, and hence, if different portions be cut from the same uniformly magnetized mass, their moments will be simply proportional to their volumes. The quotient of moment by volume, for any uniformly magnetized mass, is called *intensity of magnetization*.

113. Actual Magnets.—The definitions and laws of simple magnets are approximately applicable to actual magnets, when magnetized in the usual manner.

If an actual bar-magnet in the form of a rectangular parallelepiped were magnetized with perfect uniformity, and in the direction of its length, it might be regarded as made up of a number of simple magnets laid side by side, and its behaviour would be represented by supposing a complete absence of magnetic fluid from all parts of it except its *ends* (in the strict mathematical sense). One of these terminal faces would be covered with positive, and the other with negative fluid, and if the magnet were broken across at any part of its length, the quantities of positive and negative fluid on the broken ends would be the same as on the ends of the complete magnet. The observed fact that magnets behave as if the fluids were distributed through a portion of their substance in the neighbourhood of the ends, and not confined to the ends strictly so called, indicates a falling off in magnetization towards the extremities, and is approximately represented by conceiving of a number of short magnets laid end to end, and falling off in strength towards the two extremities of the series.[1]

[1] Thus the last magnet at the positive end being weaker than its neighbour, its negative pole will be weaker than its neighbour's positive pole, so that there will be an excess of positive fluid at this junction. Similar reasoning applies to all the junctions near the ends. There will be an excess of positive fluid at all junctions near the positive end, and an excess of negative at all junctions near the negative end.

The resultant force due to the imaginary magnetic fluids which are distributed through the terminal portions of an actual bar-magnet is, in the case of actions at a great distance, sensibly the same as if the two portions of fluid were collected at their respective centres of gravity. These two centres of gravity are the poles of the magnet for all actions between the magnet and other magnets at a great distance, and more especially between the magnet and the earth.

CHAPTER XI.

EXPERIMENTAL DETAILS.

114. The Earth's Force simply Directive.—The forces which produce the orientation of a magnet depend upon causes of which very little is known. They are evidently connected in some way with the earth, and are accordingly referred to TERRESTRIAL MAGNETISM. We have already stated (§ 110) that the combined effect of the forces exerted by terrestrial magnetism upon a magnetized needle is equivalent to a couple tending to turn the needle into a particular direction, and (§ 113) that in the case of needles magnetized in the ordinary way, there are two definite points or poles (near the two ends of the needle) which may be regarded as the points of application of the two equal forces which constitute the couple.

The fact that terrestrial magnetic force simply tends to turn the needle, and not to give it a movement of translation, in other words, that the resultant *force* (as distinguished from *couple*) is zero, is completely proved by the two following experiments:—

(1) If a bar of steel is weighed before and after magnetization, no change is found in its weight. This proves that the vertical component is zero.

(2) If a bar of steel, not magnetized, is suspended by a long and fine thread, the direction of the thread is of course vertical. If the bar is then magnetized, the direction of the thread still remains vertical. The most rigorous tests fail to show any change of its position. This proves that the horizontal component is zero, a conclusion which may be verified by floating a magnet on water by means of a cork. It will be found that there is no tendency to move across the water in any particular direction.

115. Horizontal, Vertical, and Total Intensities.—If S denote the strength of a magnet, and I the intensity of terrestrial magnetic force,

each pole of the magnet experiences a force S I, and if L denote the distance between the poles (often called the length of the magnet), the distance between the lines of action of these two parallel and opposite forces may have any value intermediate between L and zero, according to the position in which the needle is held. It will be zero when the line of poles is that of the dipping-needle; it will be L when the line of poles is perpendicular to the dipping-needle; and will be L sin α when the line of poles is inclined at any angle α to the dipping-needle.

The force SI upon either pole of the magnet acts in the direction of the dipping-needle; in other words, in the direction of the lines of force due to terrestrial magnetism. Let δ denote the dip, that is the inclination of the lines of force to the horizon, then the force SI can be resolved into SI cos δ horizontal, and SI sin δ vertical. Hence the horizontal and vertical intensities H and V are connected with the total intensity and dip I and δ by the two equations

$$H = I \cos \delta \quad , \quad V = I \sin \delta \qquad (1)$$

which are equivalent to the following two

$$\frac{V}{H} = \tan \delta \quad , \quad V^2 + H^2 = I^2. \qquad (2)$$

116. Torsion-balance.—Coulomb, in investigating the laws of the mutual action of magnets, employed a torsion-balance scarcely differing from that which he used in his electrical researches. The suspending thread carried, at its lower end, a stirrup on which a magnetized bar was laid horizontally. The torsion-head was so adjusted that one end of the magnet was opposite the zero of the divisions on the glass case when the supporting thread was without torsion. In order to effect this adjustment, the magnet was first suspended by a thread whose torsional power was inconsiderable, so that the magnet placed itself in the magnetic meridian. The case was then turned till its zero came to this position. The torsionless thread was then replaced by a fine metallic wire, and the magnet was replaced by a copper bar of the same weight. The head was then turned till this bar came into the magnetic meridian, and lastly the magnet was put in the place of the bar.

Fig. 68 shows the arrangement adopted for observing the repulsion or attraction between one pole of the suspended magnet and one pole of another magnet placed vertically. Before the insertion of the latter, the suspended magnet was acted on by no horizontal

forces except the horizontal component of terrestrial magnetism and the torsion of the wire. It was then found that the torsion requisite for keeping the magnet in any position was proportional to the sine of the displacement from the meridian.

This result is evidently in accordance with the principles stated above, for the two equal horizontal forces on the two poles being

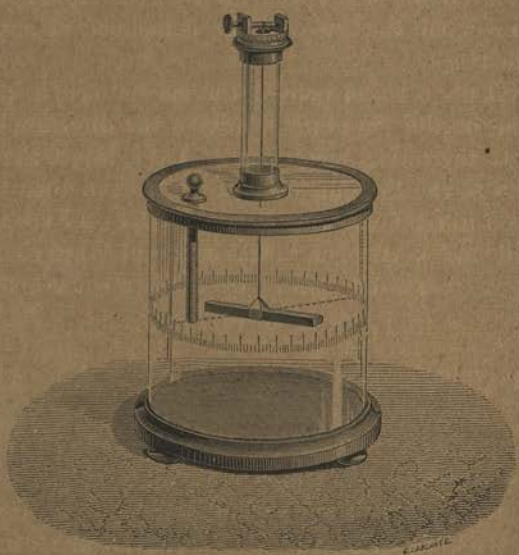


Fig. 68.—Torsion-balance.

constant for all positions, the couple which they compose is proportional to the distance between their lines of action, and this distance is evidently L sin θ, L denoting the constant distance between the poles, and θ the deviation of the needle from the meridian.

117. Measurement of Declination.—Magnetic declination has been observed with several different forms of apparatus.

At sea, the most common method of determining it has consisted in observing the magnetic bearing of the rising or setting sun, and comparing this with its true bearing as calculated by a well-known astronomical method.

At fixed observatories more accurate methods of observation are employed. Fig. 69. shows the arrangement adopted at Greenwich. A bar-magnet B carries at one end a cross of fine threads C, and at the other a lens D, the distance between them being equal to the

focal length of the lens, thus forming a kind of inverted telescope, whose line of collimation is the line joining the cross to the optical centre of the lens. The bar is suspended by means of a stirrup from a torsionless thread, and sets its magnetic axis in the magnetic meridian. The telescope E, with theodolite mounting,[1] is stationed opposite the end which carries the lens, and is so adjusted at each observation that its line of collimation is parallel to that of the

Fig. 69.—Declination Magnet.

inverted telescope carried by the magnet, an adjustment which is identified by seeing the cross C coincident with a similar cross fixed in the interior of the telescope E. When the observation has been made with the magnet in one position, it must be repeated with the magnet turned upside down as shown in the figure. Error of parallelism between the magnetic axis of the bar and the line of collimation of the inverted telescope which it carries, will affect these two observations to the same extent in opposite directions, and will therefore disappear from their mean. The readings are taken on the horizontal circle at the base of the instrument, and astronomical observations must be made once for all to determine what reading corresponds to the geographical meridian.

Another very accurate method consists in rigidly attaching to the

[1] A *theodolite* consists of a telescope mounted so as to have independent motions in azimuth and altitude, the amounts of these motions being indicated by divided circles or arcs of circles. It does not differ essentially from the larger instrument called the *altazimuth*.

bar, instead of the lens and cross, a small vertical mirror. This can either be viewed through a telescope, so as to show the reflection of a horizontal scale of equal parts, which will appear to travel across the field of view of the telescope as the magnet turns, or it can be employed to throw the image of a spot of light either upon a screen viewed by the observer, or still better upon photographic paper drawn by clock-work, which leaves a permanent record of continuous changes. Both these methods of employing mirrors for the observation of small movements of rotation are now extensively employed in many applications. They appear to have been first introduced by Gauss, who employed them for the purpose which we are now considering.

118. Measurement of Dip.—The dip-circle or inclination compass is represented in Fig. 70. It consists essentially of a magnetized needle, very accurately and delicately mounted on a horizontal axis through its centre of gravity, in the centre of a vertical circle on which the positions of the two ends of the needle can be read off. This circle can be turned with the needle into any azimuth, the amount of rotation being indicated by a horizontal circle. It is obvious that, if the vertical circle is placed in the plane of the magnetic meridian, the needle, being free to move in this plane, will directly indicate the dip. On the other hand, if the vertical circle is placed in a plane perpendicular to the magnetic meridian, the horizontal component of terrestrial magnetism is prevented from moving the needle, which, accordingly, obeys the vertical component only, and takes a vertical position. In intermediate positions of the vertical circle, the needle will assume positions intermediate between the vertical and the true angle of dip. In fact, if θ be the angle which the plane of the vertical circle makes with the magnetic meridian, the component $H \sin \theta$ of terrestrial magnetism, being perpendicular to this plane, merely tends to produce pressure against the supports, and the horizontal component influencing the position

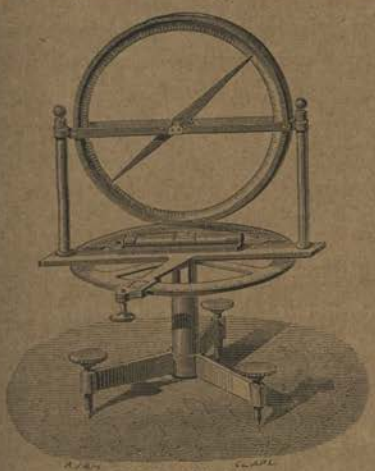


Fig. 70.—Dip-circle.

of the needle is only H cos θ, which lies in the plane of the circle. As none of the vertical force is destroyed, the tangent of the apparent dip will be $\frac{V}{H \cos \theta} = \frac{\tan \delta}{\cos \theta}$. The most accurate method of setting the vertical circle in the magnetic meridian consists in first adjusting it so that the needle takes a vertical position, and then turning it through 90°.

The instrument having thus been set, and a reading taken at each end of the needle, it should be turned in azimuth through 180°, and another pair of readings taken. By employing the mean of these two pairs of readings, several sources of error are eliminated, including non-coincidence of the axis of magnetization with the line joining the ends of the needle. One important source of error—deviation of the centre of gravity from the axis of suspension in a direction parallel to the length of the needle, is, however, not thus corrected. It can only be eliminated by remagnetizing the needle in the reverse direction so as to interchange its poles. The mean of the results obtained before and after the reversal of its magnetization will be the true dip.

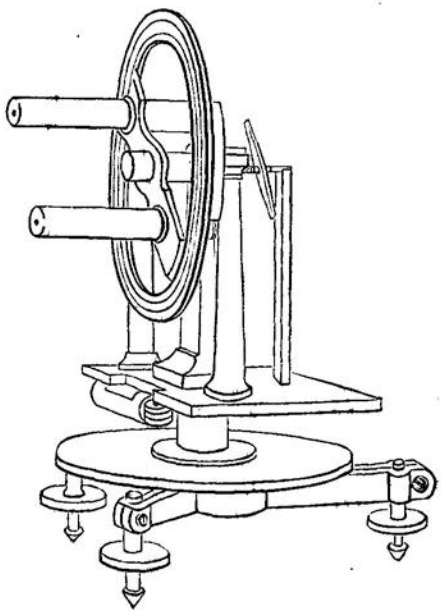


Fig. 71.—Kew Dip-circle.

A better form of instrument, known as the Kew dip-circle, is now employed. Its essential parts are represented in Fig. 71. There is no metal near the needle, and the readings are taken on a circle round which two telescopes travel. In each observation the telescopes are directed to the two ends of the needle.

119. Measurement of Intensity of Terrestrial Magnetic Force.—The complete specification of the earth's magnetic force at any place involves three independent elements. For example, if declination,

dip, and horizontal force are determined by observation, vertical force and total force can be calculated by the formulæ of § 115.

Observations of magnetic force are made either by counting the number of vibrations executed in a given time, or by statical measurements. If a magnet executes small horizontal vibrations under the influence of the earth's magnetism, the square of the number of vibrations in a given time is proportional to $\frac{HM}{\mu}$, H denoting the horizontal intensity, M the moment of the magnet, and μ its moment of inertia about the centre of suspension. Hence it is easy to observe the *variations* of horizontal intensity which occur from time to time, if we can ensure that our magnet itself shall undergo no change, or if we have the means of correcting for such changes as it undergoes. To obtain absolute determinations of horizontal intensity, the following method is employed.

First, observe the time of vibration of a freely-suspended horizontal magnet under the influence of the earth alone,—this will give the *product* of the earth's horizontal intensity and the moment of the magnet.

Secondly, employ this same magnet to act upon another also freely suspended, and thus compare its influence with that of the earth,—this will give the ratio of the same two quantities whose product was found before. Hence the two quantities themselves can easily be computed. (See § 144.)

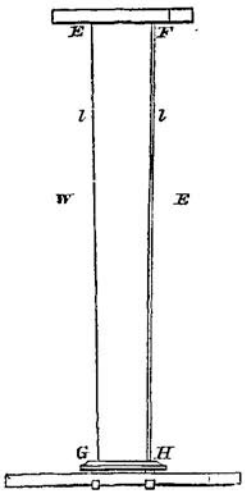


Fig. 72.—Bifilar Magnetometer.

120. Bifilar and Balance Magnetometers.—The changes of horizontal intensity are measured statically by means of the bifilar magnetometer. This consists of a bar-magnet (Fig. 72) suspended by two equal threads which would be in one vertical plane if the bar were unmagnetized; but matters are so arranged that, under the combined action of the pull of the threads, the weight of the bar, and the earth's magnetism, the bar is kept in a position nearly perpendicular to the magnetic meridian. The changes which occur in its position from time to time are due only to changes in the *intensity* of the earth's horizontal force; changes in the direction

of this force, to the extent of a few minutes of angle, having no sensible effect, on account of the near approach to perpendicularity.

Let the distance E F of the upper points of attachment of the threads be $2a$ and the distance G H of the lower points $2b$, and let the angle between the directions of E F and G H be ϕ. Also let W be the weight of the magnet, l the length of each thread, T its tension, and θ its inclination to the vertical. In Fig. 73 E′, F′ are the projections of E, F upon the horizontal plane which contains G, H; and the two lines G H, E′ F′ bisect each other at O, so that we have O E′ $= a$, O G $= b$, E′ O G $= \phi$.

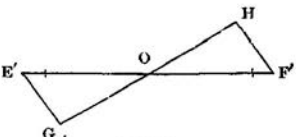


Fig. 73.

G E′ is the projection of one of the threads; we have therefore G E′ $= l \sin\theta$, and the tension of this thread can be resolved into a vertical component T $\cos\theta$ and a horizontal component which acts along G E′, and is T $\sin\theta$ or T $\frac{\text{G E}'}{l}$. The moment of the latter round O is found by multiplying by the perpendicular dropped from O upon G E′. But G E′ multiplied by this perpendicular is double the area of the triangle E′ O G, or is $ab \sin\phi$, hence the moment in question is equal to $\frac{\text{T}}{l} ab \sin\phi$; and as this is due to one thread only, the couple due to the two threads is $\frac{2\text{T}}{l} ab \sin\phi$, which is practically equal to $\frac{\text{W}}{l} ab \sin\phi$, since θ is practically so small that $\cos\theta$ may be taken as unity.

If M be the magnetic moment of the magnet, and H the earth's horizontal intensity, M H will be the horizontal magnetic couple acting on the magnet, if the axis of the latter is perpendicular to the magnetic meridian. If the deviation from perpendicularity be β the couple will be M H $\cos\beta$, and β is practically so small that $\cos\beta$ may be taken as unity. Since in the position of equilibrium the two couples balance each other, we have the equation

$$\text{W} \frac{ab}{l} \sin\phi = \text{M H},$$

which shows that H varies as $\sin\phi$.

The changes of vertical intensity are measured by the *balance-magnetometer*, which consists of a bar-magnet placed in the magnetic meridian, and suspended on knife-edges like the beam of an ordinary balance. Its deviations from horizontality are measures of the changes of vertical intensity.

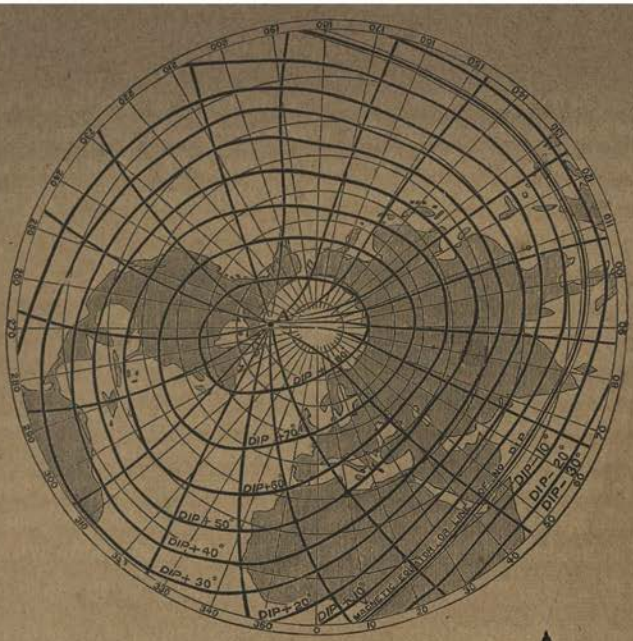


Fig. 74.—Northern Hemisphere.

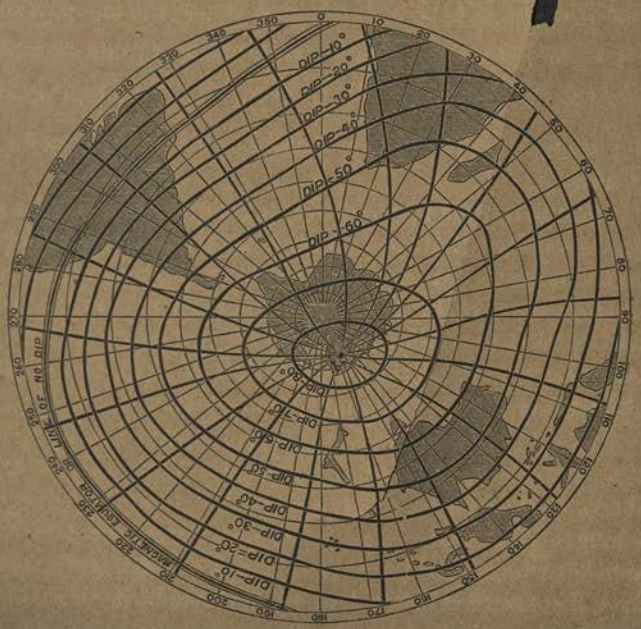


Fig. 75.—Southern Hemisphere.

MAGNETIC MERIDIANS AND LINES OF EQUAL DIP.

Both these instruments have mirrors attached to the magnet, which produce a photographic record of the movements of the magnet, on principles above explained.

The moment of a magnet varies with temperature, being diminished by something like one ten-thousandth part of itself for each degree Fahr. of increase, and increasing again at the same rate when the temperature falls. Hence magnetic observatories must be kept at a nearly uniform temperature. They must also be completely free from iron. No iron nails are allowed to be used in their construction, copper being employed instead.

121. Results of Observation.—Figs. 74, 75[1] contain an approximate representation of the magnetic meridians and lines of equal dip over both hemispheres of the earth. These two systems of lines combined, furnish a complete specification of the *direction* of magnetic force at all parts of the earth's surface; but they indicate nothing as to *intensity*. The curves of equal total intensity have a general resemblance to the lines of equal dip, the intensity being greatest near the poles, and least near the equator; but their arrangement is somewhat more complicated, there being two north poles of greatest intensity, one in Canada, and the other in the northern part of Siberia. Speaking roughly, the intensity near the poles is about double of the intensity near the equator. Curves of equal total intensity are often called *isodynamic* lines; curves of equal dip are often called *isoclinic* lines; curves of equal declination are often called *isogonic* lines; curves cutting the magnetic meridians at right angles are often called *magnetic parallels*. They are the lines which would be traced by continually travelling in the direction of magnetic east or west.

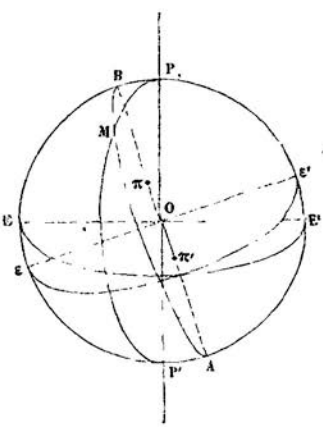


Fig. 76.—Biot's Hypothesis.

122. The Earth as a Magnet.—The intensity and direction of terrestrial magnetic force at different places may be *roughly* represented by supposing that there is a magnet $\pi\,\pi'$ (Fig. 76) at the earth's centre, having a length very small in compari-

[1] For Figs. 69, 71, 72, 74, 75 we are indebted to the publishers of Airy's *Treatise on Magnetism*.

son with the earth's radius, and making an angle of about 20° with the earth's axis of rotation. The points A and B obtained by producing this magnet longitudinally to meet the surface, would be the magnetic poles, and at any other place the magnetic meridian would be the vertical plane containing the magnetic axis A B. At places situated on the great circle whose plane contains both the axis of rotation and the magnetic axis, the magnetic meridian would coincide with the geographical meridian, and the declination would be zero. At any other place M, the two meridians would cut each other at an angle which would be the angle of declination. At all places on the great circle $\epsilon\,\epsilon'$ whose plane is perpendicular to the magnetic axis, a needle suspended at its centre of gravity would place itself parallel to this axis, and consequently the dip would be zero. This circle would be the magnetic equator.[1] It would cut the geographical equator at an angle of 20°. Proceeding from the magnetic equator towards the north magnetic pole B, the needle would dip more and more, until at B it became vertical. A declination needle at B would remain indifferently in all positions. Similar phenomena would be observed at the other magnetic pole A. The end of the needle which would dip at B, and which at other parts of the earth would point to magnetic north, is that which is similar to the southern pole π' of the terrestrial magnet $\pi\,\pi'$, and the pole which is similar to π would dip at A.

The supposition of such a central magnet is known as *Biot's hypothesis*. It leads to the same results as the supposition that the earth is a uniformly magnetized sphere. For if we have a sphere built up of a number of equal and similar small magnets with their poles pointing the same way, we may suppose all the imaginary fluid at their northern ends to be collected at one central point, and all the imaginary fluid at their southern ends at another central point, the distance between these two points being equal to the common length of the small magnets. Hence the small central magnet will have the same moment as the uniformly magnetized earth.

The actual phenomena of terrestrial magnetism are much more irregular than the results to which this hypothesis leads. It would

[1] If latitude reckoned from the magnetic equator be called magnetic latitude, and denoted by λ, it can be shown that we should have, on this theory,

$$\tan\delta = 2\tan\lambda\,;\quad I = E\sqrt{\cos^2\lambda + 4\sin^2\lambda},$$

E denoting the intensity at the magnetic equator.

appear that the earth's magnetism is distributed in a manner not reducible to any simple expression.

123. Secular Changes.—Declination and dip vary greatly not only from place to place, but also from time to time. Thus at the date of the earliest recorded observations at Paris, 1580, the declination was about 11° 30′ E. In 1663 the needle pointed due north and south, so that Paris was on the line of no declination. Since that time the declination has been west, increasing to a maximum of 22° 34′, which it attained in 1814. Since then it has gone on diminishing to the present time, its present value being about 19° W.

As to dip, its amount at Paris has continued to diminish ever since it was first observed in 1671. From 75° it has fallen to 66°, its present value. As its variations since 1863 have been scarcely sensible, it would seem to have now attained a minimum, to be followed by a gradual increase.

124. Periodic and Irregular Fluctuations. — Besides the gradual changes which occur in terrestrial magnetism, both as regards direction and intensity of force, in the course of long periods of time, there are minute fluctuations continually traceable. To a certain extent these are dependent on the varying position of the sun, and, to a much smaller extent, of the moon, with respect to the place of observation; but over and above all regular and periodic changes, there is a large amount of irregular fluctuation, which occasionally becomes so great as to constitute what is called a *magnetic storm*. Magnetic storms "are not connected with thunderstorms, or any other known disturbance of the atmosphere; but they are invariably connected with exhibitions of aurora borealis, and with spontaneous galvanic currents in the ordinary telegraph wires; and this connection is found to be so certain, that, upon remarking the display of one of the three classes of phenomena, we can at once assert

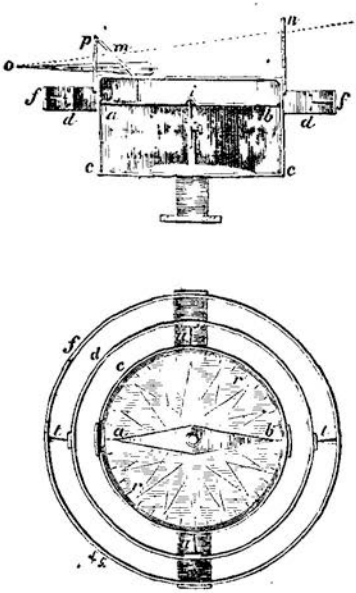


Fig. 77.—Ship's Compass.

that the other two are observable (the aurora borealis sometimes not visible here, but certainly visible in a more northern latitude)."[1] They are sensibly the same at stations many miles apart, for example at Greenwich and Kew, and they affect the direction and amount of horizontal much more than of vertical force.

125. Ship's Compass.—In a ship's compass, the box *c c* (Fig. 77) which contains the needle is weighted below, and hung on gimbals, which consist of two rings so arranged as to admit of motion about two independent horizontal axes *t t*, *u u*, at right angles to each other. This arrangement prevents it from being tilted by the pitching and rolling of the ship. The needle *a b* is firmly attached to the compass card, which is a circular card marked with the 32 points of the compass, as in Fig. 78, and also usually divided at its circumference into 360 degrees. The card with its attached needle is accurately balanced on a point at its centre. The needle, which, in actual use, is concealed from view, lies along the line N S. The box contains a vertical mark in its interior on the side next the ship's bow; and this mark serves as an index for reading off on the card the direction to which the ship's head is turned. Sometimes a reflector is employed, as *m* in the first part of Fig. 77, in such a position that an observer looking in from behind can read off the indicated direction by reflection, and can at the same time sight a distant object whose magnetic bearing is required. The origin of the compass is very obscure. The ancients were aware that the loadstone attracted iron, but were ignorant of its directive property. The instrument came into use in Europe some time in the course of the thirteenth century.

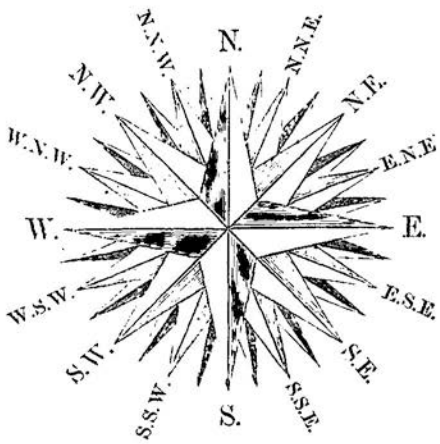


Fig. 78.—Compass Card.

126. Methods of Magnetization.—The usual process of magnetizing a bar consists in rubbing it with or against a bar already magnetized.

[1] Airy on *Magnetism*, p. 204.

Different methods of doing this, called single touch, double touch, &c., have been devised, in which magnetized bars of steel were the magnetizing agents. Much greater power can, however, be obtained by means of electro-magnetism; and the two following methods are now almost exclusively employed by the makers of magnets.

1. A fixed electro-magnet (Fig. 79) is employed, and the bar to be magnetized is drawn in opposite directions over its two poles.

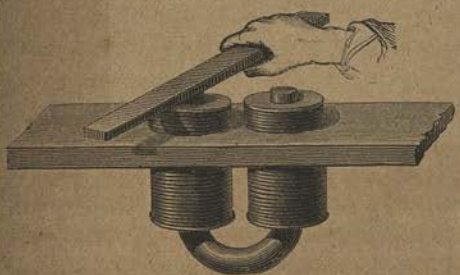


Fig. 79.

Fig. 80.

Methods of Magnetization.

Each stroke tends to develop at the end of the bar at which the motion ceases, the opposite magnetism to that of the pole which is in contact with it. Hence strokes in opposite directions over the two contrary poles tend to magnetize the bar the same way.

2. When very intense magnetization is to be produced, the electro-magnet must be very powerful, and the bar then adheres to it so strongly that the operation above described, becomes difficult of execution, besides scratching the bar. Hence it is more convenient to move along the bar, as in Fig. 80, a coil of wire through which a current is passing. This was the method employed by Arago and Ampère.

A bar of steel is said to be magnetized to *saturation*, when its magnetization is as intense as it is able to retain without sensible loss. It is possible, by means of a powerful magnet, to magnetize a bar considerably above saturation; but in this case it rapidly loses intensity.

Pieces of iron and steel frequently become magnetized temporarily or permanently by the influence of the earth's magnetism, and this action is the more powerful as the direction of their length more nearly coincides with that of the dipping-needle. If fire-irons which have usually stood in a nearly vertical position be examined by their

influence on a needle, they will generally be found to have acquired some permanent magnetism, the lower end being that which seeks the north.

It sometimes happens that, either from some peculiarity in the structure of a bar, or from some irregularity in the magnetizing

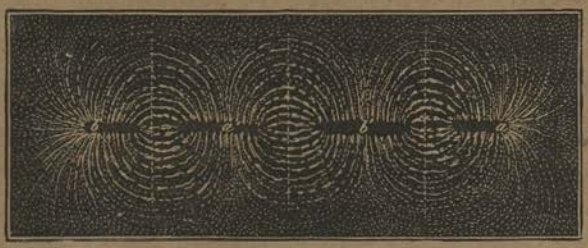


Fig. 81.—Consequent Points.

process, a reversal of the direction of magnetization occurs in some part or parts of the length as compared with the rest. In this case the magnet will have not only a pole at each end, but also a pole at each point where the reversal occurs. These intermediate poles are called *consequent points*. Fig. 81 represents the arrangement of iron-filings about a bar-magnet which has two consequent points $a'\,b'$. The whole bar may be regarded as consisting of three magnets laid end to end, the ends which are in contact being similar poles. Thus the two poles at a' and the one pole at a are of one kind, while the two poles at b' and the one pole at b are of the opposite kind.

Fig. 82.—Compound Magnet.

The lifting power (or *portative* force) of a magnet of given shape and intensity of magnetization is proportional, not to its volume, but its superficial dimensions. Thin bars can be more thoroughly magnetized than thick ones; and hence it has been found advantageous to construct compound magnets, consisting of a number of thin bars laid side by side, with their similar poles all pointing the same way. Fig. 82 represents such a compound magnet composed of twelve elementary bars, arranged 4×3. Their ends are inserted in masses of soft iron, the extremities of which constitute the poles of the system.

Fig. 83 represents a compound horse-shoe magnet, whose poles N and S support a keeper of soft iron, from which is hung a bucket for holding weights. By continually adding fresh weights day after day, the magnet may be made to carry a much greater load than it could have supported originally; but if the keeper is torn away from the magnet, the additional power is instantly lost, and the magnet is only able to sustain its original load.

Fig. 83.—Compound Horse-shoe Magnet.

Much attention was at one time given to methods of obtaining steel magnets of great power. These researches have now been superseded by electro-magnetism, which affords the means of obtaining temporary magnets of almost any power we please.

127. Molecular Changes accompanying Magnetization.—Joule has shown that, when a bar of iron is magnetized longitudinally, it acquires a slight increase of length, compensated, however, by transverse contraction, so that its volume undergoes no change.

If the magnetization is effected suddenly, by completing an electric circuit, an ear close to the bar hears a clink, and another clink is heard when the current is stopped.

These phenomena have been accounted for by the hypothesis that, when iron is magnetized, its molecules place their longest dimensions in the direction of magnetization.

The effect of heat in diminishing the strength of a magnet is another instance of the connection between magnetism and other molecular conditions. In ordinary cases, this diminution is merely transient; but if a steel magnet is raised to a white-heat, it is permanently demagnetized.

128. Action of Magnetism on all Bodies.—It has long been known that iron and steel are not the only substances which can be acted on by magnetism. Nickel and cobalt especially were known to be attracted by a magnet, though very much more feebly than iron, while bismuth and antimony were repelled. Faraday, by means of a powerful electro-magnet, showed that all or nearly all sub-

stances in nature, whether solid, liquid, or gaseous, were susceptible of magnetic influence, and that they could all be arranged in one or the other of two classes, characterized by opposite qualities. This opposition of quality is manifested in two ways.

1. As regards attraction and repulsion, iron and other *paramagnetic* bodies are attracted by either pole of a magnet, or more generally they tend to move from places of weaker to places of stronger force. On the other hand, bismuth and other *diamagnetic* bodies are repelled by either pole of a magnet, and in general tend to move from places of stronger to places of weaker force.[1]

2. As regards orientation, a paramagnetic[2] body, when suspended between the poles of a magnet, tends to set *axially;* that is to say, tends to place its length along the line joining the poles; whereas a diamagnetic body tends to set *equatorially,* that is, to place its length at right angles to the line joining the poles.

The fundamental difference is, that a piece of bismuth (or other diamagnetic substance) when it becomes a temporary magnet from the inductive influence of the field in which it is placed, has its poles opposite, end for end, to those of a piece of iron (or other paramagnetic substance) similarly placed. From this reversal of the poles, it follows that the resultant force upon the bismuth is opposite in direction to the resultant force upon the iron; and as the iron is urged from places of weaker to places of stronger force, the bismuth is urged from stronger to weaker. This is the cause of the equatorial setting of a diamagnetic bar when suspended between the poles of a magnet. It is merely a result of the tendency of the particles to move outwards into the regions of weaker magnetic action. In a uniform field, with parallel lines of force, the equatorial setting would not occur.

The axial setting of an iron bar between the poles of a magnet is jointly due to two causes, one being the tendency of its ends to move to places of stronger force, while the other cause, which we will now proceed to explain, tends to produce axial setting even in a uniform field.

[1] The motion of iron from weaker to stronger parts of the field is roughly explained in the second foot-note to § 109. The same explanation serves for bismuth if we substitute "push" for "pull" and "convex" for "concave."

[2] The nomenclature here adopted was proposed by Faraday in 1850 (*Researches*, § 2790), and is eminently worthy of acceptance. Many writers, however, continue to employ *magnetic* in the exclusive sense of *para*magnetic. To be consistent, they should call the other class *anti*magnetic, not *dia*magnetic. "The word *magnetic* ought to be general, and include *all* the phenomena and effects produced by the power."

129. Reason of Setting in a Uniform Field.—Suppose a row of iron balls placed axially, as in Fig. 84, either between the poles of a magnet, or along a line of force in uniform field; the force of the field being such as to urge a north pole from left to right. Each ball will, by induction, become a magnet with its north pole to the right, and the force which each ball experiences from its neighbours will be in the same direction as the force of the field. The mutual action of the balls, therefore, increases the induction due to the field.

Fig. 84.

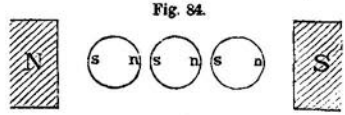
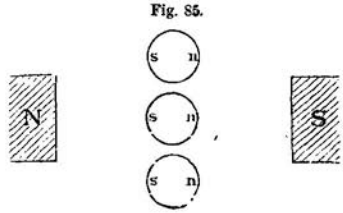


Fig. 85.



Next, suppose a row of iron balls placed equatorially in the same field (Fig. 85). The north pole of each ball will be attracted to the left by the south poles of its neighbours, and the induction due to the field will therefore be diminished.

Thirdly, let a row of iron balls be placed in a line inclined to the lines of force (Fig. 86.) The force of the field can be resolved into two components, one (which we shall call the longitudinal component) along the line of balls, and the other (which we shall call the transverse component) normal to the line of balls. Mutual induction will, as above shown, augment the longitudinal and diminish the transverse component. It will, therefore, alter the direction of the total induction, so as to make it more nearly longitudinal. The poles of any one of the balls will therefore have such positions as are shown at n and s in the figure, n being above and s below the horizontal line through the centre.

Fig. 86.

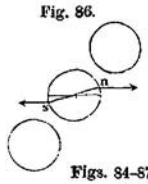
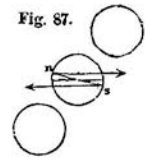


Fig. 87.



Figs. 84-87.—Reason of Setting in Uniform Field.

In estimating the forces which tend to turn the row of balls as a whole when they are rigidly connected together, we must remember that mutual actions between different parts of a rigid body do not tend to move the body as a whole. Such motion can only be produced by forces from without, that is, in the present case, by the original force of the field, which urges north poles from left to right, and south poles from right to left. The forces on each ball will constitute a couple as shown by the two arrows in the

figure, and these couples tend to turn the body into the axial position.

If we apply similar reasoning to a row of balls of bismuth, we shall find that mutual induction diminishes the longitudinal component, increases the transverse component, and in the case of the oblique row, gives the poles of each ball positions such as are shown in Fig. 87. The couple due to the external forces of the field is represented by the two arrows in the figure, and tends (just as in the case of iron) to turn the body into the axial position. This directive action is, however, excessively feeble, the forces due to mutual induction in bismuth being insensible in comparison with the external forces of the field.

130. Experimental Arrangements. Faraday's Apparatus.— Fig. 88 represents the apparatus commonly employed for experiments on

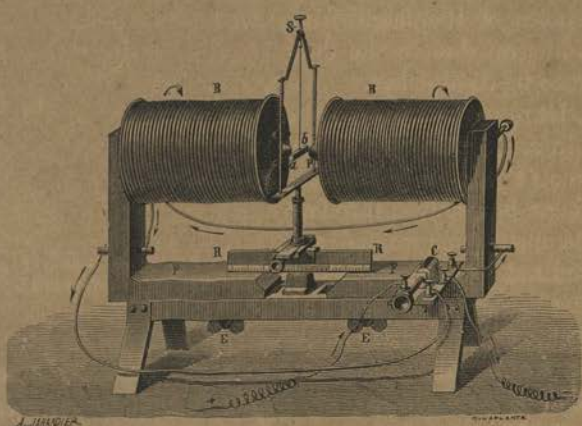


Fig. 88.—Apparatus for Diamagnetism.

this subject. B, B are two large coils of stout copper wire, wound on massive hollow cylinders of soft iron. These latter form portions of the heavy frames F, F, which can be slid to or from each other, and fixed firmly at any distance by means of the screws E, E. The pole pieces P, which can be screwed on or off, have the form of rounded cones, and produce a great concentration of force at their extremities.

The action of magnetism upon a solid can be examined by suspending a small bar of it $a\,b$, by means of a special support R S, between the poles P. When a current is passed through the coils, the bar

immediately exhibits a preference either for the axial or the equatorial position. Attraction and repulsion are most easily tested by suspending a small ball of the substance at the level of the central line of poles, but a little beside it, the poles having first been brought very near together. On passing the current through the coil, the ball will move inwards towards the line of poles if paramagnetic, and outwards if diamagnetic.

It is important, however, to remark, that experiments of this kind, unless performed *in vacuo*, are merely differential—they merely indicate that the suspended body is, in the one case, more paramagnetic or less diamagnetic, in the other case more diamagnetic or less paramagnetic, than the medium in which it moves, the comparison being made between equal volumes. Oxygen is paramagnetic, and nitrogen is nearly or quite indifferent. Air is accordingly paramagnetic, and a body suspended in air appears less paramagnetic or more diamagnetic than it really is. If more feebly paramagnetic than air, it will appear to be diamagnetic. Thus heated air, in consequence probably of its rarefaction, appears diamagnetic when surrounded by cold air, and the flame of a taper is repelled downwards and outwards from the axial line.

If, on the other hand, the body under examination is suspended in water, it will appear more paramagnetic than it really is, by reason of the diamagnetism of water.

The following metals are paramagnetic: iron, nickel, cobalt, manganese, chromium, titanium, cerium, paladium, platinum. osmium.

The following are diamagnetic: bismuth, antimony, lead, tin, mercury, gold, silver, zinc, copper.

The following substances are also diamagnetic: water, alcohol, flint, glass, phosphorus, sulphur, resin, wax, sugar, starch, wood, ivory, beef (whether fresh or dried), blood (whether fresh or dried), leather, apple, bread.

131. Magneto-crystallic Action.—The orientation of crystals in a magnetic field presents some remarkable peculiarities which were extremely perplexing to investigators until Tyndall and Knoblauch discovered the principle on which they depend. This principle is, that crystals are susceptible of magnetic induction to different degrees in different directions. Every crystal (except those belonging to the cubic system) has either one line or one plane along which induction takes place more powerfully than in any other direction; and it is this line or plane which tends to place itself axially or

equatorially according as the crystal is paramagnetic or diamagnetic. The directions of most powerful and least powerful induction are found to be closely related to the optic axes of crystals, and also to their planes of cleavage. When a sphere cut from a crystal is brought near to one pole of a magnet, it is attracted or repelled (according as it is para- or dia-magnetic) with the greatest force when the direction of most powerful induction coincides with the direction of the force.

Directions of unequal induction can be produced artificially in non-crystalline substances by applying pressure. "Bismuth is a brittle metal, and can readily be reduced to a fine powder in a mortar. Let a tea-spoonful of the powdered metal be wetted with gum-water, kneaded into a paste, and made into a little roll, say an inch long and a quarter of an inch across. Hung between the excited poles, it will set itself like a little bar of bismuth—equatorial. Place the roll, protected by bits of pasteboard, within the jaws of a vice, squeeze it flat, and suspend the plate thus formed between the poles. On exciting the magnet, the plate will turn, with the energy of a magnetic substance, into the axial position, though its length may be ten times its breadth.

"Pound a piece of carbonate of iron into fine powder, and form it into a roll in the manner described. Hung between the excited poles, it will stand as an ordinary [para]magnetic substance—axial. Squeeze it in the vice, and suspend it edgeways, its position will be immediately reversed. On the development of the magnetic force, the plate thus formed will recoil from the poles *as if violently repelled*, and take up the equatorial position."[1]

In these experiments the direction of most powerful induction is a line transverse to the thickness, and this is also the direction in which pressure has been applied. Tyndall accordingly concludes that "if the arrangement of the component particles of any body be such as to present different degrees of proximity, in different directions, then the line of closest proximity, other circumstances being equal, will be that chosen by the respective forces for the exhibition of their greatest energy. If the mass be [para]magnetic, this line will stand axial; if diamagnetic, equatorial."[2]

[The account of Magnetic Units in the Appendix should be read before commencing the next chapter.]

[1] Tyndall on *Diamagnetism*, p. 18. [2] Ibid. p. 23.

CHAPTER XII.

FURTHER DEVELOPMENT OF MAGNETIC THEORY.

132.—We shall now proceed to establish a few of the mathematical consequences of the elementary principles laid down in Chapter X.

Given the moments of two magnets which are rigidly fastened together, to find their joint moment.

Solution. From any point O, draw a line OA, representing the moment of the first, and from A draw AB, representing the moment of the second; then the straight line OB will represent the joint moment.

In representing the moment of a magnet by a straight line it is to be understood that the length of the line is proportional to the moment, and the direction in which the line is drawn is the direction in which the north-seeking end of the axis points. This being understood, the above construction is justified by the following considerations:—

The couple which a uniform field exerts on a magnet, is the same for any two magnets which have equal moments and parallel axes (pointed the same way, not opposite ways). Hence we may substitute for the first magnet a simple magnet OA, with poles of unit strength, and for the second magnet a simple magnet AB, with poles of unit strength. The two coincident poles at A will be of opposite sign, and their free magnetisms will cancel each other, leaving only two unit poles at O and B, which may be regarded as the poles of a magnet represented by OB.

The above proof can be extended to any number of joined magnets, and shows that their resultant moment is obtained by the same construction as the resultant of forces acting at a point. This is technically expressed by saying that the *moment of the whole is the vector-sum of the moments of the parts.*

The students should verify the following results which are deducible from the above rule.

(*a.*) If two magnets, each of moment M, are fastened together with their axes at right angles, the resultant moment is $M\sqrt{2}$, and the resultant axis bisects the angle between the given axes.

(*b.*) If a compound magnet consist of two magnets whose axes are exactly opposite, and whose moments are not exactly equal, its moment will be the difference of the given moments, and its axis will have the same direction as the axis of the greater.

(*c.*) If it consist of two magnets whose moments are exactly equal, and whose axes are nearly but not exactly opposite, its moment will be small, and its axis nearly perpendicular to that of either of the given magnets.

What is called an *astatic* needle consists of two magnetized needles with their moments nearly equal and their axes nearly opposite. Such a needle in the earth's field may set north and south as in (*b*), or east and west as in (*c*), or in any intermediate position. Practically, it usually sets nearly north and south.

(*d.*) If the compound magnet be a closed polygon of uniform wire, magnetized longitudinally in one direction round the polygon with uniform intensity, its moment will be zero; it will not exhibit any tendency to orientation. The same is true of a ring, which may be regarded as the limit of a polygon.

133. Surface Density of Free Magnetism.—If we have a magnet in the form of a right prism, of length l and cross section a, uniformly magnetized in the direction of its length, its moment, being the product of the intensity of magnetization I by the volume, is $I l a$. This must be equal to the product of the length by the quantity of magnetism on either end. Hence the quantity on each end is $I a$, and the quantity per unit area (called the surface-density) is I.

Next let one end of the prism be a right section, and the other an oblique section, the obliquity (or angle between the two sections) being ϕ. Then, since the free magnetisms of opposite kinds must be equal in magnitude, the quantity on the oblique section is $I a$, and since it is spread over an area $a \sec \phi$, the surface density is $I \cos \phi$.

The surface density of free magnetism at any point on the surface of a magnet is accordingly $I \cos \phi$, I denoting the magnetization at this point, and ϕ the angle between the direction of

magnetization and the normal to the surface. Our conventions as to sign are:—

North-seeking magnetism is positive.

The direction of the magnetic force H is the direction in which a north-seeking pole would be urged.

The direction of the magnetization I is the direction in which the north-seeking end of the elementary magnet points.

Accordingly, the surface density will be positive if the direction of magnetization makes an acute angle with the outward-drawn normal.

134. Volume-Density of Free Magnetism.—When the magnetization is parallel to a fixed direction, the relation between the volume-density of free magnetism in the interior and the magnetization can be investigated as follows:—

Consider a long narrow prismatic filament in the direction of the magnetization. Let a be its cross section, and let it be divided into elementary prisms of common length l. Regard each element as uniformly magnetized with the magnetization actually existing at its centre. This will give a surface-density $+\mathrm{I}$ at the forward end, and $-\mathrm{I}$ at the backward end of the element. For the consecutive element on the forward side, the surface-density will be $\mathrm{I}+\frac{d\mathrm{I}}{ds}l$ on the forward end, and $-\mathrm{I}-\frac{d\mathrm{I}}{ds}l$ on the backward end, s denoting distance measured in the forward direction. At the junction of the two elements we have to combine the two surface-densities I and $-\mathrm{I}-\frac{d\mathrm{I}}{ds}l$, and the resulting surface-density is $-\frac{d\mathrm{I}}{ds}l$. Thus we have a quantity of free magnetism $-\frac{d\mathrm{I}}{ds}la$, which we may regard as belonging to a volume la, consisting of the second half of the first element and the first half of the second. Thus the expression for the volume-density is $-\frac{d\mathrm{I}}{ds}$.

135.—Abolishing the restriction as to parallelism of magnetization, the student should verify for himself, by dividing the substance into elementary prisms by three sets of planes parallel to rectangular co-ordinate planes, and applying reasoning similar to the above, that the volume-density is $-\frac{d\mathrm{A}}{dx}-\frac{d\mathrm{B}}{dy}-\frac{d\mathrm{C}}{dz}$, A, B, C denoting the components of I parallel to the axes of x, y, z.

internal points is zero, it is said to be *solenoidal*.[1] A solenoidal magnet may either be ring-shaped and have no free magnetism anywhere, or it may have free magnetism residing on some portions of its surface.

136. Force in a Crevasse; H and B.—When a piece of iron is in a magnetic field, the magnetic force at the centre of a small cavity in the iron will depend partly on the shape and direction of the cavity. It will be a maximum for a narrow cavity with plane parallel sides perpendicular to the direction of magnetization of the iron, and will be a minimum for a similar cavity with its sides parallel to the direction of magnetization. Such cavities are briefly designated a *transverse crevasse* and a *longitudinal crevasse*.

The surface density on the sides of a transverse crevasse is I, and the repulsion of one surface layer and attraction of the other are both in the same direction as the magnetizing force to which I is due. Each of them amounts, by a well-known theorem in attractions, to 2π I. Hence if we denote the magnetizing force by H, and the force in the transverse crevasse by B, we have

$$B = H + 4\pi I,$$

whereas in a longitudinal crevasse the force is simply H. The modern name for B is *induction*, or more fully "induction at a point," or "intensity of induction." We shall call H the *magnetizing force*. It is often called the *magnetic force*. In non-magnetic matter, such as air or copper, B and H are the same, and I is zero.

When we speak of the values of B and H at a point in any medium, we mean the forces which would exist in crevasses excavated at that point. H may be more simply defined as the *force due to the actual free magnetism*.

The ratio of I to H is called magnetic *susceptibility*, and is denoted by the Greek letter κ. The ratio of B to H is called magnetic *permeability*, and is denoted by the Greek letter μ. The above equation for the value of B in terms of H and I may accordingly be written

$$\mu = 1 + 4\pi\kappa.$$

The following table gives a good idea of the values of these various quantities for very soft iron. The values will differ considerably for different specimens.

[1] Because in this case lines drawn in the direction of magnetization are analogous to lines of steady flow in a liquid, and the magnet can be divided into filaments which are analogous to *tubes* of flow. Compare § 52.

H	I	B	κ	μ
0.3	3	41	10	128
1·4	32	413	23	299
2·2	117	1460	53	670
3·5	574	7230	164	2070
4·9	917	11540	187	2350
6·7	1078	13520	161	2020
10·2	1173	14840	115	1450
22·3	1249	15710	56	705
78	1337	16900	17	215
208	1452	18500	7	89
585	1530	19800	2·6	34
24500	1660	45300	0·067	1·9

137.—In passing from the substance of a magnet of any kind into the external air at a part where the direction of magnetization is normal to the surface, there is no abrupt change in B either as regards magnitude or direction. To prove this, we shall consider the case in which H and I in the magnet are directed outwards at the surface in question, and shall regard this outward direction as the positive direction.

On account of the repulsion $2\pi I$ due to the surface layer, which reverses its direction as we pass through the surface, the value of H just outside is equal to the value of $H+4\pi I$ just inside. But H outside is the same as B outside, since I outside is zero; and $H+4\pi I$ inside is B inside. Hence B outside is equal to B inside.

138.—The student may verify for himself the more general statement that in passing out of a magnet into air at any point of the surface (which will generally be oblique to the direction of magnetization) there is *no abrupt change in the normal component of B*. This property, as we shall see hereafter, is of great importance.

139. Application to Steel Magnet.—In an ordinary steel bar-magnet, although the direction of the magnetization is everywhere from the negative to the positive (that is north-seeking) end, the magnetizing force (which is due to the free magnetism of the bar) is in the opposite direction. A compass needle held over the bar points in the opposite direction to the bar itself, and it would do the same if it could be placed in a longitudinal crevasse inside the bar. The lines of magnetizing (or magnetic) force both inside and outside the bar run from the positive to the negative end. The lines of induction, on the other hand, are closed curves running outside from positive to negative, and returning inside from negative to positive.

140. Self-Demagnetizing Force.—The opposition of direction between magnetization and magnetizing force is the chief cause of the

tendency of steel magnets to get weaker with time; and it does not exist in a ring, such as would be formed if a uniformly magnetized bar could be bent into a circle and its ends joined. There is no free magnetism in a ring thus circularly magnetized, and though the force in a transverse crevasse may be very strong, the force in a longitudinal crevasse would be evanescent. It is accordingly found that rings possess much greater retentive power than bars or open horse-shoes. The soft iron keeper which is usually kept in contact with the two ends serves to convert the horse-shoe into a temporary ring, cancelling most of the free magnetism on the terminal surfaces by the development of opposite magnetism at the surfaces of the soft iron, and thus checking the self-demagnetizing tendency.

141.—A demagnetizing tendency also exists in soft iron temporarily magnetized. When a long cylinder of iron is brought into a uniform magnetic field and placed with its axis parallel to the lines of the field, the free magnetism called out at either end exerts upon the adjoining parts a force opposite to that of the undisturbed field, but not sufficiently strong to reverse it.

The difference between H just inside and H just outside is 4π I, and this difference is not sufficient to change the sign of H; hence I at the ends is numerically smaller than the fraction $1/4\pi$ of the external H. In the middle of the cylinder, I may be some hundreds of times greater than at the ends. As the length of the cylinder is increased the intensity of magnetization in the middle is increased also. The demagnetizing tendency is nevertheless sensible even in the middle portions of long cylinders. The form which is most amenable to calculation is the ellipsoid, and Ewing calculates that for an ellipsoid of very soft iron 200 times as long as it is broad, placed longitudinally in a field of moderate strength, the demagnetizing force (which is uniform throughout the ellipsoid,) is half the undisturbed force of the field, so that the value of H in the ellipsoid is half the value of H in distant parts of the field.

142. Small Magnet at Earth's Centre.—The following is the working out of Biot's hypothesis described in § 122.

Let r be the distance of any point at or near the earth's surface from the centre, and θ its angular distance from the earth's north magnetic pole, which is determined by producing the axis of the small magnet through its south-seeking end. The southern end of the small magnet will be north-seeking; let its strength be m, and that of the northern end $-m$, their distance apart being $2l$. The

magnetic potential at the point in question will be $m/(r+l\cos\theta) - m/(r-l\cos\theta)$, which to a first approximation equals $-2lm\cos\theta/r^2$. But $2lm$ is the moment of the small magnet. Denoting this by M, and the potential by V, we have $V = -\frac{M}{r^2}\cos\theta$.

The force on a unit north-seeking pole along any short line ds is $-dV/ds$. Vertically downwards, we have $ds = -dr$. Horizontally northwards, we have $ds = -rd\theta$. These values give for the vertical force $\frac{2M}{r^3}\cos\theta$, and for the horizontal force $\frac{M}{r^3}\sin\theta$ Hence the total force is $\frac{M}{r^3}\sqrt{(1+3\cos^2\theta)}$, and $\tan \text{dip} = 2\cot\theta$.

We shall return to the subject of magnetic potentials in a later chapter, in connection with electro-magnetism.

143.—The vibrations of a horizontally balanced magnet under the action of the earth's horizontal magnetic force are closely analogous to the vibrations of a pendulum under the action of gravity. In both cases the couple acting on the body is proportional to $\sin\theta$, where θ denotes the deviation from the position of equilibrium. In the case of the pendulum this couple is $mgh\sin\theta$, m denoting the mass of the pendulum, and h the distance of the centre of gravity from the point of suspension. In the case of the magnet it is $MH\sin\theta$, M denoting the magnetic moment of the magnet. The angular acceleration in both cases is the quotient of this couple by the moment of inertia about the point of suspension, which we shall denote by mk^2. For the magnet this acceleration will be $\frac{MH}{mk^2}\sin\theta$, and for the pendulum it is $\frac{mgh\sin\theta}{mk^2}$ or $\frac{gh}{k^2}\sin\theta$ or $\frac{g}{l}\sin\theta$, l denoting the length of the equivalent simple pendulum. Thus, the time of one to-and-fro vibration, which is $2\pi\sqrt{\frac{l}{g}}$ for the pendulum, will be $2\pi\sqrt{\frac{mk^2}{MH}}$ for the magnet.

144.—To determine experimentally the value of H at a given time and place, two observations must be made. One of them consists in timing the horizontal vibrations of a hard steel magnet, and the other consists in observing the deflection which this magnet produces in a magnetic needle.

In the deflection observation the deflecting magnet is either placed "end on" or "broadside on."

In the former case it is magnetic east or west of the centre of the needle, and points directly towards it. Let r be the distance from centre to centre, and $2a$ the distance between the poles of the

magnet, the strength of each pole being P. Then the force at the centre of the needle due to the magnet is $\frac{P}{(r-a)^2} - \frac{P}{(r+a)^2} = \frac{4 P r a}{(r^2-a^2)^2}$ $= \frac{4 P a}{r^3}$ nearly, since a is small compared with r.

But $2 P a$ is the moment of the magnet; denoting this by M, the force is $\frac{2 M}{r^3}$, and its direction is perpendicular to the earth's force, H. If we regard the field as uniform, the tangent of the deflection will be the ratio of these two forces, say

$$\tan \delta = \frac{2 M}{H r^3}.$$

By combining this equation with $T^2 = 4\pi^2 \frac{m k^2}{M H}$, T denoting the time of a complete vibration, we find

$$H^2 = \frac{8\pi^2}{T^2} \frac{m k^2}{r^3 \tan \delta}.$$

In the "broadside on" position, the line of centres is magnetic north and south, and the deflecting magnet points magnetic east and west. With the same notation as above, the force at the centre of the needle due to either pole is approximately $\frac{P}{r^2}$, and its east and west component is approximately $\frac{a}{r}$ of this, or is $\frac{P a}{r^3}$. This is in the same direction for both poles of the magnet, hence the total east and west force is $\frac{2 P a}{r^3}$ or $\frac{M}{r^3}$. This is the whole disturbing force, for the north and south components destroy one another. Accordingly we have now

$$\tan \delta = \frac{M}{H r^3}, \text{ and } H^2 = \frac{4\pi^2}{T^2} \frac{m k^2}{r^3 \tan \delta}.$$

To eliminate errors arising from want of symmetry in the magnetization, the position of the disturbing magnet should be reversed in three ways, viz. by turning it upside down, by turning it end for end, and by shifting it to the opposite side of the needle, keeping the distance r unchanged. This makes 8 positions in all, and the mean of the 8 deflections obtained is to be adopted. These remarks apply alike to the "end on" and the "broadside on" positions.

The above formulæ may be regarded as the first approximations; for closer approximations we must refer to fuller treatises.

As regards the moment of inertia, $m k^2$, if the magnet is a rectangular bar, k^2 is one-twelfth of the square of the diagonal of its horizontal face. This diagonal and the distance r should be ex-

pressed in centimetres, and the mass m in grammes, the time T being in seconds. The value of H will thus be obtained in C.G.S. measure. Its average value for the British Isles is about ·17.

The observations must be made at a distance from all other magnets and from all large masses of iron.

145. Field Observations.—We shall conclude this chapter with an account of the methods of observation usually employed in a magnetic survey. The measurement of declination is made in the following manner:—

A "magnetic theodolite" is employed, having a tripod stand and a horizontal divided circle like an ordinary theodolite; but its telescope, instead of occupying a central position, is supported on a projecting arm, movable round the circumference, and is directed towards the centre of the instrument. In the centre hangs the magnet, suspended by a bundle of silk fibres and suitably protected from wind. This magnet is a hollow steel cylinder, having an achromatic lens at the near end, and a glass scale in its focus at the other. Thus the observer sees in the telescope, when properly placed, the divisions of the glass scale.

The line joining the centre of the lens to the middle division of the scale is called the line of collimation of the magnet.

The telescope is moved along the divided circle till its line of collimation coincides with that of the magnet; in other words, till its wire is seen in coincidence with the middle point of the scale. The magnet is then turned upside down and the observation repeated. The mean of the two readings will be the reading when the telescope is pointed to magnetic south. The reading which corresponds to true south is then determined by observing the sun's azimuth as indicated on the horizontal circle at a noted time. For making this observation a mirror is provided, which travels round the instrument on the opposite side from the telescope, and can be turned about a horizontal axis which is at right angles to the telescope; when it reflects the sun's rays into the telescope, the telescope is in the same vertical plane with the sun. Two observations are made, in one of which the sun is in front of the telescope, and in the other behind it.

146.—The same theodolite serves for the observations of horizontal force; the deflecting magnet (which is usually the same magnet that was employed in the observation of declination) being supported on a lateral bar attached for the purpose, and the deflection of a

second and smaller magnet of the same construction being observed in a telescope, which is moved round the divided circle till its wire coincides with the deflected position of the line of collimation of this small magnet. In a complete series of observations, the deflecting magnet is placed successively at two certain distances from the suspended magnet, on each side of it, and in each position two deflections are found—one with the N. end pointing E., and one with the N. end pointing W. The vibration observation is made by dismounting the small magnet from the centre of the instrument, and suspending the deflecting magnet in its place.

147.—For observing dip, the theodolite is dismounted, and in its place is erected an instrument with two microscopes and verniers, travelling round a vertical circle, as in Fig. 71.

The plane of the vertical circle is first made approximately perpendicular to the meridian, by setting the lower microscope and vernier to 90° (the lowest point of the circle), and turning the instrument in azimuth till the lower end of the needle is bisected by the wire of the microscope. A reading of the horizontal circle is taken. The needle is then reversed in its bearings and a second reading taken. A similar adjustment is made with the upper microscope and the upper end of the needle, giving a third and fourth reading of the horizontal circle, which will be nearly the same as the first and second. The instrument is then turned in azimuth through about 180°, and the adjustments are repeated, giving four additional readings, which will differ by about 180° from the first four. If these eight readings are added, and the sum is divided by 8, the quotient will differ by about 90° from each of the readings, and will correspond to the magnetic meridian, as will also another reading differing by 180° from this.

The instrument is now set to one of these two readings, so that the plane of the vertical circle is in the magnetic meridian, the lower microscope is set so that its wire bisects the lower end of the needle, and a first determination of dip is obtained by reading the verniers. A similar adjustment is then made with the upper microscope and the upper end of the needle, and fresh readings are taken, which will not differ much from the preceding. The needle is then reversed in its bearings so as to turn its other face to the microscopes, and the observations are repeated. The instrument is then turned in azimuth through 180°, and similar observations are made in this position. Finally, the needle is dismounted, fixed on

a wooden support provided for the purpose, and stroked with two magnets from the centre to the ends in such a way as to reverse its magnetization, and all the above observations of dip are repeated with the needle in this condition. The mean of all the readings for dip is adopted as the true dip.

CURRENT ELECTRICITY.

CHAPTER XIII.

GALVANIC BATTERY.

148. **Voltaic Electricity.**—Towards the close of last century, when the discovery of the various phenomena of frictional electricity had been followed by Coulomb's investigations, which first reduced them to an accurate theory, a new instrument was brought to light destined to effect a complete revolution in electrical science. In place of an element difficult to manage, capricious and uncertain in its behaviour, and constantly baffling investigation by the rapidity of its dissipation, the galvanic battery furnished a steady source of electricity, constantly available in all weathers, and requiring no special precautions to prevent its escape. Moreover, the electricity thus developed exhibited an entirely new set of phenomena, and opened up the way to such various and important applications, that frictional electricity at once fell into the second place, and the new agent became the main object of interest with all electrical investigators.

149. **Galvanic Element.**—If two plates, one of zinc and the other of copper (Fig. 89), are immersed in water acidulated by the addition of sulphuric acid, and are not allowed to touch each other within the acid, but are connected outside it, either by direct contact, or by a metallic wire M and binding screws, as in the figure, a continuous current of electricity flows round the circuit thus formed, the direction of the positive current being from copper to zinc in the portion external to the liquid, and from zinc to copper through the liquid. Chemical action at the same time takes place, the zinc being gradually dissolved by the acid, and hydrogen being given out at the copper plate.

If, instead of employing two metals and a liquid, we form a circuit with any number of metals alone, no current will be gene-

rated, provided that the whole circuit be kept at one temperature. If, however, some of the junctions be kept hot and others cold, a current will in general be produced.

To explain these phenomena it is necessary to suppose that there is an abrupt difference of potential in crossing one or more of the surfaces of junction of dissimilar substances; and this seems to imply the presence of two layers of opposite electricities facing each other at a very small distance. It can be shown that two layers, of surface density ρ, at distance l, would give a difference of potential $4\pi\rho l$. There has been much discussion of late years as to whether the abrupt change occurs at the junction of the zinc and copper, or at the junction of the zinc and the liquid.

If the latter view is adopted, we may suppose that the chemical action which takes place at this surface involves a continual generation of equal and opposite electricities, which refuse to combine across the junction at which the chemical action occurs, but are able to combine by flowing round in opposite directions through the circuit. On this theory the zinc is negative relative to the liquid, in other words the potential falls abruptly as we cross the boundary from the liquid to the zinc.

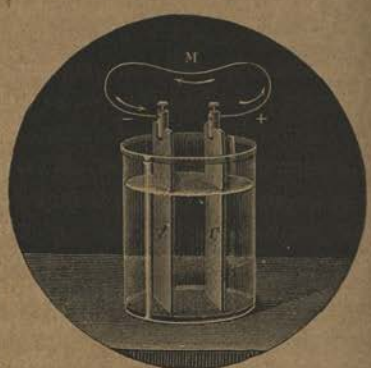


Fig. 89.—Voltaic Element.

On the opposite theory, zinc and copper in direct contact are supposed to have an abrupt difference of potential, zinc being the higher; but when these metals are immersed in an acid solution, without being connected in any other way, they are supposed to have the same or nearly the same potential. Accordingly when the circuit of the cell is completed, the highest and lowest potentials are those of the zinc and copper respectively at their junction.

Whichever theory is held, it is agreed that for keeping up the current there must be a continual running down of the potential energy of chemical affinity by the formation of chemical compounds, this energy being the source of the energy of the current.

Every electric current may be regarded as a flow of positive electricity in one direction, and of negative electricity in the opposite

direction. The direction in which the positive electricity flows is always spoken of as the *direction of the current.*

150. **Galvanic Battery.**—By connecting the plates of successive elements in the manner represented in Fig. 90, we obtain a battery. The copper of the first cell on the left hand is connected with the

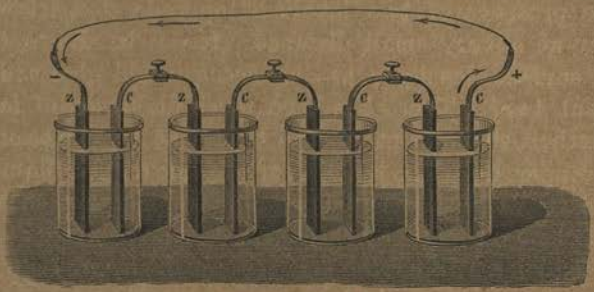


Fig. 90.—Battery of Four Elements.

zinc of the second; the copper of the second with the zinc of the third; and so on to the end of the series.

If two wires of the same metal be connected, one with the first zinc and the other with the last copper, the difference of potential between these wires is independent of the particular metal of which they are composed, and is called the *electro-motive force* of the battery. Its amount can be measured by means of Thomson's quadrant electrometer; and in applying this test, it is not necessary that the wires which connect the battery with the electrometer should be of the same metal. Whatever metals these wires may be composed of, the quadrants of the electrometer will assume the same potentials as if in direct contact with the plates of the battery.[1]

The zinc of the first and the copper of the last cell (or wires proceeding from them) are called the *electrodes* or *poles* of the battery, the zinc being the negative and the copper the positive electrode. The current flows through the connecting wire from the positive to the negative electrode, and is forced through the battery from the negative to the positive.

151. **Galvani's Discoveries.**—About the year 1780, Galvani, professor

[1] The difference of potentials of two metals must be the same whether they are in direct contact or are connected by a third metal; otherwise a circuit of the three metals would give a current, which is contrary to experimental fact. The temperature is supposed uniform.

of anatomy at Bologna, had his attention called to the circumstance that some recently skinned frogs, lying on a table near an electrical machine, moved as if alive, on sparks being drawn from the machine. Struck with the apparent connection thus manifested between electricity and vital action, he commenced a series of experiments on the effects of electricity upon the animal system. In the course of these experiments, it so happened that, on one occasion, several dead frogs were hung on an iron balcony by means of copper hooks which were in contact with the lumbar nerves, and the legs of some of them were observed to move convulsively. He succeeded in obtaining a repetition of these movements by placing one of the frogs on a plate of iron, and touching the lumbar nerves with one end of a copper wire, the other end of which was in contact with the iron plate. Another mode of obtaining the result is represented in Fig. 91, two wires of different metals being employed which touch each other at one end, while their other ends touch respectively the lumbar nerves and the crural muscles. Every time the contact is completed, the limb is convulsed.

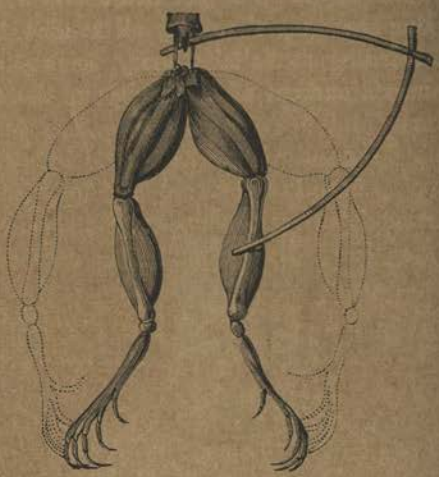


Fig. 91.—Experiment with Frog.

Galvani's explanation was, that at the junction of the nerves and muscles there is a separation of the two electricities, the nerve being positively, and the muscle negatively electrified, and that the convulsive movements are due to the establishment of communication between these two electricities by means of the connecting metals.

Volta, professor of physics at Pavia, disproved this explanation by showing that the movements could be produced by merely connecting two parts of a muscle by means of an arc of two metals; and he referred the source of electricity not to the junction of nerve and muscle, but to the junction of the two metals. Acting on this belief, he constructed in the year 1800 a voltaic pile.

152. Voltaic Pile.—This consisted of a series of discs of copper, zinc, and wet cloth, *c*, *z*, *d*, Fig. 92, arranged in uniform order, thus—

copper, zinc, cloth, copper, zinc, cloth . . . the lowest plate of all being copper and the highest zinc. The wet cloth was intended merely to serve as a conductor, and prevent contact between each zinc and the copper above it. All the contacts between zinc and copper were between a copper below and a zinc above, so that they all tended, according to Volta's theory, to produce a current of electricity in the same direction. The effects obtained from the pile were so powerful as to excite extraordinary interest in the scientific world.

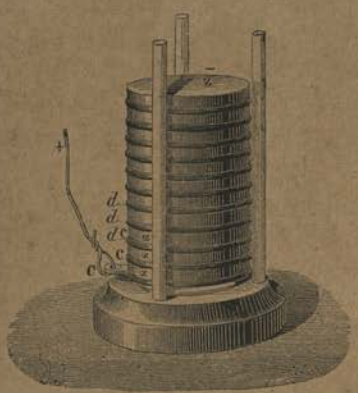


Fig. 92.—Structure of Pile.

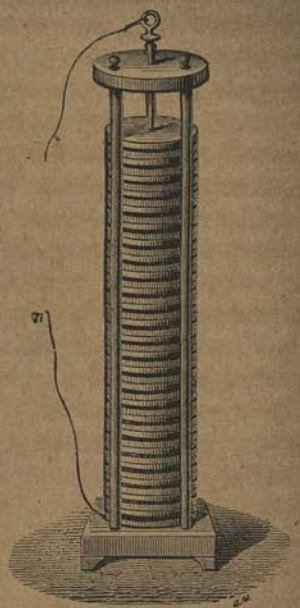


Fig. 93.—Complete Pile.

A *dry pile*, built up on the general plan of Volta's moist pile, was subsequently devised by De Luc, and improved by Zamboni. In Zamboni's construction, sheets of paper are prepared by pasting finely laminated zinc or tin on one side, and rubbing black oxide of manganese on the other. Discs are punched out of this paper, and piled up into a column, with their similar sides all facing the same way, to the number of a thousand or upwards, and are well pressed together. The difference of potential between the two ends is sufficient to produce sensible divergence of the gold-leaves of an electroscope, but the quantity of electricity which can be developed in a given time is exceedingly small. *No pile or battery can generate a sensible current, except by a sensible consumption of its materials in the shape of chemical action.*

A very delicate gold-leaf electroscope was devised by Bohnen-

berger, consisting of a single leaf suspended between the two poles of a dry pile, which for this purpose is arranged in two columns connected below, so that the poles are at the summits. If their lower ends, which form the middle of the series, be connected with the earth, one pole will always have positive, and the other negative potential. A very slight charge, positive or negative, given to the gold-leaf by means of the knob at the top of the case, suffices to make it move to the negative or the positive pole.

153. Couronne de Tasses. — Volta improved upon his invention of the pile by constructing the *couronne de tasses* (crown of cups),

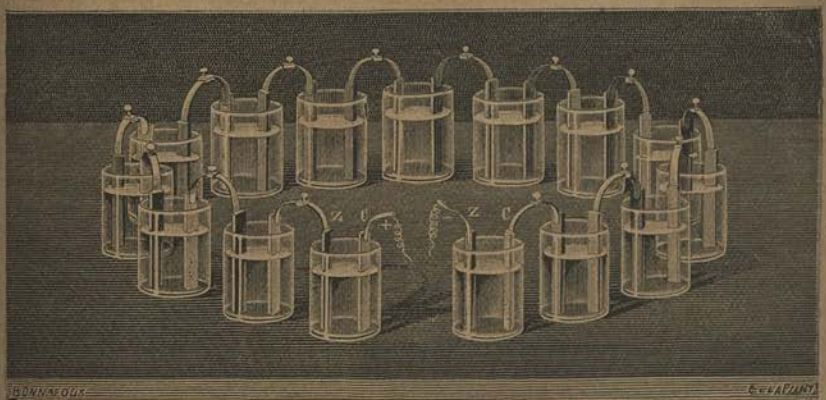


Fig. 94.—Couronne de Tasses.

consisting of a series of cups arranged in a circle, each containing salt water with a plate of silver or copper and a plate of zinc immersed in it, the silver or copper of each cup being connected with the zinc of the next.

154. Trough Battery.—More convenient arrangements, equivalent

Fig. 95.—Cruickshank's Trough.

to the couronne de tasses, were soon introduced. One of these, devised by Cruickshank, is represented in Fig. 95, consisting of a

rectangular box, called a trough, of baked wood, which is a non-conductor of electricity, divided into compartments by partitions each consisting of a plate of zinc and a plate of copper soldered together. Dilute acid is poured into these compartments.

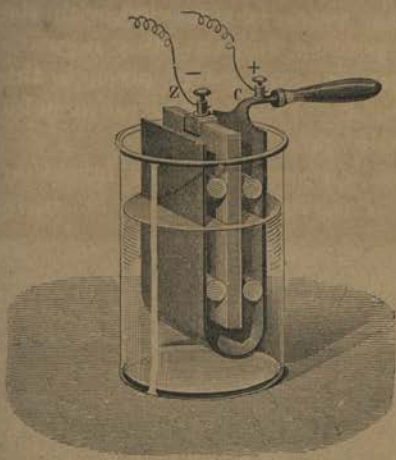


Fig. 96.—Wollaston's Cell.

155. Wollaston's Battery.—In Wollaston's battery, the plates were suspended from a single horizontal bar, by means of which they could all be let down into the acid, or lifted out of it together. The liquid was contained either in compartments of a trough of glazed earthenware, with partitions of the same material, or in separate vessels as shown in Fig. 97. The plates were double-coppered; that is to say, they consisted of a zinc plate with a copper plate bent round it on both sides (Fig. 96), contact between them being prevented by pieces of wood or cork.

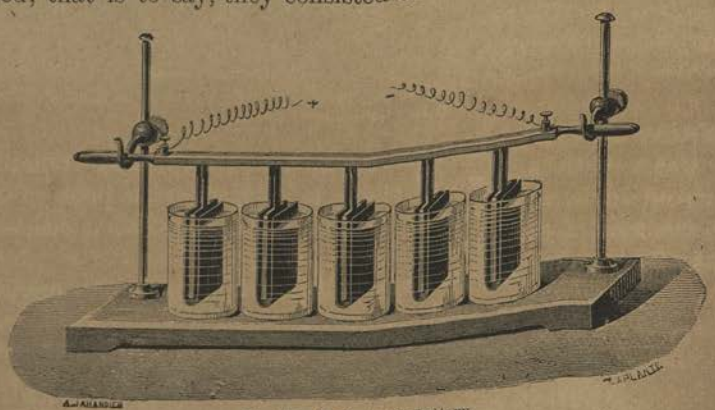


Fig. 97.—Wollaston's Battery.

156. Hare's Deflagrator.—For some purposes it is more important to diminish the resistance of a cell, or, in other words, to facilitate the conduction of electricity between the zinc and the copper plate, than to increase the electromotive force by multiplying cells. The

spiral arrangement devised by Hare of Philadelphia (Fig. 98) is specially adapted to such purposes. It consists of two very large plates of zinc and copper rolled upon a central cylinder of wood, and prevented from touching each other by pieces of cloth or twine inserted between them. It is plunged in a tub of acidulated water, as represented in the figure. From the remarkably powerful heating effects which can be obtained by the use of this cell, it is called Hare's *deflagrator*.

Fig. 98.—Hare's Deflagrator.

157. Polarization of Plates.—All the forms of battery which we have thus far described, are liable to a rapid decrease of power, owing to causes which are partly chemical and partly electrical.

The chemical action which takes place in each cell consists primarily in the formation of sulphate of zinc, at the expense of the zinc plate, the sulphuric acid, and the oxygen of the water with which the acid is diluted, the hydrogen of the water being thus liberated. As this action proceeds, the liquid becomes continually less capable of acting powerfully on the zinc. Again, a portion of the zinc which has been dissolved becomes deposited on the copper plate, thus tending to make the two plates alike, and so to destroy the current, which essentially depends on the difference between them.

But the most important cause is to be found in what is called the *polarization* of the copper plate; that is to say, in the deposition of a film of hydrogen on the surface of the plate. This film not only interposes resistance by its defect of conductivity, but also brings to bear an electromotive force in the direction opposed to that of the current.

These obstacles to the maintenance of a constant current were first overcome by Daniell.

158. Daniell's Battery.—In the cell devised by Daniell, there is a

porous partition of unglazed earthenware, separating the two liquids, which are in contact one with the zinc, and the other with the copper plate. These two liquids are not precisely alike, that which is in contact with the copper being not simply dilute sulphuric acid like the other, but containing also as much sulphate of copper as it will take up. For the purpose of keeping it saturated, crystals of sulphate of copper are suspended in it near its surface by means of a wire basket of copper. The effect of this arrangement is, that the hydrogen is intercepted before it can arrive at the copper plate, and the deposit which takes place on the copper plate is a deposit of copper, the hydrogen taking the place of this copper in the saturated solution.

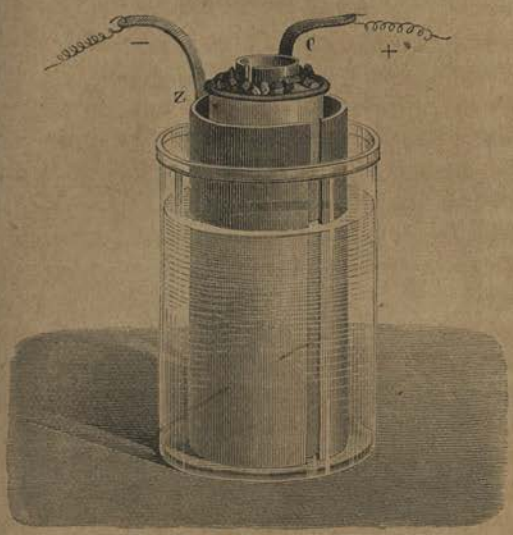


Fig. 99.—Daniell's Cell.

The current given by a battery of these cells remains nearly constant for some hours.

In the figure, the copper plate C is represented as a cleft cylinder occupying the interior, with the crystals of sulphate of copper piled up round it. The entire cylinder surrounding these is the porous partition, outside of which is the cleft cylinder of zinc Z, the whole being contained in a vessel of glass.

It is more usual in this country to dispense with the glass vessel, and interchange the places of the zinc and copper in the figure, the copper plate being a cylindrical vessel of copper containing the saturated solution. In this is immersed the porous vessel containing the other fluid with the zinc plate immersed in it. The cells thus constructed are usually arranged in square compartments in a wooden box.

159. Bunsen's Battery.—One of the best-known forms of battery is that which was invented by Bunsen in 1843, being substantially

identical with one previously invented by Grove, except that carbon is substituted for platinum.

The usual construction of its cells is very clearly represented in Fig. 100, and the mode of connecting them in Fig. 101. The cleft cylinder is the zinc plate, which is immersed in dilute sulphuric acid. Within this is the porous cylinder, similar to Daniell's, containing *strong nitric acid*, in which is immersed a rectangular prism, of a very dense kind of charcoal, obtained from the interior of the retorts at gas-works, being deposited there in the manufacture of gas.

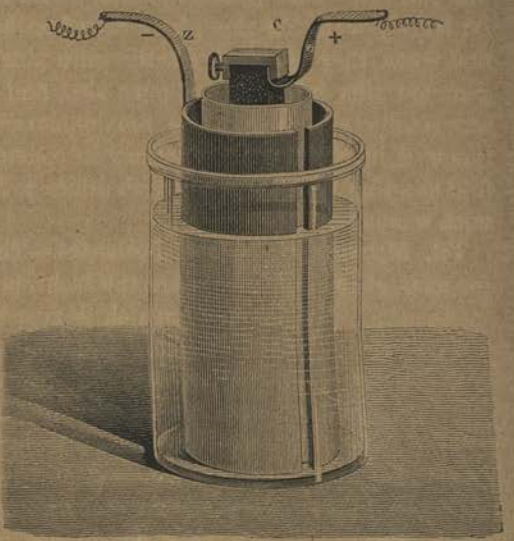


Fig. 100.—Bunsen's Cell.

In this cell the hydrogen is intercepted on its way to the carbon plate by the nitric acid, with which it forms nitrous acid.

Fig. 101.—Bunsen's Battery.

Grove's battery possesses some advantages over Bunsen's, but its first cost is much greater.

160. Amalgamated Zinc.—When the poles of a battery are insulated from one another, there ought to be no chemical action in the cells

Any action which then goes on is wasteful, and is an indication that unproductive consumption of zinc goes on when the current is passing, in addition to the consumption which is necessary for producing the current. This wasteful action, which is called *local action*, goes on largely when the zinc plates are of ordinary commercial zinc, but not when they are of perfectly pure zinc. In this respect amalgamated zinc behaves like pure zinc, and it is accordingly almost universally employed. The amalgamation, which must be often renewed in the case of a battery in constant use, is performed by first cleaning the zinc plates with dilute acid, and then rubbing them with mercury.

161. Bichromate. Fuller. Leclanché.—The most convenient cells for most class experiments are the bichromate of potash bottle-cells, one of which is represented in Fig. 102. The liquid is a solution of bichromate of potash, with a little sulphuric acid added. In this liquid two flat plates of carbon are suspended, and between them is a flat plate of zinc, which can be slid up and down by means of a rod projecting through the top of the cell. It is slid up when not in use, and is then just clear of the liquid. By pushing it down the cell is brought into full action, and as soon as the experiment for which it is required is concluded the zinc should again be raised out of the liquid. The cell is not suited for long-continued work, but it gives powerful effects when only used for a few minutes at a time. It has the conveniences of great portability and of freedom from noxious fumes.

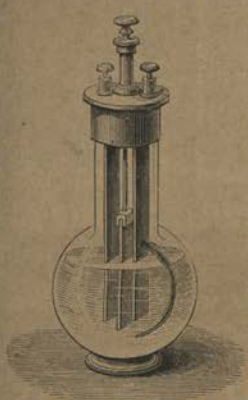


Fig. 102.—Bichromate Bottle-cell.

For long-continued use the two-fluid bichromate battery known as Fuller's is very effective. One plate is of carbon immersed in the solution above described. The other plate is a block of zinc cast on to the end of a thin copper rod, at the top of which is the binding screw. Mercury is poured in till the zinc is completely covered, and the cell is filled up with dilute acid. The copper rod is usually coated with gutta percha to protect it from the acid. If not so coated it must be amalgamated.

The favourite battery for electric bells is that of Leclanché. One of its plates is a zinc rod immersed in a solution of sal-ammoniac.

The other is a plate of carbon, tightly packed in a porous cell with a mixture of peroxide of manganese and gas-carbon, moistened with water, or with the same liquid as that in the outer vessel. It is suitable for sending brief currents at any time during many months.

CHAPTER XIV.

GALVANOMETER.

162. Œrsted's Experiment.—The discovery by the Danish philosopher Œrsted, in 1819, that a magnetized needle could be deflected by an electric current, was justly regarded with intense interest by the scientific world, as affording the first indication of a definite relation existing between magnetism and electricity.

Œrsted's experiment can be repeated by means of the apparatus represented in Fig. 103. Two insulated metallic wires are placed in the magnetic meridian, one of them above, and the other below a magnetized needle. If a current be sent through one of these wires, the needle will be deflected; and if the current be strong, the deflection will amount nearly to a right angle. The direction of the deflection will be reversed if the current be passed through the lower instead of the upper wire. It will also be reversed by reversing the direction of the current. In the figure, the current is supposed to be passing above the needle from south to north. In this case the north end of the needle moves to the west, and the south end to the east. On making the current pass in various directions, either horizontally, vertically, or obliquely, near one pole of the needle, it will be found that deviation is always produced except when the plane containing the pole and current is perpendicular to the length of the needle.

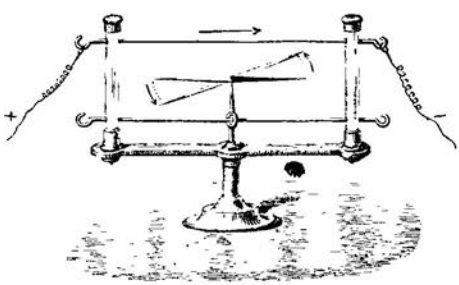


Fig. 103.—Œrsted's Experiment.

163. Ampère's Rule.—The direction in which either pole of a needle is deflected by a current, whatever their relative positions may be, is given by the following rule, which was first laid down by Ampère.

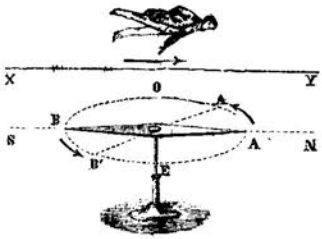


Fig. 104.

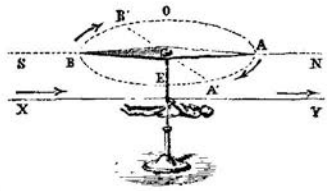


Fig. 105.

Ampère's Rule.

Imagine an observer to be so placed that the current passes through him, *entering at his feet* and leaving at his head (or that he is swimming with the current and looking at the needle), then the deflection of a *north-seeking pole* will be *to his left.* The deflection of a south-seeking pole will be in the opposite direction. The two figures 104, 105 illustrate the application of this rule to the two cases just considered. A is the austral or north-seeking pole of the needle, and B the boreal or south-seeking pole.[1]

164. Lines of Magnetic Force due to Current.—The relation between currents and magnetic forces may be more precisely expressed by saying that a current flowing through a straight wire produces circular lines of force, having the wire for their common axis. A pole of a magnet placed anywhere in the neighbourhood of the wire, experiences a force tending to urge it in a circular path round the wire, and the direction of motion round the wire is opposite for opposite poles. Fig. 106 represents three of the lines of force for a north-seeking pole, due to a current flowing through a straight wire from the end marked + to the end marked −. The lines of force are circles (shown in perspective as ellipses), having their centre

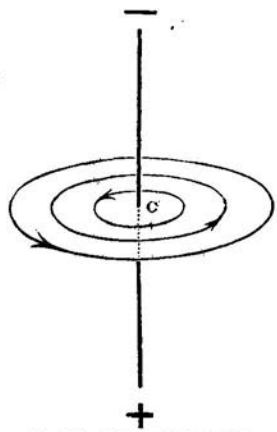


Fig. 106.—Lines of Force due to Current.

[1] A corkscrew advancing in the direction of the current turns the same way as the north-seeking pole.

at a point C in the wire, and having their plane perpendicular to the length of the wire. The arrows indicate the direction in which a north-seeking pole will be urged.[1] This direction is from right to left round the wire as seen from the wire itself by a person with his feet towards + and his head towards −, according to Ampère's rule. The figure may be turned upside down, or into any other position, and will still remain true.

165. Reaction of Magnet on Current.—While the wire, in virtue of the current flowing up through it, urges an austral pole from A towards A′ (Fig. 107), it is itself urged in the opposite direction C C′. If an observer be in imagination identified with the wire, the current being supposed, as in Ampère's rule, to enter at his feet, and come out at his head, the force which he will experience from a north-seeking pole directly in front of him will be a force to his right. It will be noted that the magnetic influence which thus urges him to the right, would urge a north-seeking pole from his front to his back. *A conductor conveying a current is not urged along lines of magnetic force, but in a direction which is at right angles to them, and at the same time at right angles to its own length.*[2]

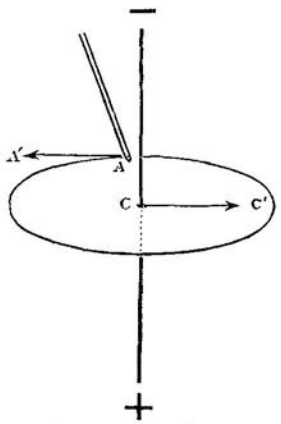


Fig. 107.—Reaction on Current.

166. Numerical Estimate of Currents.—The numerical measure of a current denotes the quantity of electricity which flows across a section of it in unit time. It is sometimes called *strength* of current, sometimes, especially by French writers, *intensity* of current, sometimes simply *current* or *amount* of current. If a thin and a thick wire are joined end to end, it has the same value for them both; just as the same quantity of water flows through the broad as through the contracted parts of the bed of a stream. Hence the name *intensity* is obviously inappropriate, for, with the same total quantity of

[1] The *direction of a line of magnetic force* is by general agreement taken to mean the direction in which a *north-seeking* pole would be urged.

[2] The following brief rules, the second of which will be established hereafter, are collected here for convenience of reference:—

In field from front to back, current from foot to head is urged to right.

In field from front to back, motion to right produces current from head to foot.

electricity flowing through both, the current is, properly speaking, more *intense* in the thin than in the thick wire

Currents may be measured experimentally by various tests, which are found to agree precisely. The most convenient of these for general purposes is the deflection of a magnetized needle. The force which a given pole experiences in a given position with respect to a wire conveying a current, is simply proportional to the current. Hence the name *strength* of current admits of being interpreted in a sense corresponding to that in which we speak of the strength of a pole. Instruments for measuring currents by means of the deflections which they produce in a magnetized needle are called *galvanometers*.

167. Multiplier.—The idea of carrying a wire several times round a needle in a vertical plane is due to Schweiger. The form of apparatus designed by him, called *Schweiger's multiplier*, is represented in Fig. 108. The name *multiplier* is derived from the fact that, if the current is not sensibly diminished by increasing the number of convolutions of wire through which it has to pass, the force exerted on the needle is n times as great with n convolutions as with only 1, since each convolution exerts its own force on the needle independent of the rest. Cases, however, frequently occur in which the increased *resistance* introduced by increasing the number of convolutions outweighs the advantage of multiplication, so that a short thick wire with few convolutions gives a more powerful effect than a long thin wire with many. This is especially the case with thermo-electric currents.

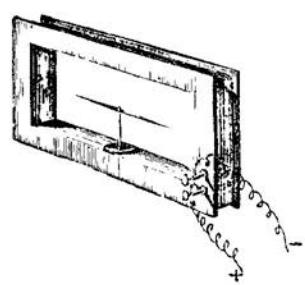


Fig. 108.—Schweiger's Multiplier.

The difference between the rectangular and the circular form is merely a matter of detail. Whichever form be adopted, all parts of the coil contribute to make the needle deviate in the same direction. For instance, in Fig. 109, if the current proceeds in the direction indicated by the arrows, the application of Ampère's rule to any one of the four sides of the rectangle shows that the austral pole a will be urged towards the front of the figure. When the coil is circular, and the needle so small that

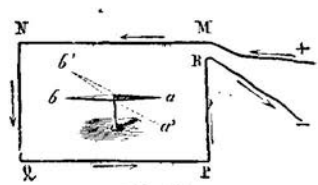


Fig. 109.

each pole is nearly in the centre, equal lengths of the current, in whatever parts of the circle they may be situated, exert equal forces upon the needle, and all alike urge the poles in directions perpendicular to the plane of the coil.

168. **Sine Galvanometer.**—The sine galvanometer, which was invented by Pouillet, is represented in Fig. 110. The current which is to be measured traverses a copper wire, wrapped round with silk for insulation, which is carried either once or several times round a vertical circle; and this circle can be turned into any position in azimuth, the amount of turning being indicated on a horizontal circle. In the centre of the vertical circle, a declination needle is mounted, surrounded by a horizontal circle for indicating its position, this circle being rigidly attached to the vertical circle. Suppose that, before the current is allowed to pass, both the needle and the vertical circle are in the magnetic meridian, and that the needle consequently points at zero on its horizontal circle. On the current passing, the needle will move away. The vertical circle must then be turned until it overtakes the needle; that is, until the needle again points at zero. This implies turning the circles through an angle α equal to that by which the needle finally deviates from the magnetic meridian. In this position the terrestrial couple tending to bring back the needle to the meridian is proportional to $\sin \alpha$ (§ 116). The forces exerted upon the two poles by the current are perpendicular to the plane of the vertical circle, and are simply proportional to the current. Hence, in comparing different observations made with the same instrument, the amounts of current are proportional to the sines of the deviations.

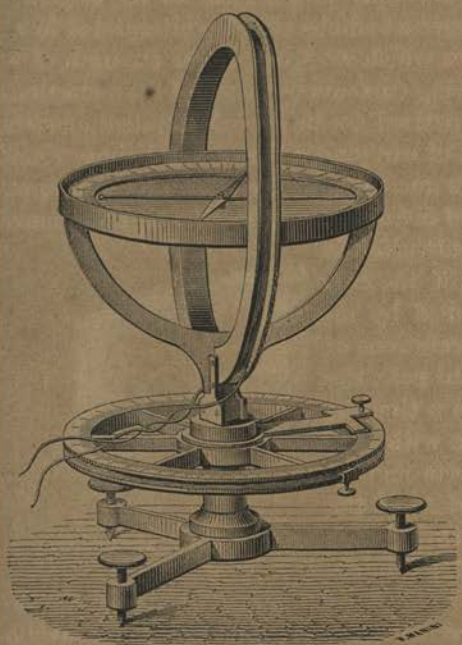


Fig. 110.—Sine Galvanometer.

169. Tangent Galvanometer.—The tangent galvanometer, which is simpler in its construction and use, and is much more frequently employed, consists of a declination needle mounted in the centre of a vertical circle whose plane always coincides with the magnetic meridian, the length of the needle being small in comparison with the radius of the circle.

Let o (Fig. 111) be the centre of suspension, $a\,b$ the initial position of the needle, and $a'\,b'$ its deflected position. The force F exerted on either pole by the current is sensibly the same at a' as at a, on account of the smallness of the needle, and it acts in the direction $l\,k$, while the horizontal force of the earth upon the pole acts along $a'\,m$; and these two forces give a resultant along $o\,a'$. Hence, taking the triangle $o\,l\,a'$ as the triangle of forces, the force exerted by the current is to the horizontal force exerted by the earth as $l\,a'$ to $o\,l$, or as *tan* α to unity; that is, the current is proportional to the tangent of the deflection.

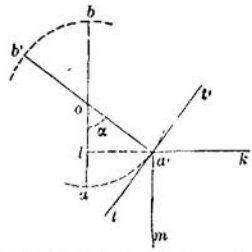


Fig. 111.—Principle of Tangent Galvanometer.

In order to permit the deviation of the short needle to be accurately read, a long pointer is attached to it, usually at right angles, the two ends of which move along a fixed horizontal circle.

When the needle is deflected, it is no longer in the plane of the coil, and this circumstance complicates the relation between current and deflection. Helmholtz has overcome this difficulty by placing the needle midway between two equal and parallel coils, whose distance apart is equal to the radius of either, the two being connected in series so that the same current flows through both. The lines of magnetic force in the intervening region can be shown to be very nearly straight.[1]

170. Differential Galvanometer.—The coil of a galvanometer sometimes consists of two distinct wires, having the same number of convolutions, and connected with separate binding-screws. This arrangement allows of currents from two distinct sources being sent at the same time round the coil either in the same or in opposite directions. In the latter case, the resultant effect upon the needle will be that due to the difference of the two currents; and if they

[1] A drawing of the lines will be found in Maxwell's *Electricity and Magnetism*, vol. ii. fig. xix.

are not exactly equal, the direction of the deflection will indicate which of them is the greater. An instrument thus arranged is called a *differential galvanometer.*

171. Astatic Needle.—The sensibility of the galvanometer is greatly increased by employing what is called an *astatic* needle. It consists of a combination of two magnetized needles *with their poles turned opposite ways.* The two needles are rigidly attached at different heights to a vertical stem, and the system is usually suspended by a silk fibre, which gives greater freedom of motion than support upon a point. On account of the opposition of the poles, the directive action of the earth on the system is very feeble. If the magnetic moments of the two needles were exactly equal, and their axes exactly parallel, the resultant moment would be zero, and the system would remain indifferently in all azimuths.

One of the needles $a\,b$ (Fig. 112) is nearly in the centre of the coil C D E F through which the current passes. The other $a'\,b'$ is just above the coil. When a current traverses the coil in the direction of the arrows, the action of all parts of the current upon the lower needle tends to urge the austral pole a towards the back of the figure, and the boreal pole b to the front. The upper needle $a'\,b'$ is affected principally by the current in the upper part CD of the coil, which urges the austral pole a' to the front of the figure, and the boreal pole b' to the back. Both needles are thus urged to rotate in the same direction by the current, and as the opposing action of the earth is greatly enfeebled by the combination, a much larger deflection is obtained than would be given by one of the needles if employed alone.

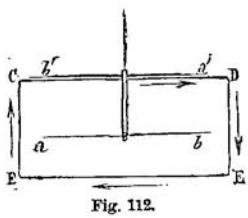


Fig. 112.

If the two needles had rigorously equal moments, the system would be said to be *perfectly astatic.* The smallest current in the coil would then suffice to set the needles at right angles to the meridian, and no measure would be obtained of the amount of current.

Fig. 113 represents an astatic galvanometer, as usually constructed. The coil is wound upon a wooden frame, which supports the divided circle in whose centre the upper needle is suspended. The ends of the coil are connected with two binding-screws, for making connection with the wires which convey the current to be measured.

The needles are usually two sewing-needles, and the upper one often carries a light pointer. The suspending fibre is attached at its upper end to a hook, which can be raised or lowered, and when the instrument is not in use this is lowered till the upper needle rests upon the plate beneath it, so as to relieve the fibre from strain. In using the instrument care must be taken to adjust the three levelling-screws so that the needle swings free.

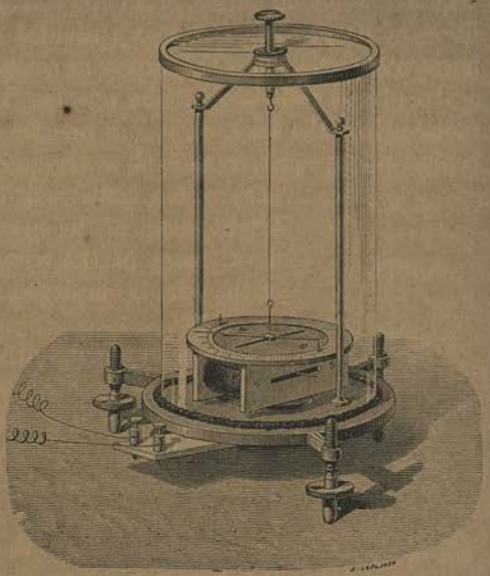


Fig. 113.—Astatic Galvanometer.

172. Thomson's Mirror Galvanometer.—A more sensitive instrument is the mirror galvanometer of Sir W. Thomson. Its needle, which is very short, is rigidly attached to a small light concave mirror, and suspended in the centre of a vertical coil of very small diameter by a silk fibre. A divided scale is

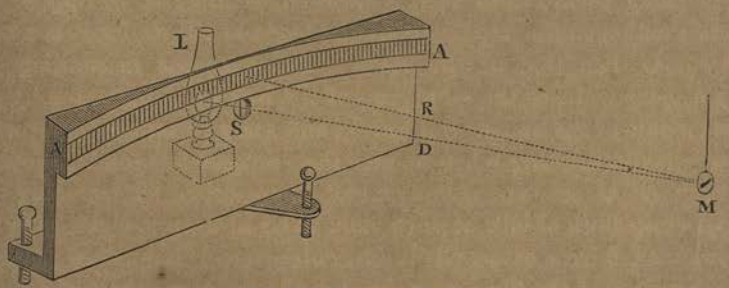


Fig. 114.—Mirror and Scale.

placed in a horizontal position in front of the mirror, at the distance of about a yard, and the image of an illuminated slit, which is thrown by the mirror upon this scale, serves as the index. The arrangement of the mirror and scale, which is the same as in the case of the quadrant electrometer described in a previous chapter, is exhibited in

Fig. 114. M is the mirror of silvered glass, slightly concave, with a small piece of magnetized watch-spring attached to its back, the two together weighing only a grain and a half, and suspended by a few fibres of unspun silk. AA is a divided scale forming an arc of a horizontal circle about the mirror as centre. Immediately below the centre of this scale is a circular opening S with a fine wire stretched vertically at the back of it. A paraffin lamp L is placed directly behind this opening, so as to shine through it upon the mirror, which is at such a distance as to throw upon the screen a bright image of the opening with a sharply-defined dark image of the wire in its centre. The image of the wire is employed as the index in taking the readings.

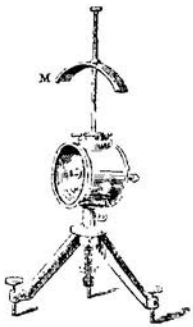


Fig. 115.—Thomson's Mirror Galvanometer.

In order to obviate the necessity of keeping the needle in the meridian, with the lamp east or west of it, and to admit of other positions, a controlling magnet M (Fig. 115) is provided, which can be raised or lowered, and can also be turned round. When it is low down it overpowers the earth's magnetism, and compels the needle to take any position that may be required.

For maximum sensitiveness, the magnetic field around the needle should be made as weak as possible. For this purpose the needle should be placed in or not far from the meridian, and the magnet, after being turned into such a position as directly to oppose the earth's action on the needle, should be lowered till its force is a very little less than that of the earth. The operator in making the adjustment watches the vibrations of the needle, as indicated by the movements of the image on the scale, and knows that the force on the needle is diminishing when he sees the vibrations becoming slower.

For use at sea the galvanometer is modified by fastening the supporting fibre of silk at both ends, so as to keep it tight, with the needle and mirror attached at its centre, care being taken to make the direction of the fibre pass through the common centre of gravity of the needle and mirror, in order that the rolling of the ship may not tend to produce rotation. In this form it is called the *marine galvanometer*.

173. Direct-reading Galvanometers. Ammeters.—In many galvanometers the connection between strength of current and magnitude of deflection cannot be expressed by a simple law.

It is becoming customary in instruments for commercial use to place the graduations not at equal distances, but at such distances that the reading shall be simply proportional to the current. The instruments are then called *direct-reading* galvanometers.

This mode of graduation is especially common in instruments intended for measuring very strong currents, such as those produced by dynamos. These instruments have a controlling magnet, whose field is so strong as completely to overpower that of the earth. Their readings usually give the current in "amperes," in which case the instrument is called an *ammeter* (short for ampere-meter).

174. General Law for Magnetic Force due to a Current.—In every case, the magnetic force at a given point due to a current, can be computed by dividing the current into elementary portions, each sensibly straight, and compounding by the parallelogram of forces the effects due to these separate elements. The force due to each element is normal to the plane drawn through the element and the given point, and is proportional to $C\frac{l}{r^2}\sin\theta$, where C denotes the strength of the current, l the length of the element, r the distance between the element and the given point, and θ the angle between the joining line and the element. The force at the centre of a single circular current of radius a is therefore $C\frac{2\pi a}{a^2}=C\frac{2\pi}{a}$, and the force at the centre of a circular galvanometer-coil of n convolutions, if all can be regarded as in one plane and of the same radius a, is $C\frac{2\pi n}{a}$.

175. Instantaneous Current; Ballistic Galvanometer.—When the duration of a current is small in comparison with the time of vibration of the needle, and the total deflection small, the velocity which the current gives the needle is jointly proportional to the duration of the current and its average strength. It is, therefore, simply proportional to the quantity of electricity which passes. This velocity is equal to that acquired in the return movement to zero; and this latter obviously follows the same law as the motion of a simple pendulum, for in both cases the effective force is proportional to the sine of the displacement. In the case of the pendulum, the square of the velocity acquired in the whole descent is proportional to the vertical height descended, and this vertical height multiplied by the diameter of the circle in which the pendulum moves, is equal to the square of the chord; hence, the velocity acquired is propor-

tional rigorously to the chord, and approximately to the arc of descent if small. The same rule must hold for the needle; that is to say, the velocity acquired must be proportional to the extreme displacement. . *The quantity of electricity transmitted through the galvanometer coil by an instantaneous discharge is* therefore *proportional to the distance to which the needle swings.*

Galvanometers specially intended for this use are called *ballistic galvanometers.*

176. The Galvanometer a True Measurer of Current.—This reasoning assumes the principle that the force exerted by a current on a needle is a true measure of the strength of the current (defined as the quantity of electricity conveyed per unit of time); and conversely, the observed fact that, when known quantities of electricity are discharged through a galvanometer, the swings produced are proportional to these quantities, establishes the principle. The experiment has frequently been made by discharging a condenser (§ 73) which has been charged by a galvanic battery; and Faraday obtained a similar result with Leyden-jars which had been charged by a powerful frictional machine, the jars being discharged through a wet thread or string leading to the galvanometer. He found that the swing was independent of the length and thickness of the thread or string, as well as of the number of jars employed, and was proportional to the number of turns that had been given to the electrical machine in charging the jars.

The proportionality of force to current might have been inferred *à priori* from the consideration that, if we have two parallel wires close together, conveying equal currents, the resultant force on a pole will be the sum of the forces due to each, and will therefore be double of the force due to one alone. The force will not be altered by allowing the wires to touch each other all along their length; and in this position they form a single conductor conveying a double current.

177. Needle Deflected by Motion of a Charged Body.—The question has been raised whether the carrying of electricity by the motion of a charged body produces effects similar to those of a current flowing through a conductor, and in particular, whether it is capable of deflecting a magnetized needle. The matter was put to the test by Professor Rowland, in an experiment performed at the laboratory of the Berlin University.[1]

[1] See *Phil. Mag.* September, 1876, pp. 211-216.

The carrier of the electricity was a rapidly revolving horizontal disc of ebonite, gilt on both sides, and maintained in a high state of electrification by means of a fixed discharging point connected with one of the coatings of a battery of Leyden-jars. The needle to be deflected was suspended over it near its circumference, the length of the needle being perpendicular to the radius of the disc, so that the motion of the electricity beneath the needle was parallel to its length. Between the needle and the revolving disc, a larger fixed disc of glass, gilt on one side and connected with the earth, was interposed; and there was a similar disc on the lower side. The needle was one of an astatic pair, the other needle being at a much greater height; and both were inclosed in a brass case, to protect them from electrostatic influences. The deflection was observed by means of a mirror attached to the stem of the needles, and a telescope for viewing in the mirror the reflected image of a scale. The disc, which was $8\frac{1}{3}$ inches in diameter, revolved at the rate of about 60 turns per second, and the deflections observed amounted to from 5 to $7\frac{1}{2}$ divisions of the scale, the deflection being to the one side or the other, according as the charge of the disc was positive or negative. The observations extended over several weeks, and conclusively proved, subject to small errors of observation and reduction, that the magnetic effect of carrying a charge of electricity is the same as that of the flow of the same quantity of electricity in the same time through a conductor.

CHAPTER XV.

ELECTRO-CHEMISTRY.

178. **Electrolysis.**—When a current is passed through a compound liquid, decomposition is frequently observed, two of the component substances being separated, one at the place where the current enters and the other at the place where it leaves the liquid. This decomposition is called *electrolysis*, and the substance decomposed or *electrolysed* is called the *electrolyte*. The action only occurs in the case of liquids, and these must be conductors.

The process may be illustrated by the decomposition of water as represented in Fig. 116. The apparatus consists of a vessel con-

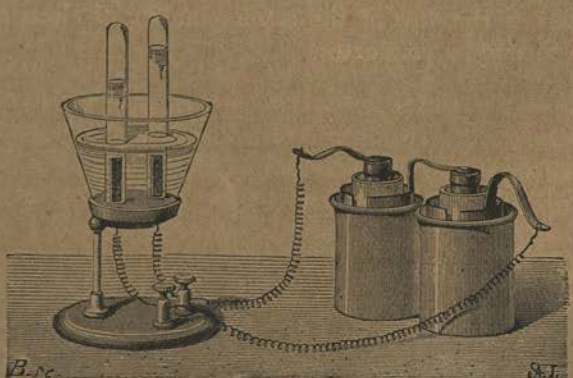


Fig. 116.—Voltameter.

taining water to which a little sulphuric acid has been added, and in which two strips of platinum are immersed, connected respectively with the two poles of a battery. When the connections are completed, bubbles make their appearance at the surfaces of the two

strips and rapidly rise to the surface. If two tubes filled with the liquid are inverted over the two strips, the gases will bubble up through the liquid into the upper part of these tubes and the level of the liquid will gradually fall as shown in the figure. It will be found that the volume of the hydrogen is about double that of the oxygen.

The two strips of platinum are called the poles or *electrodes* of the decomposing cell; the one in connection with the positive pole of the battery is called the *anode* (literally the *way up*) and the one in connection with the negative pole the *cathode* or *kathode* (*way down*). The direction of the current through the liquid is from the anode to the kathode. The gas which is given off at the anode (in the present case oxygen) is called the *anion* (*that which goes up*) and that which is given off at the kathode the *kathion* or *cation* (*that which goes down*). The anion is often called the *electro-negative* element, because it moves as if attracted by the positive and repelled by the negative pole. For a similar reason the kathion is called the *electro-positive* element.

In many cases, the separation effected by the direct action of the current is followed by secondary actions due to chemical affinities.

Thus, in the decomposition of acidulated water above described, the first effect, according to modern theory, is a breaking up of the sulphuric acid ($S O_3$, $H_2 O$) into hydrogen and sulphion (2 H and $S O_4$), the latter being a substance which has never been obtained by itself. The hydrogen travels to the negative pole and there escapes. The sulphion goes to the positive pole, but instead of escaping enters into combination with the hydrogen of the liquid, forming again the primitive compound ($S O_3$, $H_2 O$) and leaving the oxygen of the liquid to escape.

179. Transport of Elements.—It is a remarkable fact that the separated elements never make their appearance except at the electrodes. Nothing is seen of them, nor is any action exhibited, at intermediate points. The appearance is as if the gases could vanish from one extremity and appear at the other without passing through the intermediate space. The only possible explanation of this phenomenon seems to be what is known as Grotthus'

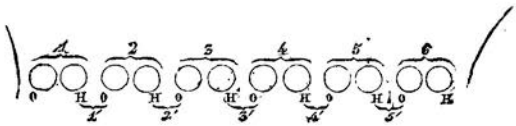


Fig. 117.—Grotthus' Hypothesis.

hypothesis, that all the particles of the water in the course of the current undergo continual decomposition and recomposition. Thus if Fig. 117 represent a line of particles traversed by the current from left to right, there will be a continual stream of hydrogen particles along this line from left to right, and a stream of oxygen[1] particles from right to left. The hydrogen of molecule 1 will combine with the oxygen of molecule 2 to form a new molecule 1′; the hydrogen of molecule 2 will combine with the oxygen of molecule 3 to form a new molecule 2′, and so on. The oxygen of molecule 1 is given off at the left-hand extremity, which we suppose to be the point of contact with one of the strips of platinum, and the hydrogen of molecule 6 at the other strip. The molecules 1′, 2′, 3′ . . . are then in their turn decomposed to form a new set. In actual cases, the number of molecules, instead of being only six as represented in the figure, is of course many millions.

180. Discovery of Potassium.—When a compound formed by the union of a metal with a non-metallic substance is submitted to electrolysis, the metal always comes to the negative pole. It was in this way that several of the metals were first obtained from their oxides by Sir Humphry Davy. Potassium, for example, was obtained by placing a piece of potash on a platinum disc connected with the negative pole of a battery of 250 cells, and then applying a platinum wire connected with the positive pole to its upper surface. The potash, which had been allowed to contract a little moisture from the atmosphere, in order to give it sufficient conducting power, soon began to fuse at the points of contact of the electrodes. A violent effervescence occurred at the upper or positive electrode; while at the lower surface small globules appeared resembling quicksilver, some of which instantly burst into flame, while others merely became tarnished and afterwards coated over with a white film.

181. Electrolysis of Salts.—When a salt of any of the less inflammable metals is submitted to electrolysis, a continual deposition of the metal is observed on the negative electrode; while, at the positive electrode, oxygen is disengaged, and acid set free. These effects occur, for example, if platinum electrodes are plunged in a solution of sulphate of copper. If copper is employed as the positive electrode, the oxygen will combine with it instead

[1] Or sulphion particles according to modern theories. The explanation as it stands in the text represents the views held by Faraday and his contemporaries.

of being given off, and the oxide thus formed will be dissolved by the acid.

182. **Voltameters.**—The quickness with which a given electrolyte is decomposed is simply proportional to the strength of the current; and any apparatus designed for measuring the strength of a current on this principle is called a *voltameter*. The apparatus in Fig. 116 is one of its forms. The commonest form consists of a cell containing a solution of sulphate of copper, with two copper plates for electrodes. As the current passes, the anode is gradually dissolved away, and an equal quantity of copper is deposited on the kathode. The quantity of electricity that has passed through the cell can accordingly be determined by weighing the plates. Copper, however, has the drawback that it is to some extent acted on by the solution when no current is passing. When the greatest possible accuracy is required, two plates of silver immersed in a solution of nitrate of silver are preferred.

183. **Quantitative Laws of Electrolysis.**—The following table gives, in the column headed "chemical equivalent," the quantities of the several elements deposited by one and the same quantity of electricity. It will be seen from the table that the same element in different states of combination may have different equivalents. The general law for such cases is that the equivalent is equal to the atomic weight (which is definite for each element) divided by the "valency." The "valency" expresses the number of atoms of hydrogen which are replaced by one atom of the substance.

Electro-positive.	Atomic weight.	Valency.	Chemical equivalent.
Hydrogen,	1	1	1
Potassium,	39·1	1	39·1
Sodium,	23·	1	23·
Gold,	196·6	3	65·5
Silver,	108·	1	108·
Copper (Cupric),	63·	2	31·5
„ (Cuprous),	63·	1	63·
Mercury (Mercuric),	200·	2	100·
„ (Mercurous),	200·	1	200·
Tin (Stannic),	118·	4	29·5
„ (Stannous),	118·	2	59·
Iron (Ferric),	56·	3	18·7
„ (Ferrous),	56·	2	28·
Nickel,	59·	2	29·5
Zinc,	65·	2	32·5
Lead,	207·	2	103·5
Aluminium,	27·	3	9·

	Atomic weight.	Valency.	Chemical equivalent.
Electro-negative.			
Oxygen,	16·	2	8·
Chlorine,	35·5	1	35·5
Iodine,	127·	1	127·
Bromine,	80·	1	80·
Nitrogen,	14·	3	4·3

The quantity of a substance that is deposited by the passage of a unit of electricity is called the *electro-chemical equivalent* of the substance. The electro-chemical equivalent of silver, according to the latest and best determinations, is ·01118 of a gramme for one C.G.S. unit of electricity, or ·001118 of a gramme for one "coulomb." The electro-chemical equivalents of the other substances mentioned in the above table can be calculated from this by simple proportion.

The above table holds good also for battery cells, provided that there is no wasteful "local action." The quantity of zinc dissolved in each cell of a battery is chemically equivalent to the quantity of any metal that is deposited in an electrolytic cell by the same current.

184. Chemical Relations of Electro-motive Force.—The energy of the current produced by a battery is equal to the potential energy of chemical affinity which runs down in its production; and this latter is measured by the heat of combination due to the chemical action which goes on in the battery. If there are decomposing cells in circuit, they contribute negative heat of combination, and there can be no current unless the total heat of combination for the whole circuit be positive.

The energy of the current for a given time is equivalent to the total heat of combination due to the action which takes place in this time; and if this energy be divided by the quantity of electricity conveyed by the current in the time, the quotient is called the *electro-motive force* of the circuit.

The quantity of electricity conveyed bears a definite relation of equivalence to the zinc dissolved in any one cell; and hence the electro-motive force of a cell is proportional to the heat of combination due to the action which takes place during the consumption of a given quantity of zinc. This heat is about twice as great for a Grove's cell as for a Daniell, and hence the electro-motive force of the former is about double that of the latter.

185. Secondary Batteries.—We have stated in § 157 that in some

forms of battery a reverse electromotive force is produced by the deposition of gas on the surface of the plates, and that this action is called *polarization*. A similar effect occurs in the voltameter of Fig. 116; and if the oxygen and hydrogen, as they are given off, are collected in separate tubes, each platinum plate being in contact both with one of the gases and with the liquid, the voltameter, after being disconnected from the source of the original current, can be used as a battery cell giving a current in the opposite direction. This is the principle of *Grove's Gas Battery*, in which the tubes may be filled with oxygen and hydrogen either by electrolysis or in any other manner, and any number of the cells may be connected in series. During the flow of the current which this battery gives, the two gases of each cell gradually unite through the medium of the acidulated water between them.

Any cell which is first subjected to electrolysis by a current from an external source, and then gives of itself a current in the opposite direction, is called a *secondary cell*, and a combination of such cells is a *secondary battery*. Recent improvements in their construction have called public attention to them as an important means of storing up energy, to be used when and where it is wanted. Hence they have received the name of *storage batteries* or *accumulators*.

186. Faure's Accumulator.—Faure's Accumulator consists of two leaden plates of large surface, covered with minium (red oxide of lead), rolled up together with flannel between them, and immersed in dilute sulphuric acid. The primary current, which is usually supplied by a dynamo machine, changes a portion of the minium on the anode into peroxide of lead, and reduces a portion of the minium on the cathode to metallic lead in a spongy condition. It is probably to the presence of these two substances with dilute sulphuric acid between them, that the secondary current is due. A battery of these cells freshly charged by a dynamo machine will give a powerful current for some hours; and an interval of a few hours or even of a few days between charging and using does not involve very much loss.[1]

Planté had previously constructed secondary cells consisting in like manner of two lead plates spirally coiled in dilute sulphuric acid, and had greatly improved their action by coating them with peroxide, which he did, not by mechanical means, but by sending

[1] The favourite accumulator at present (1893) is that of Crompton and Howell. The plates are of a porous kind of lead, and are not in the form of rolls but of a number of parallel plates. No minium is used.

through the cell a succession of currents in opposite directions, with intervals of rest between.

Planté also constructed a "rheostatic machine" containing some hundreds of these cells each connected to a separate condenser, and with an arrangement by which the connections of the cells can be instantaneously altered. During the charging process, poles of the same name in the secondary cells are all connected together, one set being connected with the positive pole of the charging battery, and the other set with the negative pole, the charging battery being usually composed of two large Bunsen cells. A small movement suffices to alter the connections so as to arrange the secondary cells in series, and a battery is thus obtained whose electromotive force is hundreds of times greater than that of the charging battery. The process of charging is instantaneous, and a quick succession of discharges of the secondary battery can be obtained by turning a handle, the effects being somewhat similar to the discharges of a Ruhmkorff coil (§ 275).

187. Applications of Accumulators.—Accumulators serve two important purposes. They can be used either like a fly-wheel, to smooth down irregularities in the supply of energy, or, like a mill-reservoir, to store up a large quantity of energy which is running to waste, and give it out later.

For example, a gas-engine usually gives alternately a quick stroke and a slow one; and when it is employed to drive a dynamo for supplying incandescent lamps, a corresponding flickering is seen in the lamps. This evil can be effectually cured by connecting the two terminals of the dynamo with the two terminals of a storage battery. The battery, after it has been charged up to a certain point, absorbs energy during each quick stroke, and gives it out again during the slow stroke, thus keeping the electromotive force of the main circuit nearly constant.

On the other hand, storage cells can be charged during the day by employing the surplus power of a factory engine to drive a dynamo, and can be used at night to supply electricity for lighting the factory. Or they can be charged by a dynamo driven by a fixed engine, and can then be placed in a tram-car to supply motive power for its journey.

188. Electro-metallurgy.—Electrolysis is largely used in connection with the reduction of metals from their ores, especially in the case of copper. Electrolytically deposited copper is remarkable for its

purity, and is almost universally employed when very high conductivity is required.

Electrolysis is also used still more extensively for the two kindred purposes of electro-plating and electrotype. In the former the electrolytic deposit is intended as a permanent covering, and should adhere perfectly so as to form one mass with the body which it covers. In the latter, the adhesion is temporary, and must not be too close, the object being merely to obtain an exact copy of the original form.

189. Electro-gilding and Electro-plating. — The deposition of a coating of gold or silver on the surface of a less precious metal is merely an example of the electrolysis of a salt, as described in § 181. The metal in solution is always deposited on the negative electrode; hence we have merely to make the negative electrode consist of the article which we wish to coat. The only points to be decided practically relate to the means of making the deposit solid and firmly

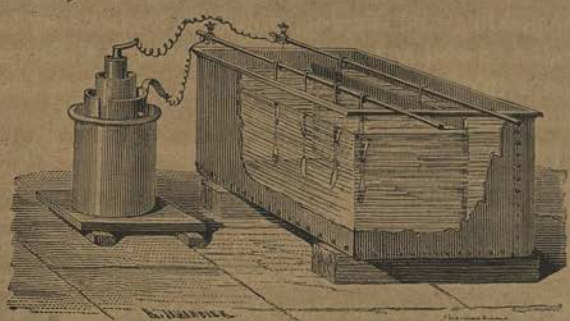


Fig. 118.—Apparatus for Electro-gilding.

adherent. These ends have been completely attained by the methods patented about 1840 by Elkington in England and Ruolz in France.

The solutions are always alkaline, and usually consist of the cyanide or chloride of the metal, dissolved in an alkaline cyanide.

To prepare the *gold bath*, 50 grammes of fine gold are dissolved in aqua regia; and the solution is evaporated till it has the consistence of syrup. Water is then added, together with 50 grammes of cyanide of potassium, and the mixture is boiled. The quantities named give about 50 litres of solution.

The negative electrode consists of the article to be gilded. The positive electrode is a plate of fine gold, which constitutes a soluble

electrode, and serves to keep the solution at a constant strength. In order that the gilding may be well done, the bath must be maintained, during the operation, at a temperature of from 60° to 70° Centigrade.

Fig. 118 represents a form of apparatus which is very frequently employed. The poles of the battery are connected with two metallic rods resting on the top of the cistern which contains the bath. The articles to be gilded are hung from the negative rod. From the positive rod is hung a plate of gold, whose size should be proportional to the total surface of the articles which form the negative electrode.

The *silver bath* is a solution containing 2 parts of cyanide of silver, 10 of cyanide of potassium, and 250 of water. The operation of plating is the same as that of gilding, except that the apparatus is usually on a larger scale, and that the temperature may be lower.

In both cases the surfaces to be coated must be thoroughly cleansed from grease. For this purpose they are subjected to the processes of pickling and dipping, which we cannot stay to describe.

Other bodies, as well as metals, can be coated, if their surfaces are first covered with some conducting material. Baskets, fruits, leaves, &c., have thus been gilded or silvered.

Similar processes are employed for depositing other metals, of which copper is the most frequent example.

190. Electrotype.—Electrotyping consists in obtaining copper casts or facsimiles of medals, engraved plates, &c., by means of electrolytic deposition. The first successful attempts in this direction were made about 1839 by Jacobi at St. Petersburg and Spencer in England. The art is now very extensively practised.

If a fac-simile of a medal is required, a cast is first taken of it, either in fusible alloy, plaster of Paris, or gutta percha softened by heating to 100° C., this last material being the most frequently employed. The fusible alloy is a conductor; the other materials are not, and their surfaces are therefore rendered conducting by rubbing them over with plumbago. The mould thus prepared is made to serve as the negative electrode in a bath of sulphate of copper, a copper plate being used as the positive electrode. When the current passes, copper is deposited on the surface of the mould, forming a thin sheet, which, when detached, is a fac-simile of one side of the original medal. A similar process can be applied to the other side, and thus a complete copy can be obtained.

In operations of this kind, the bath itself is often made to serve as the battery. Fig. 119 represents such an arrangement.

In the interior of a vessel containing a saturated solution of sulphate of copper, a second vessel is supported, consisting either of porous earthenware or of a glass cylinder closed below by a membrane. In this second vessel is placed acidulated water, with a cylinder of zinc suspended in it. The mould is placed in the outer vessel under the bottom of the porous cylinder, and is connected with

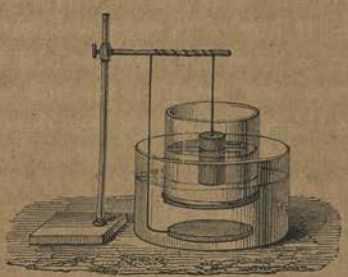


Fig. 119.—Bath and Battery in one

the zinc by a stout wire which completes the circuit. The arrangement is evidently equivalent to a Daniell's cell. The current passes through the liquids from the zinc to the mould, electrolysing the solution of sulphate of copper; and as the metal travels with the current, it is deposited on the surface of the mould. The strength of the solution is kept up by suspending in it crystals of sulphate of copper contained in a vessel pierced with holes.

191. Applications of Electrotype.—One of the commonest applications of electrotype is to the production of copies of wood engravings. The original blocks, as they leave the hand of the engraver, could not yield a large number of impressions without being materially injured by wear. When many impressions are required, they are not taken directly from the wood, but from an electrotype taken in copper from a gutta-percha mould. The process of deposition is continued only for twenty-four hours, and the plate of copper thus obtained is very thin. It is strengthened by filling up its back with melted type-metal. Such plates will afford about 80,000 impressions, and it is from them that nearly all the illustrations in popular works are printed. Postage stamps, which must be exactly alike in order to prevent counterfeits, are also printed from electrotypes; and, on account of the great number of impressions required, the electrotypes

themselves need frequent renewal; but the operations necessary for this purpose do not sensibly injure the original.

Copperplate engravings and even daguerreotypes can be very accurately reproduced in copper. No preparation of the surface is necessary, as the thin film of oxide which is present is quite sufficient to prevent the deposit from adhering too closely.

Gasaliers are usually of cast-iron coated with copper by electrolysis. The copper is not, however, deposited on the surface of the iron, as the contact of the two metals would greatly promote the oxidation of the iron, if any of it were accidentally exposed to the air. The iron is first painted over with red-lead, which, when dry, is covered with a very thin layer of plumbago to render it conducting; and it is on this that the copper is deposited.

CHAPTER XVI.

OHM'S LAW.

192. Brief Statement.—The strength of the current which traverses a circuit depends partly on the electromotive force of the source of electricity, and partly on the resistance of the circuit. For equal resistances, it is proportional to the whole electromotive force tending to maintain the current, and for equal electromotive forces it is inversely as the whole resistance in the circuit. Hence, when proper units are chosen for expressing the current C, the resistance R, and the electromotive force E, we have

$$C = \frac{E}{R},$$

or *the current is equal to the electromotive force divided by the resistance*. This is Ohm's law, so called from its discoverer.

193. Explanation of the term Electromotive Force.[1]—We may define electromotive force as *the quotient of the energy of a current by the quantity of electricity which it conveys.* This definition implies that electromotive force is a quantity of the same nature as difference of potential; for when electricity passes from one conductor to another, the work done in the passage is equal to the quantity of electricity multiplied by the difference of potentials of the two conductors.

When a steady current is flowing through a galvanic circuit, there must be a gradual fall of potential in every uniform conductor which forms part of the circuit; since, in such a conductor, the direction of a current must necessarily be from higher to lower potential. These gradual falls are exactly compensated by the abrupt rises (diminished by the abrupt falls, if any) which occur at the various places of contact of dissimilar substances.

[1] The abbreviation *e.m.f.* is commonly used for *electromotive force.*

If we imagine a large and deep trough of water of annular form, divided into compartments by transverse partitions; and suppose that circulation in one direction is maintained by pumping water over the partitions, we have a rough representation of the distribution of potential in the cells of a battery; the rise of level in passing across a partition being analogous to the rise of potential in traversing a surface of junction.

The electromotive force of a galvanic battery may be defined as the *algebraic sum of the abrupt differences of potential which occur at the junctions of dissimilar substances.* In a battery consisting of a number of similar cells arranged in series, it is of course proportional to the number of cells.

194. Explanation of the term Resistance.—When the current of a circuit is taken through the coil of a galvanometer, it is found that, by introducing different lengths of connecting wire, very different amounts of deflection can be obtained. The longer the wire which connects either pole of the battery with the galvanometer, the smaller is the deflection; and a small deflection indicates a feeble current. The current is in like manner weakened by introducing a fine instead of a stout wire, if their length and material be the same, or by introducing an iron wire instead of a copper wire of the same dimensions. These differences in the properties of the different wires are expressed by saying that they have different resistances.

It is found that, to produce no change in the deflection, the length of the wire must vary directly as its cross-section; that is to say, if $l, l', l'' \ldots$ be the lengths of different wires employed, and $s, s', s'' \ldots$ their sectional areas, their resistances will be equal, if

$$\frac{l}{s} = \frac{l'}{s'} = \frac{l''}{s''} \ldots .$$

This is on the supposition that the wires are all of precisely the same material. Every substance has its own specific resistance, the reciprocal of which is its electrical conductivity and is precisely analogous to thermal conductivity. Denoting specific resistances by $r, r', r'', \ldots$ the condition of equal resistances, when the materials are different, is

$$\frac{rl}{s} = \frac{r'l'}{s'} = \frac{r''l''}{s''} \ldots .$$

and the resistance of any wire is expressed by the formula $\frac{rl}{s}$, l denoting its length, s its sectional area, and r the specific resistance of its material.

To express, in terms of the equivalent length of one wire, the resistance of a circuit composed of several, we can employ the relation

$$\frac{rl}{s}=\frac{r'l'}{s'};\ \text{whence}\ l=\frac{s}{s'}\,\frac{r'}{r}\,l'.$$

l denoting the length of one kind of wire equivalent to the length l' of the other. The length l is called the reduced length of the wire whose actual length is l'.

195. Fuller Statement.—The following more complete statement of Ohm's law will now be intelligible.

When a current is flowing steadily through a conductor which contains no source of electromotive force, it flows from sections of higher to sections of lower potential; and the *difference of potential between two sections is equal to the intervening resistance multiplied by the current.*

The resistance may vary with the temperature and other physical conditions of the conductor, but is independent of the strength of current, so that at a given temperature it varies in the exact ratio of the difference of potential of the two bounding sections. This proportionality has been tested with extreme rigour by experiments devised for the purpose by Maxwell.

In travelling round a complete circuit in the direction of the current, the *total fall of potential* computed by the above rule (in other words the *current multiplied by the whole resistance*) must be *equal to the whole electromotive force.*

196. Rheostat. Resistance Coils.—Wheatstone's *rheostat* was one of the earliest instruments contrived for the comparison of resistances. It consists (Fig. 120) of two cylinders, one of brass, and the other of non-conducting material, so arranged that a copper wire can be wound off the one on to the other by turning a handle. The surface of the non-conducting cylinder B has a screw-thread cut in it, for its whole length, in which the wire lies, so that its successive convolutions are well insulated from each other. Two binding-screws are provided for introducing the rheostat

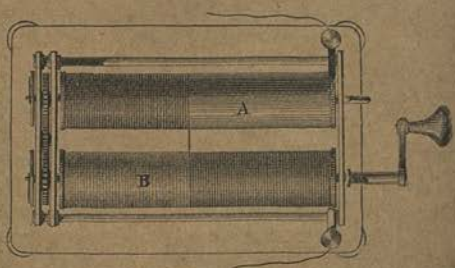


Fig. 120.—Rheostat.

into a circuit; and the resistance which is thus introduced depends on the length of wire which is wrapped upon the non-conducting cylinder, for the brass cylinder A has so large a section that its resistance may be neglected. The amount of resistance can thus be varied as gradually as we please by winding on and off. The handle can be shifted from one cylinder to the other. The figure shows it in the position for winding wire off A on to B. The number of convolutions of wire on B can be read off on a graduated bar provided for the purpose, and parts of a revolution are indicated on a circle at one end.

Fig. 121 represents a very direct mode of measuring resistances by the rheostat. The current traverses a galvanometer B, a rheostat R, and the conductor m, whose resistance is to be measured, the whole of the wire of the rheostat being wound on the brass cylinder. The deflection of the galvanometer having been observed, the conductor m is taken out of circuit, the two wires at a and b are directly connected, and as much of the rheostat wire is brought into circuit as suffices to reduce the deflection to its former amount.

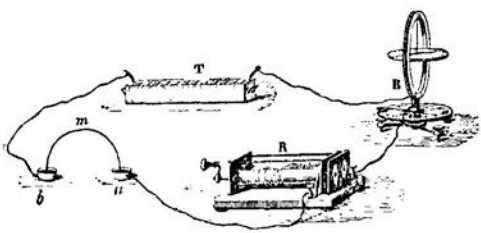


Fig. 121.—Measurement of Resistance.

Measurements of resistance are now usually made by comparison

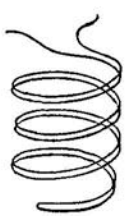


Fig. 122.—Resistance Coil.

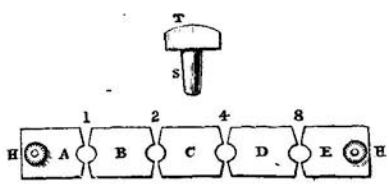


Fig. 123.—Mode of Connecting Resistance Coils.

with standard coils of wire which at a certain specified temperature have known resistances. The most usual material for the wire is German silver, this being a metal whose resistance is comparatively little affected by temperature. The coil is always wound double, as in Fig. 122, because this arrangement prevents it from influencing

or being influenced by magnets and currents in the neighbourhood. In fact, the influences exerted or received by the two strands of the coil cancel one another.

The coils are usually fixed in a box in such a manner that, by combining separate coils, resistances of 1, 2, 3 or any integer number of *ohms* up to 5000 or 10,000 can be obtained. The mode of combining them will be understood by reference to Fig. 123.

A, B, C, D, E are stout plates of brass, which can be connected together by the insertion of plugs in the hollows between them. The plugs (one of which is shown in the figure) are of brass, with ebonite handles, and exactly fit the hollows, so as to give contact over a large surface. The ends of one coil are attached to A and B, the ends of the next to B and C, and so on, the resistances of the coils being marked in figures above them. When the hollow between two plates is open, the current can only pass from one to the other by going through the intervening coil; but when the plug is inserted, the resistance between the plates is inappreciable.

Hence the resistance between the two binding-screws H, H, attached to the extreme plates, will be the sum of the unplugged resistances.

197. Specific Resistances and Conductivities.—Numerous experimenters have compared the specific resistances of the different metals. Though the results thus obtained exhibit some diversity, they all agree in making silver, gold, and copper the three best conductors. Slight impurities, especially in the case of copper, have a very great effect in diminishing conductivity, or, in other words, in increasing resistance. Resistance is also increased, in the case of metals, by increase of temperature; but the opposite rule holds for insulators, such as gutta-percha and india-rubber.

The order of the metals as regards their conductivity for heat is nearly the same as for electricity. The effects of impurity and of change of temperature are also, upon the whole, alike in the two cases.

The following are E. Becquerel's determinations of specific electrical resistance at the temperature 15° C., the resistance of silver at 0° C. being denoted by 100:—

SPECIFIC RESISTANCES AT 15° C.

Silver,	107	Palladium,	715
Copper,	112	Iron,	825
Gold,	155	Lead,	1213
Cadmium,	407	Platinum,	1243
Zinc,	414	Mercury,	5550
Tin,	734		

On comparing these results with the thermal conductivities obtained by Wiedemann and Franz, it will be observed that the order is precisely the same as far as the comparison extends, and that the numerical values are nearly in inverse proportion, showing that electrical and thermal *conductivities* are nearly in direct proportion.

198. **Resistance of Liquids.**—In measuring the resistance of a liquid which is decomposed by a current, care must be taken to avoid complications arising from the reverse e.m.f. called out by the decomposition. The liquid may be contained in a cylindrical tube of glass, closed at both ends by metal plates, and having two small holes in its side near the ends, through which wires (which must both be of one metal) pass into the interior of the liquid, in a direction perpendicular to the length of the tube.

Let a measured current pass steadily through the whole column of liquid, the metallic plates at the ends serving as terminals; and let the difference of potential of the two wires be measured. This may be regarded as the difference of potential of the two cross sections in which the wires lie; and the resistance of the column of liquid between these cross sections will be equal to the difference of their potentials divided by the strength of the current. Arrangements are sometimes made for including a variable and known resistance, such as a box of resistance coils, in the same circuit. When there is the same difference of potential between the ends of this known resistance as between the two wires above mentioned, the resistance of the liquid column between the cross sections is equal to the known resistance, and no measurement of current is necessary.

The resistance even of the best conducting liquids, except mercury, is enormously greater than that of metals. For instance, in round numbers, the resistance of dilute sulphuric acid is a million times, and that of solution of sulphate of copper ten million times greater than that of pure silver. The resistance of pure water is very much greater than either of these.

In the cells of a galvanic battery, the current has to traverse liquid conductors, and the resistance of these is sometimes a large part of the whole resistance in circuit. It is diminished by bringing the plates nearer together, and by increasing their size, since the former change involves diminution of length, and the latter increase of sectional area in the liquid conductor to be traversed. This is the only advantage of large plates over small ones, the electromotive force being the same for both. The advantage of the double coppers

in Wollaston's battery (§ 155) is similarly explained, the resistance with this arrangement being about half what it would be with copper on only one side of the zinc, at the same distance.

199. Choice of Galvanometer.—When stout wire is employed for a galvanometer coil the resistance is small, but it is not practicable to multiply convolutions to any great extent. Short coils of thick wire are accordingly employed in connection with thermopiles, the resistance in the pile itself being so small that the resistance of the galvanometer may be regarded as the total resistance in circuit.

When, on the other hand, the resistance in the other parts of the circuit is very great, the resistance of the galvanometer coil becomes comparatively immaterial, so that, within moderate limits, the force which deflects the needle is nearly proportional to the number of convolutions, and a coil composed of a great length of wire will give the maximum effect.

In both cases, for a given length and diameter of wire, the sensibility increases with the conductivity of the metal composing the wire. Copper is the metal universally employed, and its purity is of immense importance for purposes of delicacy, as impurities often increase its resistance by 50 or even 100 per cent.

200. Resistances in Series and in Parallel.—When two or more wires are connected *in series*, so that whatever flows through one must flow through all, *the resistance of the whole is the sum of the resistances of the wires composing it.*

On the other hand, when two or more wires are arranged *in parallel circuit*, so as to constitute so many independent channels of communication between the same two points, the joint resistance is evidently less than the resistance of any one of the wires. A circuit is said to be *divided* when such an arrangement occurs in any part of it, and the current is also said to be divided. Thus, in Fig. 124 the current from the positive source s to the negative source s' is divided between the two wires acb, adb, which connect the small mercury cups a, b.

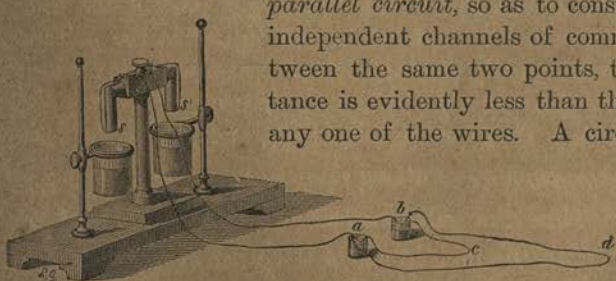


Fig. 124.—Divided Circuit.

Let E denote the difference of potential between the two points

a, b, which are thus joined, and r_1, r_2, &c., the resistances of the separate paths; then the currents through the separate paths will be $\frac{E}{r_1}$, $\frac{E}{r_2}$, &c. The total current between the two points is the sum of these, or $E\left(\frac{1}{r_1}+\frac{1}{r_2}+\&c.\right)$, and this must be equal to $\frac{E}{R}$, where R denotes the resistance of a single wire equivalent to the system. Hence we have

$$\frac{1}{R}=\frac{1}{r_1}+\frac{1}{r_2}+\&c.;$$

or, *the reciprocal of the joint resistance is the sum of the reciprocals of the separate resistances.*

201. Currents in Parallel Branches. Shunts.—If two points, A, B, are connected by any number of parallel branches, in which the currents are i_1, i_2, &c., and the resistances r_1, r_2, &c., we have

$$i_1 r_1 = i_2 r_2 = \&c.;$$

each of these equal quantities being the difference of potential between A and B. Hence the currents through parallel branches are inversely as the resistances.

If there are 3 branches, the current in the first branch will be to the whole current as $\frac{1}{r_1}$ to $\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}$. If there are only 2 branches, the ratio will be $\frac{1}{r_1}$ to $\frac{1}{r_1}+\frac{1}{r_2}$, or r_2 to r_2+r_1.

One of the two branches of a divided circuit is often called a *shunt*, and a portion of the whole current is said to be shunted through it. Thus, a delicate galvanometer is often accompanied by an adjustable shunt, through which the larger part of the whole current can be diverted, in order to avoid injury to the galvanometer. The shunt usually contains three resistances, any one of which can be brought into use at pleasure by inserting a brass plug in the proper hole. They are usually $\frac{1}{9}$, $\frac{1}{99}$, and $\frac{1}{999}$ of the resistance of the galvanometer. Putting $r_2=\frac{1}{999}r_1$, we shall have $\frac{i_1}{i_1+i_2}=\frac{\frac{1}{999}}{1+\frac{1}{999}}=\frac{1}{1000}$. Thus the galvanometer will only get $\frac{1}{1000}$ of the whole current. With the plug in one of the other holes the galvanometer would get $\frac{1}{10}$ or $\frac{1}{100}$ of the whole current.

202. Grouping of Cells in Battery.—Suppose that we have a number n of precisely similar cells, each having electromotive force e and resistance r, and that we connect them in a series, as in Figs. 90, 101, with a conductor of resistance R joining their poles. The whole electromotive force in the circuit will then be $n\,e$, and the

whole resistance will be $nr+R$; hence the strength of current will be

$$C=\frac{ne}{nr+R}.$$

This formula shows that, if the external resistance R is much greater than the resistance in the battery nr, any change in the number of cells will produce a nearly proportional change in the current; but that when the external resistance is much less than that of one cell, as is the case when the poles are connected by a short thick wire, a change in the number of cells affects numerator and denominator almost alike, and produces no sensible change in the current. It is impossible, by connecting any number of similar cells *in a series*, to obtain a current exceeding $\frac{e}{r}$, which is precisely the current which one of the cells would give alone if its plates were well connected by a short thick wire.

It is possible, however, by a different arrangement of the cells, to obtain a current about n times as strong as this, namely, by connecting all the zinc plates to one end of a conductor, and all the

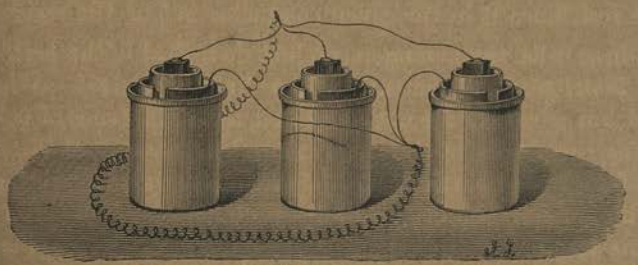


Fig. 125.—Cells with similar Plates connected.

carbons or coppers to the other end, as in Fig. 125. In the arrangement of three cells here figured, the current which passes through the spiral connecting wire is the sum of the currents which the three cells would give separately. The arrangement is equivalent to a single cell with plates three times as large superficially, and at the same distance apart. The electromotive force with n cells so arranged is simply e, but the resistance is only $\frac{r}{n}+R$, so that the current is

$$C=\frac{e}{\frac{r}{n}+R}=\frac{ne}{r+nR}.$$

This system of arrangement may be called *arranging the cells as*

one element, or arranging them *in parallel*. It has sometimes been called the *arrangement for quantity*, the arrangement in a series being called the *arrangement for intensity*.

If in Fig. 125 we substitute for each of the three cells a *series* consisting of four cells, the electromotive force in circuit will be $4e$, and the resistance in circuit will be $\frac{4r}{3}+R$, for each series has a resistance of $4r$, and three parallel series connected at the ends are equivalent to a single series, of the same electromotive force as one of the component series, and of one-third the resistance. The current will therefore be

$$C=\frac{4e}{\frac{4r}{3}+R}=\frac{12e}{4r+3R}=\frac{e}{\frac{r}{3}+\frac{R}{4}}.$$

The question often arises, What is the best manner of grouping a given number of cells in order to give the strongest possible current through a given external conductor? The answer is, they should be so grouped that the internal and external resistance should be as nearly as possible equal; for example, if we have 12 cells as above, and the resistance R in the given conductor is $\frac{4}{3}$ of the resistance of one of these cells, the arrangement just described is the best.[1]

203. Distribution of Potential in a Voltaic Circuit.—When the electrodes of a battery are not connected, their difference of potential, supposing them to be of the same metal, is a measure of the electromotive force of the battery. On joining them by a connecting wire, their difference of potential will be diminished, and will (§ 195) be the same fraction of the whole electromotive force that the resistance in the connecting wire is of the whole resistance.

In a battery of four cells, like that represented in Fig. 90, when the extreme plates are connected by a wire whose resistance is double that of the battery, the fall of potential in the connecting wire will be two-thirds, and the fall of potential in the battery will be one-third, of the whole electromotive force. To avoid fractions,

[1] Instead of 3 and 4, put x for the number of series, and y for the number of cells in a series. Then the current will be $\frac{e}{\frac{r}{x}+\frac{R}{y}}$, and will vary inversely as $\frac{r}{x}+\frac{R}{y}$. Now the product of $\frac{r}{x}$ and $\frac{R}{y}$ is given, being the quotient of rR by the whole number of cells; and when the product of two variables is given, their sum is least when they are equal, and increases as they are made more and more unequal. As x and y must be integers, exact equality cannot generally be obtained.

let the electromotive force of each cell be denoted by 3. Then the total electromotive force will be 12, the fall of potential in the connecting wire will be 8, in the battery 4, and in each cell 1.

The potential, as indicated by an electrometer connected by a copper wire with different parts of the circuit in succession, will have the under-mentioned values, according as the circuit is open or closed. The quantity x may have any value whatever, either positive or negative, but will have the same value for the whole circuit. It will be zero if the negative terminal of the battery is to earth. If any one point of the circuit is to earth, the potential of this point will be zero, and the value of x will thus be determined,

OPEN (Break being at positive terminal).		CLOSED.	
Connecting wire,	x	Connecting wire,	$x+8$ to x
1st Zinc,	x	1st Zinc,	x
Acid,	$x+3$	Acid,	$x+3$ to $x+2$
1st Copper,	$x+3$	1st Copper,	$x+2$
2d Zinc,	$x+3$	2d Zinc,	$x+2$
Acid,	$x+6$	Acid,	$x+5$ to $x+4$
2d Copper,	$x+6$	2d Copper,	$x+4$
3d Zinc,	$x+6$	3d Zinc,	$x+4$
Acid,	$x+9$	Acid,	$x+7$ to $x+6$
3d Copper,	$x+9$	3d Copper,	$x+6$
4th Zinc,	$x+9$	4th Zinc,	$x+6$
Acid,	$x+12$	Acid,	$x+9$ to $x+8$
4th Copper,	$x+12$	4th Copper,	$x+8$

The distribution of potential in the closed circuit is graphically represented by the crooked line A 3 2 5 4 7 6 9 C (Fig. 126); resistances being represented by horizontal, and potentials by vertical distances. A C represents the total resistance in circuit; A B being the resistance of the battery, and B C that of the connecting wire. A D represents the total electromotive force. The points C and A are to be regarded as identical; in other words, the diagram ought to be bent round a cylinder so as to make one of these points fall upon the other.

204. Measurement of Resistance of Battery.—The resistance of a battery may be measured in various ways, of which we shall begin with the simplest.

Let the poles of the battery be directly connected with a galvanometer whose resistance is either very small or accurately known, and let the deflection be noted. Then let a wire of known resistance be introduced into the circuit, and the deflection again noted. The two

currents thus measured will be inversely as the resistances, since the electromotive force is the same in both cases. Let the resistance of the galvanometer coil and connections be denoted by G, that of the wire introduced in the second case by W, and that of the battery by x. Then if the amounts of current be denoted by C_1, C_2, we have $\frac{C_1}{C_2} = \frac{x+G+W}{x+G}$; whence x can be determined.

This mode of determination is not very accurate; in the first place, because the electromotive force of a battery is not a constant

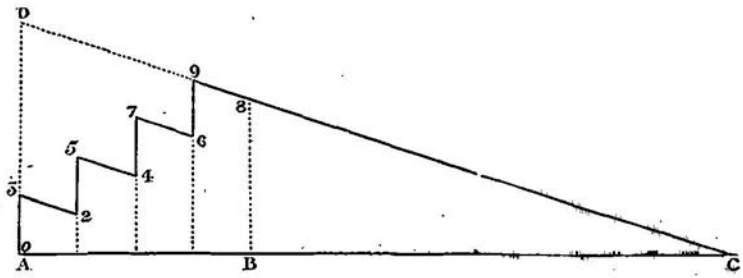


Fig. 126.—Curve of Potential for Closed Circuit.

quantity but usually diminishes as the current increases; and secondly, because the measurement of a current by a galvanometer is not a very exact operation. Some better methods will be described later.

205. **Measurement of Electromotive Force.**—The most direct mode of comparing the electromotive forces of cells of different kinds, would be to observe how many cells of the one kind arranged in series must be opposed to a given number of the other kind, in order that the resultant electromotive force may be nil as indicated by the absence of deflection in a galvanometer forming part of the circuit. For example, if two Daniell's cells and one Grove's cell be connected with each other and with a galvanometer, in such a manner that the current due to the Daniell is in one direction, and that due to the Grove is in the opposite direction, the current actually produced will be in the direction of the greater electromotive force. It will thus be shown whether the electromotive force of a Grove's cell is more or less than double that of a Daniell's. This method has not been much used.

Another method of comparison consists in first connecting the two cells to be compared, so that their electromotive forces tend the same

way, and then again connecting them so that they tend opposite ways, the resultant current being observed in both cases with the same galvanometer. The resistance in circuit is the same in both cases, being the resistance of the galvanometer plus the sum of the resistances of the cells; hence the currents will be simply as the electromotive forces, that is to say, as $E_1 + E_2$ to $E_1 - E_2$, if E_1 and E_2 denote the electromotive forces of the cells. Hence the ratio of E_1 to E_2 is easily computed.

This method is liable to the objection that increase of current gives increase of polarization (§ 157) and consequent diminution of electromotive force; besides the objection that the measurement depends upon the reduction of the indications of a galvanometer to proportional measure. The first objection can be obviated by introducing large resistance into the circuit so as to render the currents feeble.

Another and very effective method is to employ a galvanometer whose coil is so long and fine that its resistance is hundreds of times greater than that of the cell to be tested. The current given by a cell will then be practically independent of the resistance of the cell, and will be proportional simply to its e.m.f. Thomson's "potential galvanometer" is intended for this purpose, and gives the e.m.f. in absolute measure.

206. Determination by Electrometer.—The statical electromotive force of a battery or cell (that is, its electromotive force when no current is passing, the poles being disconnected) can be directly observed by connecting its poles to opposite quadrants of Thomson's electrometer; for the instrument will then show the difference of potential between them, and this difference of potential is the electromotive force.

207. Latimer Clark's Potentiometer.—Another mode of statically comparing electromotive forces is illustrated by Fig. 127. A battery of greater electromotive force than either of those which are to be compared is employed to send a current round a circuit containing a long uniform wire. By Ohm's law, the difference of potential between two points in this wire is proportional to the intervening length. In the method now to be explained we find two points in it whose difference of potential is equal to the statical difference of

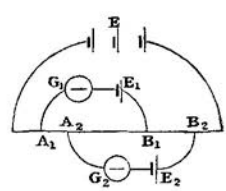


Fig. 127.—Latimer Clark's Method.

potential between the poles of one battery; and also two points whose difference of potential is that of the other battery. Then the ratio of the electromotive forces will be the ratio of the resistances between these points, which is known. E_1 and E_2 in the figure are the two batteries which are to be compared, and E is the third battery, more powerful than either, which, alone of the three, gives a current when the adjustments are complete. G_1 is a galvanometer which shows what current is passing through the battery E_1, and this current is to be reduced to zero by properly choosing the points A_1 B_1. In like manner the current through E_2 and G_2 is to be reduced to zero by proper choice of the points A_2 B_2. Then the electromotive force of E_1 is to that of E_2 as the resistance A_1 B_1 is to the resistance A_2 B_2, and these are simply as their lengths.

The points A_1 A_2 may coincide with each other and with one end of the uniform wire.

In the *Potentiometer* designed by Latimer Clark for carrying out this principle, the uniform wire is coiled in a helical groove on the surface of a non-conducting cylinder, and is made to travel past a contact-maker by turning a handle.

By this method Mr. Clark has found that the statical electromotive forces of a cell of Grove, Bunsen, Daniell, and Wollaston are approximately as 100, 98, 56, and 46. The last of these, being a one-fluid battery, is liable to fall off 50 per cent or more when in action, owing to the deposition of hydrogen on the copper plate.

Mr. Clark has designed a standard cell which is largely employed for comparisons of e.m.f. by statical methods such as the above. "It is formed by employing pure mercury as the negative element, the mercury being covered by a paste made by boiling mercurous sulphate in a thoroughly saturated solution of zinc sulphate, the positive element consisting of pure distilled zinc resting on the paste." It must not be used for producing a current; but its statical electromotive force is very constant and permanent.

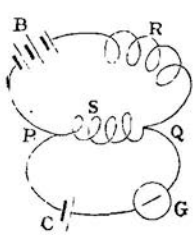


Fig. 128.

208. Bosscha's Method.—The following method gives the electromotive force of a battery through which a current is passing; and also gives the resistance of the battery.

B (Fig. 128) is the battery, which sends a current through the circuit P S Q R, containing two variable resistances, R and S. The

resistance S is to be varied until the difference of potential between its ends P and Q is equal to the statical difference of potential between the poles of a standard cell C, as shown by the absence of a current through the galvanometer G.

R is then to be increased by any convenient amount r, and S is to be increased by an amount s such that the galvanometer again indicates no current.

Then if e denote the statical electromotive force of the standard cell, and E the electromotive force of the battery B during the experiment, the strength of the current through S in the first observation is $\frac{e}{S}$, and is also $\frac{E}{B+S+R}$. We may therefore equate these two expressions, and by alternation we have

$$\frac{E}{e}=\frac{B+S+R}{S}. \tag{1}$$

Similarly, from the second experiment, we have

$$\frac{E}{e}=\frac{B+S+s+R+r}{S+s}. \tag{2}$$

By taking the differences of numerators and denominators we deduce

$$\frac{E}{e}=\frac{s+r}{s}, \tag{3}$$

an equation which determines the value of E, since e, s, and r are known.

Again by equating the right-hand members of (1) and (3) and subtracting unity, we obtain $\frac{B+R}{S}=\frac{r}{s}$, whence $B=\frac{Sr}{s}-R$; thus the resistance B of the battery is determined. It is advantageous to make R zero.

The fundamental idea which underlies these last two methods—the idea of measuring the resistance of a battery by finding two points in a circuit such that no current flows through the battery when its terminals are connected with them—was first suggested and carried out by Poggendorff.

209. Wheatstone's Bridge.—The method usually employed for measuring the resistances of wires was first contrived by Christie, but became generally known through a famous paper of Wheatstone's in the *Philosophical Transactions* (in which due credit is given to Christie); and the special instrument employed in it is called Wheatstone's Bridge. It depends on the principle that in two portions of a conductor traversed by the same current the falls of potential are proportional to the resistances.

The poles P, N of a battery (Fig. 129) are connected by two independent channels of communication A C B, A D J E B. The former is a uniform wire; the latter consists of the wire D, whose resistance is to be determined, and of a standard resistance-coil E.

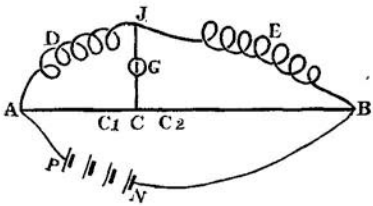


Fig. 129.—Wheatstone's Bridge.

The observation has for its immediate object to find what point in the uniform wire A B has the same potential as the junction J of the other two. When this point C is found, and connected with J through a galvanometer G, no current will pass across, and the needle of the galvanometer will not move. If a point C_1 on the positive side of C were connected with J, a current would run from C_1 to J, and if a point C_2 on the negative side were connected, the current would be from J to C_2. The deflection diminishes as the right point C is approached, and becomes reversed in passing it. When it is found we know that the resistances in A C and C B have the same ratio as those of D and E, each of those ratios being in fact equal to the fall of potential between A and J C divided by the fall between J C and B. As the resistance of E is known, and the resistances of A C, C B are as their lengths, which are indicated on a divided scale, the resistance of D can be computed by simple proportion.

In Wheatstone's original arrangement, the resistances of the two portions A C, C B were equal, and the resistances of the other two portions A D J, J E B were made equal by the help of a rheostat.

210. Conjugate Branches.—Wheatstone's bridge may be otherwise described as consisting of six branches connecting four points, two and two, in every possible way, the four points being A, B, C, J in Fig. 129. A battery is inserted in the branch which connects two of these points, A and B, and a galvanometer is inserted in the branch which connects the other two, C and J. These two branches may be called *opposite*, and in like manner A C is opposite to B J, and B C to A J. The condition of no current going through the galvanometer is expressed in § 209 as a proportion. Multiplying extremes and means, and writing the names of the branches for the resistances in them, the condition is

$$A C \cdot B J = B C \cdot A J,$$

where each member of the equation is the product of the resistances in opposite branches. When this condition is fulfilled, the remaining pair of opposite branches A B and C J are conjugate, that is to say, a battery in one produces no current in the other. The symmetry of the relations shows that the battery may change places with the galvanometer.

211. Loop Test.—The following method of finding the position of a fault in a telegraph wire is an application of the principle of Wheatstone's bridge. We suppose the fault to consist in loss of insulation at some point of the wire, so that the resistance between this point and the ground is much less than it ought to be, though it may still be as great as that of some miles of wire.

The fault is known to be between two given stations. At one of these stations let the end of the faulty wire be joined to the end

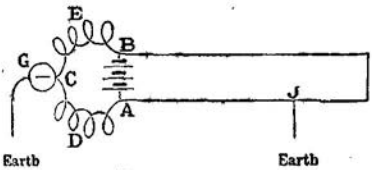


Fig. 130.—Loop Test.

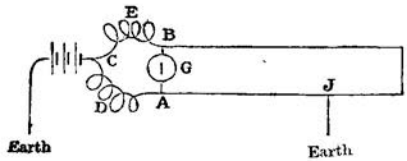


Fig. 131.—Loop Test.

of another wire; and at the other station let the ends A, B of the same two wires be put in connection with the two poles of a battery (Fig. 130). Also let A and B be connected by a circuit containing two variable resistances D, E, and let an intermediate point C be connected, through a galvanometer G, with the ground. Let the resistances D, E be made such that no current goes through the galvanometer. Then we know that the point C has the same potential as the earth, and in these circumstances the faulty point J of the wire will also be at the potential of the earth; for if there were a current flowing through the fault to the earth the battery would be steadily giving off electricity of one sign while having no outlet for electricity of the opposite sign, and this cannot be. The points J and C are therefore at the same potential, like the points J and C in Fig. 129; and a comparison of the two figures shows that the same reasoning applies to both. The loop A J B formed by the two telegraph wires is therefore divided by the point J in the ratio of the two known resistances D and E. That is, we have $\frac{\text{resistance of A J}}{\text{resistance of B J}} = \frac{D}{E}$. This determines the position of the point J.

The positions of the battery and galvanometer may be interchanged, as in Fig. 131, and the equation above obtained will still apply; for when no current flows through the galvanometer in this new arrangement, the two paths C D A J and C E B J, which lead from C to J, must be divided proportionally at A and B.

212. Usual Forms of Bridge.—One of the commonest forms of Wheatstone's bridge is that known as the *metre bridge*. Its construction will be understood from Fig. 132.

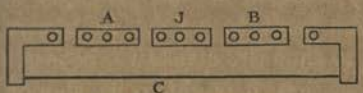


Fig. 132.—Metre Bridge.

A uniform wire of platinum, a metre long, has its ends attached to two stout strips of copper. These can be connected by other similar strips so as to form a closed circuit; but gaps can be left at four places when desired. The small circles represent binding-screws which can be used either for attaching the ends of wires or for holding short strips of copper to fill the gaps. A sliding piece not shown in the figure strides across the wire, and carries a binding-screw attached to a key, by depressing which the binding-screw is put in connection with a point of the wire. The position of this point is read off on a metre scale divided to millimetres, along which the sliding piece travels.

The wires from the battery are attached to A and B, and the wires from the galvanometer to J and the sliding piece, which we will suppose makes contact at C. In the ordinary use of the bridge the two outer gaps are filled up by their copper strips, and the two resistances to be compared are inserted in the two inner gaps. When the sliding piece has been adjusted so as to give a balance, the resistances in the two inner gaps are proportional to the lengths of the two portions into which the point C divides the uniform wire; the resistances of the copper strips being negligible unless C is very close to one end. The two lengths are read off on the metre scale.

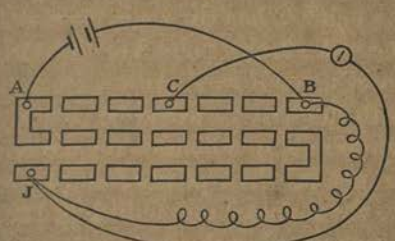


Fig. 133.

Fig. 133 shows the commonest arrangement of a box of resistance coils intended to be used as a Wheatstone's bridge.

The coils, which are out of sight in the interior of the box, and

their brass connecting pieces, which are visible at the top, are arranged in three rows, so connected at the ends as to form a single series of which B and J are the terminals. The actual forms of the brass connecting pieces were described and figured in § 196. We now indicate them merely by rectangles with gaps between. There are single binding-screws at A and C, and double binding-screws at B and J.

The wire whose resistance is required is connected to B and J. As regards the battery and galvanometer, one must be connected to A and B, and the other to C and J. Their positions may be interchanged.

The relations of the parts are best understood by regarding A, B, C, J as four points connected by six branches. The branches A B and C J are to be conjugate. This requires that the product of A C and B J shall equal the product of B C and A J. In practice, the resistance A C is either made equal to B C or 10 or 100 times B C. The required resistance B J will then be either equal to A J or one-tenth or one-hundredth of A J. A J can be adjusted to be any integer number of ohms, as described in § 196.

213. Foster's Method.—The following method, due to Professor Carey Foster, is much used for measuring the small differences between standard coils.

The two coils whose difference is required are inserted in the two *outer* gaps of the metre bridge, and two other resistances nearly equal to one another are inserted in the two inner gaps. The position of C for balance is observed. Call it C_1.

The two coils to be compared are then interchanged, and the new position of C is found. Call it C_2.

In the notation of § 210, we have

$$\frac{AJ}{JB}=\frac{AC}{CB}; \text{ whence } \frac{AJ}{AJ+JB}=\frac{AC}{AC+CB}.$$

But A J and J B are unchanged, and the changes in A C and C B do not affect their sum. Hence A C is unchanged in magnitude; that is, the difference of the two coils which have been inserted in the gap in A C is equal to the resistance of the portion $C_1 C_2$ of the metre wire.

214. Network of Conductors. Kirchhoff's Laws.—In investigating the steady currents in a network of conductors when the resistances and electromotive forces are given, the two following obvious principles, which are known as Kirchhoff's laws, will give the necessary equations.

I. *At any point of meeting of different branches, the algebraic sum of the currents is zero, opposite signs being attributed to inward and outward currents.*

II. *In travelling round any portion that forms a closed circuit, the algebraic sum of the electromotive forces is equal to the algebraic sum of the products of current by resistance.*

The first law merely expresses that there is no accumulation (positive or negative) going on at the point.

The second law expresses that the total increase of potential in travelling round the closed circuit is zero. A positive e.m.f. is a sudden rise. A positive product of current by resistance is a gradual fall.

215.—The following is an example (see Fig. 134):—

A coil of resistance R connects the terminals of a battery of resistance R_1 and electromotive force E_1, and also connects the terminals of a second battery R_2, E_2, like terminals being connected to the same end of the coil. Required the currents in the system.

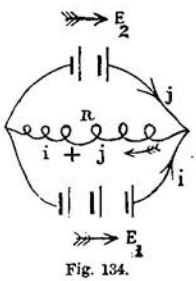


Fig. 134.

Let i denote the current through the first battery, reckoned positive when it has the same direction as E_1; and j the current through the second battery, reckoned positive when it has the same direction as E_2. Then the first law shows that the current through the coil will be $i+j$, reckoned positive when it has the direction due to E_1 and E_2. The second law, applied to the circuit formed of the first battery and the coil, gives

$$E_1 = iR_1 + (i+j)R,$$

and for the circuit formed of second battery and coil,

$$E_2 = jR_2 + (i+j)R.$$

These two equations suffice to give i and j. A third equation might be written down for the circuit formed of the two batteries; but it would be an equation derivable from the other two.

Solving the equations, we find

$$i = \frac{(E_1 - E_2)R + E_1R_2}{RR_1 + RR_2 + R_1R_2},\quad j = \frac{(E_2 - E_1)R + E_2R_1}{RR_1 + RR_2 + R_1R_2},\quad i+j = \frac{E_1R_2 + E_2R_1}{RR_1 + RR_2 + R_1R_2}.$$

Supposing E_1 to be greater than E_2 the expression for j shows that the current through the second battery will be reversed if E_2 is less than $E_1R/(R+R_1)$.

216. **Conjugate Branches when there are Several Batteries.**—When

there are batteries in more branches than one, the current in any branch will be the algebraical sum of the currents due to the several batteries considered separately. Hence when there is equality between the two products of opposite resistances, as in last section, the current in either of the two remaining branches will be independent of the electromotive force of the battery in the other; and these two branches are still said to be conjugate. In estimating the resistance of any branch which contains a battery, the resistance of the battery must of course be included.

Thus far we have not discussed the effect of change of resistance in one of two conjugate branches. The introduction of additional resistance into any branch can affect the current in the rest only by altering the difference of potentials between the ends of this branch; and the same remark applies to the introduction of a source of electromotive force into any branch. Two changes, one of resistance, and the other of electromotive force, in a branch, will have the same effect on the rest of the circuit, if they have the same effect on the difference of potentials of the ends of this branch. Hence if the current in one of two branches be independent of the electromotive force in the other, it must also be independent of the resistance in the other.

As this reasoning may appear doubtful to some of our readers, we subjoin a formal investigation leading to the same result.

217. Investigation of Condition of Conjugateness.—Let A, B, C, J (Fig. 129 or Fig. 135), be four points connected, two and two, by six branches. Let the resistance in the branch connecting A and B be denoted by A B or B A, and the electromotive force in it (positive if tending from A to B) by ab. Let the current in this branch (positive if from A to B) be denoted by γ, and the currents in the branches B C, C A, by α, β. Then the current in the branch A J (positive if from A to J) will be $\beta - \gamma$; for this current together with γ carries off from A the supply brought by β. Similarly, the currents in B J, C J, will be $\gamma - \alpha$, $\alpha - \beta$. Then, since the sum of the falls of potential in travelling round a circuit must equal the sum of the rises, we have, for the circuit J B C, the equation

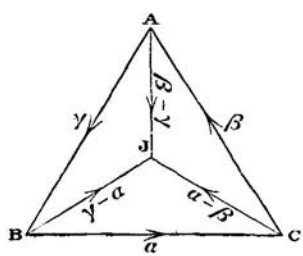


Fig. 135.—Theory of Conjugate Branches.

$$\alpha \,.\, BC+(\alpha-\beta)\,CJ+(\alpha-\gamma)\,JB=bc+cj+jb.$$

Similarly, for the circuits J C A, J A B, we have

$$\beta \,.\, CA+(\beta-\gamma)\,AJ+(\beta-\alpha)\,JC=ca+aj+jc$$
$$\gamma \,.\, AB+(\gamma-\alpha)\,BJ+(\gamma-\beta)\,JA=ab+bj+ja.$$

These three equations are sufficient to determine the three currents α, β, γ, in terms of the electromotive forces and resistances. Multiplying the equations in order, by 1, l, m (l and m being multipliers to be afterwards determined), and adding; the coefficients of α, β, γ will be

$$BC+(1-l)\,CJ+(1-m)\,BJ,$$
$$l \,.\, CA+(l-m)\,AJ+(l-1)\,CJ,$$
$$m \,.\, AB+(m-1)\,BJ+(m-l)\,AJ,$$

and the second member of the equation will be

$$bc+l \,.\, ca+m \,.\, ab+(l-m)\,aj+(m-1)\,bj+(1-l)\,cj.$$

To find the value of α, we must equate the coefficients of β and γ to zero, and then divide the second member by the coefficient of α. The electromotive force aj appears only in the term $(l-m)\,aj$, and the resistance A J only in the terms $(l-m)$ A J and $(m-l)$ A J. Hence the equality of l to m is the condition alike of the disappearance of aj and of A J. Putting $l=m$, and equating the coefficients of β and γ to zero, we have

$$l=\frac{CJ}{CA+CJ}=\frac{BJ}{AB+BJ},$$

whence

$$CJ \,.\, AB=BJ \,.\, CA,$$

which is, accordingly, the condition of conjugateness. That is to say, *if the product of one pair of opposite resistances be equal to the product of another pair, the remaining pair of branches will be so related that the current in each is independent of the electromotive force and resistance in the other.*

218. Thomson's Method of Measuring the Resistance of a Galvanometer.—The resistance of a galvanometer can be measured without the use of another galvanometer, by the following method due to Sir Wm. Thomson.

In a system of six branches joining four points, let a battery and a contact key respectively be in one pair of opposite branches. Then, if the products of the resistances in opposite branches be equal for the four remaining pairs, we know by § 210 that no current will pass through the branch containing the key, and hence making or breaking contact with the key will be nugatory; hence the galvano-

meter will not have its deflection altered by making or breaking contact with the key. The experiment is to be conducted by altering the resistance in one of the branches until the key has no effect on the galvanometer. The resistance of the galvanometer is then calculated from the equality of the products of opposite pairs.

This method was suggested by the following.

219. Mance's Method of Finding the Resistance of a Battery.— In this method a galvanometer and a contact key are in a pair of opposite branches, and the battery is in one of the four remaining branches, while the other three contain known resistances. The observation is made by varying one of these resistances till the galvanometer is not affected by the key. The branches containing the key and the galvanometer are then conjugate (and the resistance of the battery can be calculated), if the putting down of the key does not alter the electromotive force of the battery. This condition is seldom fulfilled.

In this method, as well as in that described in the preceding section, the galvanometer does not stand at zero, but shows a steady deflection, which is unaltered by opening or closing the branch containing the key.

CHAPTER XVII.

RELATIONS BETWEEN ELECTRICITY AND HEAT.

220. Heating of Wires.—The heating of a wire by the passage of a current may conveniently be exhibited by the aid of the apparatus represented in Fig. 136. Two uprights mounted on a stand are furnished, at different heights, with pairs of insulated binding-screws $a\,a'$, $b\,b'$, $c\,c'$, having wires stretched between them. A current can thus be sent through any one of the wires, by connecting the terminals of a battery with the binding-screws at its extremities. When this is done with a battery of suitable power, the wire is first seen to droop in consequence of expansion, then to redden, and finally to melt, becoming inflamed if the metal is sufficiently combustible.

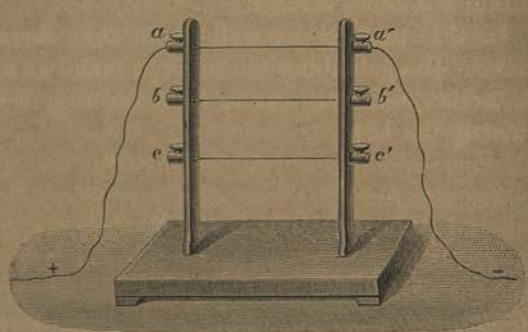


Fig. 136.—Stand for Heating Wires.

If a file is attached to one of the terminals of a battery, and the other terminal is drawn along the file, a rapid succession of sparks will be obtained; and if the battery be sufficiently powerful, globules of incandescent metal will be scattered about with brilliant effect.

221. Joule's Law.—The energy of a current is equal to the product of the quantity of electricity that passes and the electro-motive force that drives it. As the numerical measure of a current is the quantity of electricity which passes in unit time, it follows that the energy of a current C lasting for a time t, is $\mathrm{E}C t$, E denoting the electro-motive

force. But again, by Ohm's law, E is equal to the product of the current C and the whole resistance R. The expression for the energy therefore becomes

$$C^2 R t, \qquad (1)$$

and this energy is all transformed into heat in the circuit, subject to a small correction for the Peltier and Thomson effects which will be described in a later section (§ 228). It has accordingly been found, first by Joule, and afterwards by Lenz, Becquerel, and others, that the formula C^2Rt represents the quantity of heat generated by a current under ordinary circumstances. The experiments have usually been conducted by passing a current through a spiral of wire immersed in water or alcohol, and observing the elevation of temperature of the liquid.

This law of Joule's, like that of Ohm, may be applied to any part of a circuit, as well as to the circuit considered as a whole; that is to say, if the circuit consists of parts whose resistances are r_1, r_2, . . . , the quantities of heat generated in them are respectively $C^2 r_1 t$, $C^2 r_2 t$, . . . , and are therefore proportional to the resistances r_1, r_2 . . . of the respective parts, since C and t are necessarily the same for all.

222. Relation of Heat in Circuit to Chemical Action in Battery.— The energy of a current, and consequently the heat developed in the circuit, is the exact equivalent of the potential energy of chemical affinity which runs down in the cells of the battery. This fact, first verified approximately by Joule, has been more accurately confirmed by the experiments of Favre, who introduced into the muffle of his mercurial calorimeter, described and figured in Part II., a small voltaic cell with its poles connected by a fine wire. He found that the consumption of 33 grammes of zinc in the cell corresponded to a generation of heat amounting to 18,796 gramme-degrees. But the chemical action in the cell is complex. The 33 grammes of zinc unite with 8 grammes of oxygen, and in so doing generate 42,451 gramme-degrees. The combination of these 41 grammes of oxide of zinc with 40 grammes of sulphuric acid, produces 10,456 gramme-degrees, making in all 52,907. But an equivalent of water undergoes decomposition, and this *absorbs* 34,463, which must be subtracted from the above sum, leaving 18,444 gramme-degrees as the balance of heat generated in the whole complex action. The heat actually observed in the experiment agrees almost precisely with this calculated amount. (Compare § 184.)

223. Distribution of Heat in Circuit.—These experiments also served to verify the application of Joule's law to each part of the circuit considered separately. By introducing the cell into the muffle whilst a spiral of fine wire connecting the poles was outside, and then introducing the spiral while the cell was outside, Favre was able to measure separately the heat generated in the cell and in the spiral, and these were found to be proportional to their resistances.

If wires of different diameter or of different electrical conductivity form parts of the same circuit, so as to be traversed by the same current, the bad conductors will become more heated than the good, and the fine wires more than the coarse. All parts of the length of a uniform wire will be uniformly heated. The specific resistance of platinum is ten times greater than that of copper; hence ten times as much heat will be generated in a platinum as in a copper wire by a given current, if the diameters of the two wires be the same.

The *elevation of temperature* is greater in a fine than in a coarse wire, not only because of its greater resistance, which leads to the development of a greater quantity of heat in it, but also on account of its smaller capacity for heat, and its smaller surface. When the current is passed for so short a time that the heat emitted may be neglected, the elevation of temperature varies directly as the resistance per unit length, and inversely as the capacity per unit length. The resistance varies inversely and the capacity directly as the section of the wire, and hence the elevation of temperature is inversely as the square of the section, or as the fourth power of the diameter.

On the other hand, if the current be continued till the permanent temperature is attained, capacity ceases to have any influence, and the heat emitted in unit time must be equal to the heat received. If x denote the elevation of temperature, the heat emitted is approximately $2\ \pi\ r\ \mathrm{B}\ x$ by Newton's law, r being the radius of the wire, and B a constant. The heat received is $\frac{\mathrm{A}}{\pi r^2}$, A being another constant. By equating these two expressions, we find that $r^3 x$ is equal to a constant, and hence x varies inversely as r^3, that is, the elevation of temperature is inversely as the cube of the diameter.

To obtain the most rapid production of heat in the circuit considered as a whole, we must reduce the resistance to a minimum; for the heat produced in unit time is EC, which, by Ohm's law, is the same as $\frac{\mathrm{E}^2}{\mathrm{R}}$, and therefore varies inversely as R the total resistance.

224. Mechanical Work done by Current.—Favre's experiments also furnished a confirmation of the fact, that when a current is called upon to perform mechanical work, the amount of heat generated in the circuit is diminished by the equivalent of this work. He inclosed a battery of five cells in the muffle of one calorimeter, and an electro-magnet in another calorimeter; the connections between the coil of the electro-magnet and the poles of the battery being made by short thick wires whose resistance could be neglected. The electro-magnet attracted an armature, and thus raised a weight by means of external pulleys.

It was found that when the armature was fixed, so that no mechanical work could be performed, the heat developed was the precise equivalent of the chemical action which took place in the battery; but when the electro-magnet was allowed to raise the weight, the amount of heat indicated by the calorimeters was sensibly less. The difference was measured, and compared with the work done in raising the weight. The comparison indicated 444 kilogrammetres of work for each kilogramme-degree of heat that disappeared, a result which agrees sufficiently well with the established value of Joule's equivalent (425 kilogrammetres).

225. Thermo-electric Currents.—Electric currents can be produced by applying heat or cold to one of the junctions in a circuit composed of two different metals. This was first shown by Seebeck of Berlin in 1821. It may be illustrated by employing a rectangular frame (Fig. 137), having three sides formed of a copper plate, and the fourth of a cylinder of bismuth. It must be placed in the magnetic meridian, with a magnetized needle in its interior. On heating one of the junctions with a spirit-lamp, the needle will be deflected in such a direction as to indicate the existence of a current, which in the copper portion of the circuit, flows from the hot to the cold junction, and in the bismuth portion from the cold to the hot. If cold instead of heat be applied to one junction, the direction of the current will still be from the warmer junction through the copper to the colder junction, and from this through the

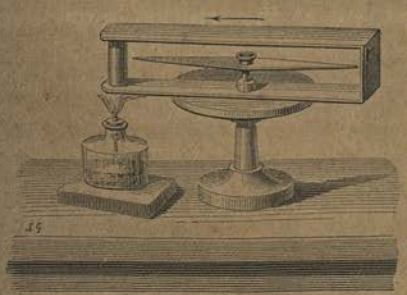


Fig. 137.—Thermo-electric Current.

bismuth to the warmer junction. Antimony, if employed instead of copper, gives a still more powerful effect.

226. Though a circuit composed of bismuth and antimony is specially susceptible of thermo-electric excitation, the property is possessed, in a more or less marked degree, by every circuit composed of two metals, and even by circuits composed of the same metal in different states. If, for example, a knot or a helix (as in Fig. 138), be formed in a piece of platinum wire, and heat applied at one side of it, a current will be indicated by a delicate galvanometer. In metals which are usually heterogeneous in their structure, such as bismuth, it is not uncommon to find currents produced by heating parts which appear quite uniform. If the ends of two copper wires be bent into hooks, and one of them be heated; on placing them in contact, a current will be produced due to the presence of a thin film of oxide on the heated wire. With two platinum wires, no such effect is obtained.

Fig. 138.—Current with one Metal.

227. **Thermo-electric Order.**—According to Becquerel's experiments, the metals may be ranged in the following order, as regards the direction of the current produced by heating a junction of any two of them:—*Bismuth, platinum, lead, tin, copper, silver, zinc, iron, antimony;* that is to say, if a junction of any two of these metals be heated, the direction of the current at the junction in question will be from that which stands first in the list to the other. His experiments have also established the important fact that the current obtained by heating all the junctions B, C, D, E, F, of a chain of dissimilar metals to one common temperature, is the same as that obtained by uniting the two extreme bars A B, F G, directly to each other, and heating their junction to the same temperature.[1]

Fig. 139.

[1] The more accurate statement is, that the *electro-motive force* is the same in the two cases. The *current* will be sensibly the same if the resistance in B C D E F is insignificant in comparison with the rest of the circuit. In order that there may be a current, the circuit must of course be completed, and not left open as in Fig. 139. In the case of an open circuit, the result of the heating will simply be to produce difference of potential between the ends.

228. Source of Thermo-electric Force.—The source of the electro-motive force in a thermo-electric arrangement is to be found in two effects called, from the names of their discoverers, the Peltier effect and the Thomson effect. They may both be described as *thermal effects of a current which are reversed by reversing the direction of the current.*

Peltier found that if two wires of different metals are joined at one end so as to form a single conductor, the junction becomes hotter when a current is sent through the conductor in one direction than when it is sent in the opposite direction.

Thomson found that a wire of one metal, if it is not at a uniform temperature, but is hotter at one end than at the other, is more heated by a current in one direction than by a current in the opposite direction.

In both cases, the quantity of heat generated in the conductor in unit time by a current C is represented by the expression

$$C^2R \pm CS,$$

R denoting the resistance of the conductor, S a quantity which we shall suppose to be essentially positive, and the positive or negative sign being taken according to the direction of the current.

The term $\pm CS$ represents an amount of energy which is converted from the electrical to the thermal form by a current in one direction, and from the thermal to the electrical form by a current in the other direction. When the conversion is from the thermal to the electrical form, the current is aided by an electro-motive force S; and when the conversion is from the electrical to the thermal form, the current is opposed by an electro-motive force S. S is the measure of the thermo-electric force, and the direction of this force is always that direction for which the sign of the term C S is negative.

These statements are true both of a thermo-electric circuit as a whole, and of any portion of it considered separately. If the portion considered be an indefinitely short portion containing a junction, the term C^2R will vanish, and the heating effect will be $\pm CS$. A junction would therefore be cooled by sending through it a current in the proper direction, if we could prevent the conduction of heat to it from neighbouring parts.

229. Thermo-electric Diagram.—The statement of the quantita-

tive laws of thermo-electricity is greatly facilitated by the use of what is called a thermo-electric diagram.

In a thermo-electric diagram each metal has its own line. Let A A and B B (Fig. 140) be the lines of two metals, which we will call A and B. Horizontal distances measured from the vertical line Ap qB represent absolute temperatures. Let pP and its equal qQ represent any absolute temperature. Then the area of the rectangle Pp qQ represents the thermo-electric force at a junction of the two metals at this temperature. If P be above Q, as in the figure, the force tends from Q to P. The temperature corresponding to the point of intersection N is the *neutral point* of the two metals. At temperatures below N the thermo-electric force at a junction is from the metal B to the metal A, but at temperatures above N it is from A to B, being in both cases directed upwards in the diagram.

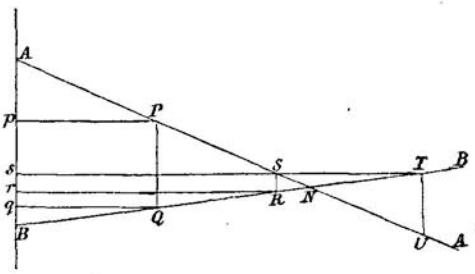


Fig. 140.

Again if pP and sS represent the temperatures of two points of the metal A, the thermo-electric force in the intervening portion is represented by the area Pp sS, which is bounded on three sides by straight lines, and on the fourth side by the line of the metal, which is not in all cases straight. The direction of this force is from the lower point S to the higher P, so that if the line of the metal slopes like A A′ in the figure, the force tends from the hot to the cold end. In the portion of the metal B represented by Q R, the force tends from Q to R, that is from the cold to the hot end. In every case the thermo-electric force in each portion of a circuit is directed up-hill in the diagram; and conversely a current generates more heat by running downhill than by running uphill.

230. **Thermo-electric Pair.**—If the circuit consist simply of a wire of the metal A, joined at both ends to a wire of B, one junction being at the temperature pP or qQ, and the other at the higher temperature sS or rR, both being below the neutral point, the thermo-electric forces in Q R, R S, and S P, all tend in one direction round the circuit, namely, the direction Q R S P Q. Their joint force is represented by the area qQ R S Pp. The force at the

junction P Q is from Q to P, and opposes the other three. It is represented by the area $qQPp$, which must accordingly be subtracted. The remainder is the area P Q R S, which accordingly represents the resultant thermo-electric force of the circuit.

231. Properties of the Neutral Point.—Supposing the lower of the two temperatures to be fixed, and to be below the neutral point N, the area P Q R S will be a maximum when S and R coincide with N, being then identical with the triangle P Q N. If the higher

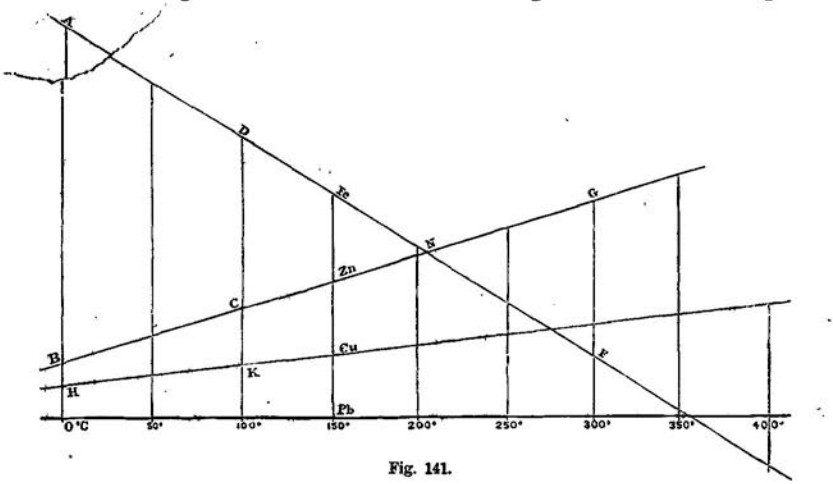


Fig. 141.

temperature be above the neutral point, and be represented by sT, the area which represents the thermo-electric force will be the difference of the two triangles P Q N and N T U. If the lines of the two metals are straight, these two triangles will be equal when one junction is as much above the neutral point as the other is below it, and the thermo-electric force will then be *nil*. At higher temperatures of the hot junction the triangle N T U is larger than the triangle P Q N, and the thermo-electric force is accordingly reversed.

232. General Formula.—Denoting absolute temperature by x, and heights above any fixed horizontal line by y, the thermo-electric force in any part of the circuit is represented by the sum of the elementary strips $x\,dy$ taken along the corresponding portion of the diagram, and the resultant thermo-electric force round a circuit is the sum of these strips taken round the whole boundary which represents the circuit. This sum $\Sigma x\,dy$ is the area enclosed by the boundary, and is positive when the boundary is traced in the

opposite direction to watch hands. The direction of the resultant force will be that which makes this area positive. If one part of the boundary crosses another, we must mark arrows along the boundary in directions corresponding to one direction round the circuit; then areas encircled by arrows against watch hands are to be reckoned positive, and those encircled by arrows with watch hands negative.

233. Application to several Metals.—Fig. 141 is a thermo-electric diagram containing portions of the lines of four metals: Iron (Fe), Zinc (Zn), Copper (Cu), and Lead (Pb).

The Thomson effect is nil in lead, and the lead line is accordingly horizontal. The lines for zinc and copper slope in the direction of ordinary writing; in these the Thomson effect is said to be positive. The line for iron slopes the contrary way, and the Thomson effect in this metal is said to be negative. In a metal for which the Thomson effect is positive, the thermo-electric force due to inequality of temperature tends from the cold to the hot portions of the metal, and conversely the metal would be more heated by a current from hot to cold, than by a current from cold to hot. In a metal in which the Thomson effect is negative the thermo-electric force is from hot to cold, and more heat would be produced by a current from cold to hot than by one from hot to cold.

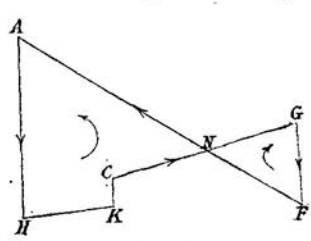


Fig. 142.

The line of lead, being horizontal, is usually employed as the axis of x, and temperatures are laid off along it. In the figure the area A B C D represents the thermo-electric force of an iron-zinc couple, A H K D that of an iron-copper couple, B H K C that of a zinc-copper couple, and so on, the junctions in each case being at 0° and 100° C.

For a circuit composed of three wires of iron, zinc, and copper respectively, with junctions at the following temperatures:

Iron-zinc 300°, Zinc-copper 100°; Copper-iron 0°,

we must trace the boundary A H K C G F A, marking our course with arrows, as shown in Fig. 142. We shall thus obtain the area A H K C N encircled by arrows against watch hands, and the area N G F encircled by arrows with watch hands. The resultant

thermo-electric force is represented by their difference, and is in the direction A H K C G F A.

Numerous experiments, of which the most important are those of Professor Tait, have shown that the lines of most metals are straight within the limits of temperature to which the experiments extended. The line of iron is straight up to a temperature just below redness, but exhibits some remarkable bends before a white heat is reached.

For a further discussion of the laws of thermo-electricity, see the Note at the end of this chapter.

234. **Nomenclature.**—It is convenient to call the ordinates in the diagram "*thermo-electric height.*" Thus A B is the difference of the heights of iron and zinc at 0°, or more definitely, it is the height of iron above zinc at 0°.

The difference of the ordinates at two points of one of the lines, divided by the difference of the abscissas, may be called the "*tangent of the slope.*" Its sign is the same as that of the Thomson effect.

The student is likely in the course of his reading to meet with the phrases "electric convection of heat" and "specific heat of electricity." We will therefore explain these somewhat misleading terms.

In a uniform linear conductor along which a current is flowing, there is, in addition to the frictional heating, which is proportional to the square of the current, a warming or cooling effect proportional (at given temperature) to the steepness of the thermometric gradient at the point which is warmed or cooled, changing sign with the gradient, and vanishing at points of maximum or minimum temperature, where the gradient vanishes. This is the Thomson effect. Let us compare it with what happens when a stream of liquid flows through a pipe surrounded at alternate points in its length with hot and cold jackets, the average temperature of the liquid being the same as the average temperature of the pipe. It will carry heat from the hotter to the colder portions, thus cooling the hottest parts, warming the coldest parts, and at the same time carrying forward the points of maximum and minimum temperature. If, at each point of the pipe (supposed straight and horizontal), we erect an ordinate to represent its temperature, and call the curve of which they are the ordinates "the temperature curve;" the effect of the flow of liquid on this curve will be twofold: (1) It will carry the

temperature curve forward; (2) It will make the temperature curve flatter.

In the Thomson effect, an electric current carries the temperature curve forward in copper, and backward in iron; but in neither case does it make the temperature curve flatter. Erroneous statements on this point[1] will be found in some text-books. They have arisen from taking the phrase "electric convection" too literally. In speaking of electric convection, it is customary to say that the "specific heat of electricity" is positive in a metal in which the temperature curve is carried forward, and is negative in a metal in which the temperature curve is carried backward.

235. Thermo-electric Pile.—If a thermo-electric chain be composed

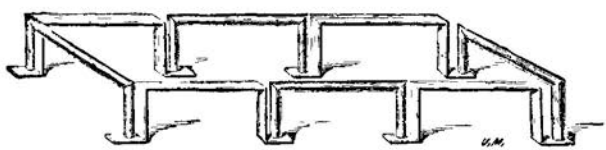


Fig. 143.—Pouillet's Thermo-pile.

of two metals occurring alternately (as in Fig. 143), no effect will be obtained by equally heating *two consecutive* junctions; for the current which would be generated by heating the one is in the opposite direction to that due to the heating of the other. If we number the junctions in order, we shall obtain a current in one direction by heating any junction which bears an odd number, and in the opposite direction by heating any one that bears an even number. The thermo-electric pile, or *thermo-pile*, whose use has been described in Part II. in connection with experiments on radiant heat, is an arrangement of this kind, in which all the odd junctions are presented together at one end, and all the even junctions at the other, the two metals composing the pile being antimony and bismuth. The electro-motive force obtained with a given difference of temperature between the ends of the pile is proportional to the number of junctions, except in so far as accidental differences may exist between different junctions.

Of late years some thermo-piles have been constructed which have sufficient electro-motive force for the deposition of metals from solutions, and they have to some extent been employed commer-

[1] See a discussion in *Nature*, vol. 34, pp. 75, 120, 143.

cially for this purpose. The number of pairs of metals is usually about 70, the materials being in one instance iron and type-metal, in another iron and galæna. These piles have the form of a hollow cylinder with junctions facing alternately inwards and outwards. The inner junctions are exposed to the flame of a Bunsen gas-burner, while the outer junctions are kept cool by the contact of the air.

236. Observations of Temperature by Thermo-electric Junctions.—Thermo-electric currents may be employed either for comparing small differences of temperature (which is the function of the thermo-pile), or for testing equality of temperature. As an example of the latter application, suppose a circuit to be formed of two long wires, one of iron and the other of copper, connected at both ends, and covered with gutta-percha or some other insulator except at the two junctions. Let one junction be lowered to the bottom of a boring, or any other inaccessible place whose temperature we wish to ascertain, and let the other junction be immersed in a vessel of water containing a thermometer. If one of the wires be carried round a galvanometer, the direction in which the needle is deflected will indicate whether the upper or lower junction is the warmer, and if we alter the temperature of the water in the vessel till the deflection is reduced to zero, we know that the two junctions are at the same temperature, which we can read off by the thermometer immersed in the water.

237. Pyro-electricity.—Another relation between heat and electricity may here be mentioned, though it belongs rather to electrostatics than to current electricity.

When a crystal of tourmaline is heated or cooled, observation shows that, while the crystal is gaining heat, one end of it has a charge of positive and the other of negative electricity; and while it is losing heat these charges are reversed. This phenomenon is called pyro-electricity, and it is always associated with a departure from symmetry in crystalline form, which enables us to distinguish one end of the crystal from the other.

238. Effect of Light on Electrical Resistance.—Mr. Willoughby Smith has discovered that the electrical resistance of crystalline selenium is diminished by the action of light. A strong instantaneous effect is observed at the moment when light first falls upon the substance, and the effect gradually increases for some time if the exposure to light is continued. Professor W. G. Adams found that exposure to the light of an ordinary wax taper at a distance

of 20 centims. diminished the resistance of a plate of selenium by about one-eighth part of the whole.

Selenium is a very bad conductor, its resistance being more than a thousand million times that of iron.

The most striking effects are obtained by constructing a so-called "selenium cell" on the following plan. A strip of mica or some other substance of high insulating power is notched at both edges, and a copper wire is wound round it leaving alternate notches vacant. Its ends are secured, one of them being attached to a binding-screw. A second wire is then wound in the intervening notches and similarly secured. It must not touch the first, but must be everywhere very near it. The face of the plate is then thinly covered with selenium, which must be melted on and allowed to cool slowly so as to assume the crystalline form. The selenium affords the only medium of electrical communication between the two wires, and if the two binding-screws are connected with a battery, a high-resistance mirror-galvanometer being also introduced into the circuit, the exposure of the face of the cell to various degrees of light will give strongly marked effects on the galvanometer. If a disc of cardboard be cut away in sectors and rapidly rotated between the face of the cell and the sun or any strong light, in such a manner that the cell is alternately in light and shadow as the sectors pass, the fluctuations of current thus produced can be detected by means of a telephone, which gives a very audible hum. This is a severe test of the quickness of the action, for a thermo-pile gives no sound under the same circumstances. This combination of a selenium cell with a telephone is called a *photophone*, and with a modified form of it articulate sounds have been transmitted to a distance by light. The invention is due to Professor Graham Bell, the inventor of the telephone, and the form of cell above described was introduced by Mr. Shelford Bidwell.

NOTE.

Mathematical Investigation relating to Thermo-electricity.

Let $\pi_1 (a\,b)$ denote the *e.m.f.* tending from the metal A to the metal B at their junction at temperature t_1. It will follow from this definition that $\pi_1 (b\,a) = -\pi_1 (a\,b)$. The experimental fact that, in a closed circuit of three metals

A, B, C at uniform temperature, there is no resultant thermo-electric force, proves that

$$\pi_1(a\,b)+\pi_1(b\,c)+\pi_1(c\,a)=0, \quad \text{or} \quad \pi_1(a\,c)=\pi_1(a\,b)+\pi_1(b\,c). \tag{1}$$

Again, the experimental fact that the resultant *e.m.f.* of a closed circuit is independent of the temperatures of all parts except the junctions, shows that the resultant *e.m.f.* due to difference of temperature in a single metal depends on the temperatures of its ends only, and is independent of its length. The same conclusion might be drawn from the fact that no current is produced in a closed circuit of one homogeneous metal by any distribution of temperature in its various portions. Hence the *e.m.f.* in a short portion of nearly uniform temperature varying from t at one end to $t+dt$ at the other, may be expressed as $\sigma\,dt$, where σ is a function of t having different values for different metals. The coefficient σ (called the Thomson coefficient) is regarded as positive when the *e.m.f.* tends from the colder to the warmer end. We shall denote its value in the metal A by σ_a, its value in B by σ_b, and so on.

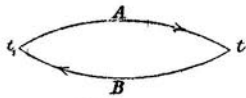


Fig. 144.

Consider a closed circuit (Fig. 144), consisting of a wire of one metal A joined at both ends to a wire of another metal B, one junction being at the temperature t_1, and the other at t_2. The resultant *e.m.f.*, regarded as positive when it is in the direction of the arrows, is

$$\pi_1(b\,a)+\int_{t_1}^{t_2}\sigma_a\,dt+\pi_2(a\,b)+\int_{t_2}^{t_1}\sigma_b\,dt, \tag{2}$$

$$\pi_2(a\,b)-\pi_1(a\,b)+\int_{t_1}^{t_2}(\sigma_a-\sigma_b)\,dt. \tag{3}$$

In any reversible thermo-dynamic engine, if we divide each portion of heat taken in or given out by the absolute temperature at which the exchange occurs, and reckon heat given out opposite in sign to heat taken in, the sum of the quotients is zero. Applying this principle to the present case, we have

$$\frac{\pi_2(a\,b)}{t_2}-\frac{\pi_1(a\,b)}{t_1}+\int_{t_1}^{t_2}\frac{(\sigma_a-\sigma_b)}{t}\,dt=0,$$

or, writing y as an abbreviation for $\frac{\pi(a\,b)}{t}$,

$$y_2-y_1+\int_{t_1}^{t_2}\frac{\sigma_a-\sigma_b}{t}\,dt=0, \tag{4}$$

By supposing a small change in t_2, while t_1 remains constant, we deduce from (4)

$$\frac{dy}{dt}+\frac{\sigma_a-\sigma_b}{t}=0, \tag{5}$$

which gives by integration

$$\int_{t_1}^{t_2}(\sigma_a-\sigma_b)\,dt=-\int_{y_1}^{y_2}t\,dy=t_1y_1-t_2y_2+\int_{t_1}^{t_2}y\,dt.$$

But t_1y_1 is $\pi_1(a\,b)$ and t_2y_2 is $\pi_2(a\,b)$. Hence expression (3) for the resultant thermo-electric force in the circuit reduces to

$$\int_{t_1}^{t_2}y\,dt, \tag{6}$$

which denotes an area such as A B C D or B H K C in Fig. 141, if y denote the length intercepted between the lines of the two metals on the ordinate corresponding to the abscissa t.

Writing $y\ (a\,b)$ instead of y for greater explicitness, we have, by dividing equation (1) by t_1,

$$y\,(a\,b) + y\,(b\,c) = y\,(a\,c), \tag{7}$$

a property which enables us to combine the lines of more than two metals in the same diagram.

The quantity y in the above investigation is the difference of the ordinates of the two metals A and B in a thermo-electric diagram. The line of some one standard metal is arbitrary both in form and position, and its choice will determine the rest. Adopting as the standard a metal (lead) for which σ is zero at all temperatures, and assigning to it a line which is straight and horizontal, we find from (5), by identifying the standard with A, that for any other metal (identified with B)

$$\frac{\sigma}{t} = \frac{dy}{dt}. \tag{8}$$

Hence the line of a metal slopes one way or the other according to the sign of σ. The *e.m.f.* due to difference of temperature in a small portion of one metal is $\sigma\,dt$, which by (8) is equal to $t\,dy$. It always tends in the direction in which y increases.

The *e.m.f.* at a junction with the standard metal, reckoned positive when it tends from the standard to the other metal, is ty, and is represented by the area of the rectangle whose sides are the two co-ordinates.

If y_a and y_b are the ordinates of the lines of two metals A and B, $y_b - y_a$ will be the quantity denoted by y in (4), (5), and (6), and by $y\,(a\,b)$ in (7).

If we travel completely round any thermo-electric circuit consisting of any number of metals, and trace the corresponding course on the thermo-electric diagram, this latter course will consist partly of vertical movements corresponding to the junctions, and partly of slanting or horizontal movements along the lines of the metals. In each of these two kinds of movement the *e.m.f.* (reckoned positive if it tends in the direction in which we are travelling) is the sum of the elementary areas $t\,dy$. The resultant *e.m.f.* is therefore $\Sigma\,t\,dy$ taken all round the course, and this is identical with the area which the course encloses.

"The specific heat of electricity" is a name which has been given to the Thomson coefficient σ for the reason explained in § 234, but it is not desirable that it should be retained, the analogy to convection of heat being very imperfect and misleading. The other coefficient π (more commonly printed Π) is called the Peltier coefficient.

CHAPTER XVIII.

ELECTRO-DYNAMICS.

[This chapter, with the exception of the last three sections, is of little importance, and should be omitted at first reading.]

239. Meaning of Electro-dynamics.—A wire through which a current is passing is found to be capable of producing movements in other wires also conveying currents. The theory of these movements, or more generally, of *the mechanical actions of currents upon one another*, constitutes a distinct branch of electrical science, and is called *electro-dynamics*. It stands in very close relation to electro-magnetism; and if the laws of either of the two sciences are given, those of the other may be deduced as consequences.

The science of electro-dynamics was founded by Ampère. Figs. 145, 146 represent an arrangement which he devised for rendering a conductor movable without interruption of the current conveyed by it.

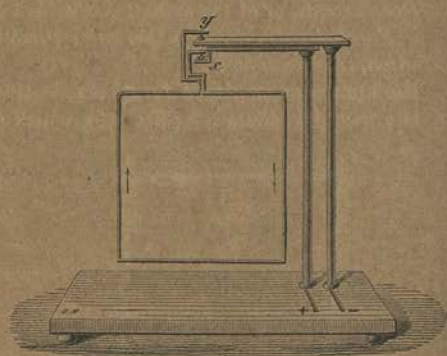


Fig. 145.—Ampère's Stand.

A wire is bent into the form of a nearly complete rectangle, and its two ends terminate in points, one above the other, so arranged that a vertical through the centre of gravity passes through them both. Accordingly, if either or both of these points be supported, the wire can turn freely about this vertical as axis. The points dip into two small metallic cups x y containing mercury, and the weight is usually borne by the upper point alone, which touches the bottom of its cup. The cups are attached to two horizontal arms of metal, supported on metallic pillars, which can be connected with the two terminals of a battery. The wire thus forms part of the circuit, the current being down one side of the rectangle and up the other. Instead of the rectangular the circular form may be employed, as in Fig. 147.

If a magnet be placed beneath, as in Fig. 146, the wire frame will set its plane perpendicular to the length of the magnet, the relative position assumed being

the same as if the wire frame were fixed, and the magnet freely suspended, if we neglect the disturbing effect of the earth's magnetism.

240. Mutual Forces between Conductors conveying Currents.—The following elementary laws, regarding the mutual forces exerted between conductors through which currents are passing, were established by Ampère. For

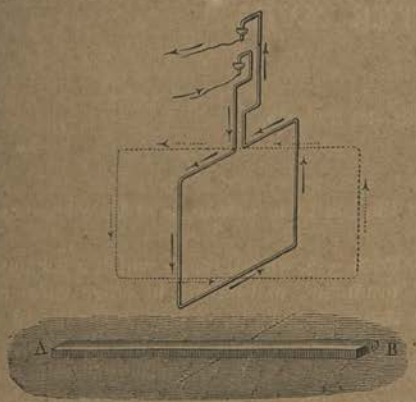


Fig. 146.—Action of Magnet on Movable Circuit.

Fig. 147.

brevity of expression, it is usual to speak, in this sense, of the *mutual forces between currents*, or of the *mutual mechanical action of currents*.

I. *Parallel currents, if in the same direction, attract, and if in the opposite direction, repel each other.*

The apparatus employed for demonstrating this twofold proposition, consists of two metallic pillars *t*, *v* (Fig. 148), which are respectively connected at their

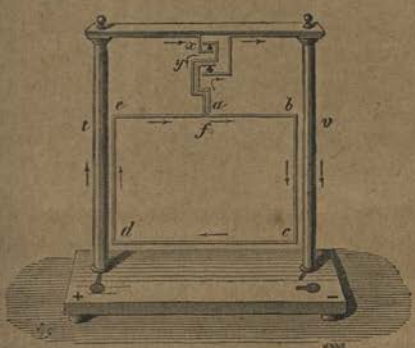


Fig. 148.—Attraction of Parallel Circuits.

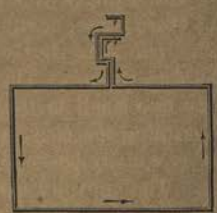


Fig. 149.—Apparatus for Repulsion.

upper ends with the two cups of mercury *x*, *y*. The rectangular conductor *a b c d e* is suspended with its terminal points in these cups so as to complete the circuit between the pillars. When the current is passed, this movable conductor

always places itself so that its plane coincides with that of the two pillars, and so that currents in the same direction in the pillars and in the wire are next each other, as shown in the figure.

For establishing repulsion, a slightly different form of wire is employed, which is represented in Fig. 149. When this is hung from the cups, in the position which the figure indicates, the currents in the pillars are in opposite directions to those in the neighbouring portions of the movable conductor, and the latter accordingly turns away until it is stopped by collision with the wires above.

II. *Currents whose directions are inclined to each other at any angle, attract each other if they both flow towards the vertex of the angle,*[1] *or if they both flow from it, and repel each other if one of them flows towards the angle, and the other from it.*

A consequence of this law is that two currents, as A B, D C (Fig. 150), crossing one another near O in different planes, tend to set themselves parallel, and so that their directions shall be the same. For there is attraction between the portions A O and D O, and also between the portions O B and O C; whereas there is repulsion between A O and O C, and between O B and O D. Accordingly, if the movable conductor of Fig. 148 or 149 be traversed by a current, and another wire carrying a current be placed horizontally at any angle underneath its lower side, the movable conductor will turn on its point of suspension till it becomes parallel to the wire below it; and in the position of stable equilibrium

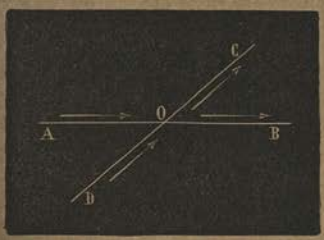


Fig. 150.—Tendency to set Parallel.

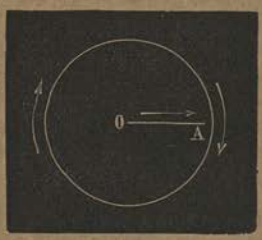


Fig. 151.—Continuous Rotation of Radial Current.

the current in its lower side will have the same direction as that in the influencing wire.

241. Continuous Rotation produced by a Circular Current.—Suppose we have a current flowing round a circle (Fig. 151), and also a current flowing along O A, which is approximately a radius of this circle. First let the current in O A be from the centre towards the circumference, as indicated in the figure. Then, by law II., O A is attracted on one side and repelled on the other, both forces combining to make O A sweep round the circle in the opposite direction to that in which the circular current is flowing. If the current in O A were from circumference to centre, the tendency would be for O A to sweep round the circle in the same direction as the circular current.

The reasoning still holds if O A is in a plane parallel to that of the circular

[1] If the currents are not in the same plane, we must substitute *their common perpendicular* for *the vertex of the angle*, in the enunciation of this law.

current, O being a point on the axis of the circle and the length of O A being not greater than the radius.

A circular current may also produce continuous rotation in a conductor parallel to the axis of the circle, and movable round that axis. Fig. 152 represents an arrangement for obtaining this effect.

A coil of wire through which a current can be sent, is wound round the copper basin E F, its extremities being connected with the binding-screws *m*, *o*. From

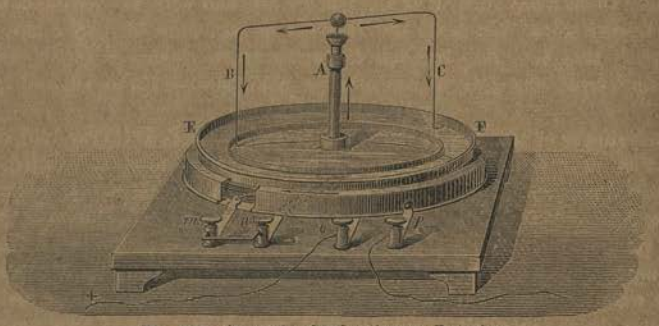


Fig. 152.—Apparatus for Continuous Rotation.

the centre of the basin rises the little metallic pillar A, terminating above in a cup containing mercury. This pillar is connected with the binding-screw *n*. The basin, which is connected with the binding-screw *p*, contains water mixed with a little acid to improve its conducting power, and a movable conductor B C rests, by a point, on the bottom of the cup of mercury, while its lowest portion, which consists of a light hoop, dips in the acidulated water. By connecting *m* and *n* a single circuit is obtained, of which *o* and *p* are the terminals, so that if *o* is connected with the positive and *p* with the negative pole of a battery, the current entering at *o* first traverses the wire coil, then ascends the pillar A, returns down the sides B, C to the floating ring and liquid, and so escapes to *p*. As soon as these connections have been completed, the movable conductor commences continuous rotation in the direction opposite to that of the current in the coil.

If, instead of connecting *m* and *n*, we connect *n* and *o*, and lead the positive wire from the battery to *p* and the negative wire to *o*, the course of the current will be from *p* to the acid, thence up the sides B, C, and inwards along the top of the movable conductor to the mercury cup, then down the pillar to *n*, thence to *o*, and through the coil from *o* to *m* in the same direction as in the former experiment; but the rotation of the movable conductor will now occur in the opposite direction to that before observed, and therefore in the same direction as the current.

242. Action of an Indefinite[1] Rectilinear Current upon a Finite Current movable around one Extremity.—A finite current movable about one extremity may also be caused to rotate continuously about this extremity by the action of an indefinite rectilinear current. This is clearly indicated by Fig. 153.

[1] The word *indefinite*, in this application, simply means *of great length in comparison with the distance and length of the movable current*.

In the right-hand diagram, the current O A flowing outwards from the centre of motion O, and acted on by the indefinite current M N, is first attracted into the position O A′. In this new position it is repelled by n N, and attracted by M n. It is thus brought successively into the positions O A″, O A‴, O A^{IV}. In this last-

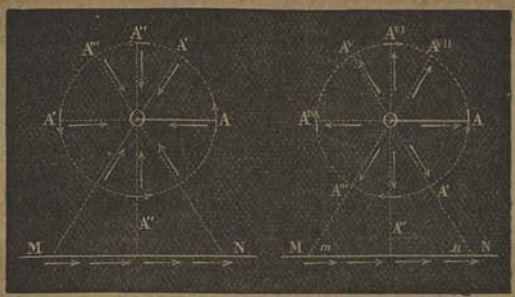


Fig. 153.—Rotation of Radial Current.

mentioned position, the two currents being parallel and opposite, there is repulsion; and after passing it, there is again repulsion on one side and attraction on the other, till it is carried round to its first position O A. It is thus kept in continual rotation. If the movable current flows inwards to the centre of motion O, as in the left-hand diagram, while the direction of the indefinite current is the same as before, the direction of rotation will be reversed.

243. Action of an Indefinite Rectilinear Current on a Finite Current Perpendicular to it.—Let M N, in the upper half of Fig. 154, be an indefinite rectilinear current, and A D a portion of another current either in the same or in any other plane. In the latter case let DC be the common perpendicular. Then, if the currents have the directions represented by the arrows, an element at p will attract an element at m with a force which we may represent by a line mf'; and an element at p' equal to that at p and situated at the same distance from C on the other side, will repel the element at m with an equal force, represented by mf. Constructing the parallelogram of forces, the resultant force of these two elements upon m is represented by the diagonal m F, which is parallel to M N, and in the opposite direction to the indefinite current. As this reasoning applies to all the elements of both currents, it follows that the current A B will experience a force tending to give it a motion of translation parallel to M N. This motion will be opposite to the direction of the indefinite current when the direction of the finite current is towards the common perpendicular D C, as in the upper diagram, and will be in the same direction

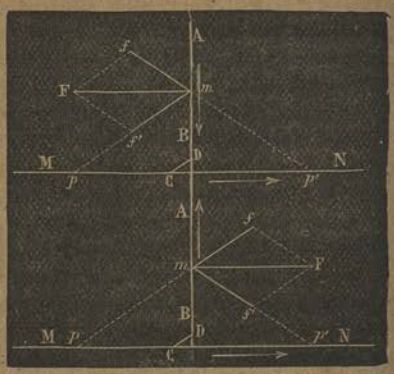


Fig. 154.—Translation Parallel to Indefinite Current.

as the indefinite current when the direction of the finite current is from the common perpendicular, as in the lower diagram.

244. Action upon a Rectangular Current movable about an Axis Perpendicular to an Indefinite Current.—It follows from the preceding section that if a finite current A B (Fig. 155), perpendicular to an indefinite

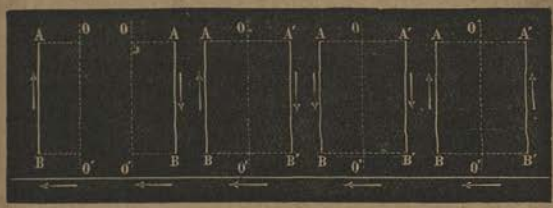


Fig. 155.—Position assumed by Perpendicular Current.

current, is movable round an axis O O′ parallel to itself, the plane A B O O′ will place itself parallel to the indefinite current, and A B will place itself in advance or in rear of the axis according as the current in A B is from or towards the indefinite current.

If a pair of parallel and opposite currents B A, A′ B′, rigidly connected together, and movable round the axis O O′ lying between them, are submitted to the action of the indefinite current, the forces upon them will conspire to place the system in the position indicated in the figure. If the two currents A B, A′ B′ are both in the same direction, their tendencies to revolve round the axis O O′ will counteract each other.

The action upon the near side of the rectangle (Fig. 156) contributes to produce the same effect, since this side tends to set itself parallel to the influencing current, and so that the directions of the two shall be the same.

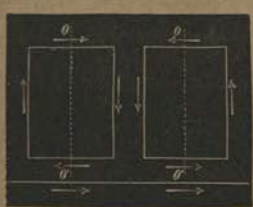


Fig. 156.—Position assumed by Rectangular Current.

The action upon the further side of the rectangle tends to produce an opposite effect; but, in consequence of the greater distance, this action is feebler than that upon the near side. The system accordingly tends to take the position of stable equilibrium represented in the right-hand half of the figure. The diagram on the left hand represents a position of unstable equilibrium.

What is here proved for a rectangular current, is true for any closed plane circuit movable round an axis of symmetry perpendicular to an indefinite rectilinear current; that is to say, any such circuit tends to place itself so that the current in the near side of it is in the same direction as the indefinite current.

These results can be verified experimentally by the aid of the apparatus represented in Fig. 157. C C, D D are two cups (shown in section) surrounding the metallic pillar A B at its upper and lower ends, and containing a conducting liquid. The lower cup is insulated from the pillar, and connected with the binding-screw *g*. The liquid in the upper cup C C is connected with the upper end of the pillar by the bent arm *d m*. *o* K is a light horizontal rod supported

on a point at B, and carrying a counterpoise K at one end, while the other carries a wire $m\,n\,o\,p$, whose two ends $n\,m$ and $o\,p$ descend vertically into the two cups, the middle portion of the wire being wrapped tightly round the rod. The binding-screw f is connected with the lower end of the pillar. If a current enters at f and leaves at g, its direction in the long vertical wire $o\,p$ will be descending; and it will be ascending if the connections are reversed. By sending a current at the same time through a long horizontal wire in the neighbourhood of the system, movements will be obtained in accordance with the foregoing conclusions.

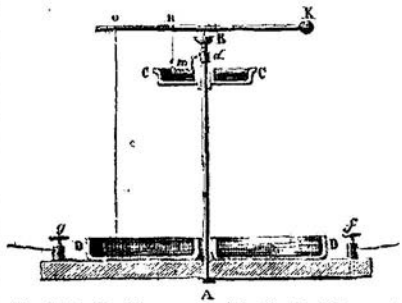


Fig. 157.—Position assumed by Vertical Current.

245. Sinuous Currents.—A sinuous current exhibits the same action as a rectilinear current, provided that they nowhere deviate far from each other. This principle can be exemplified by bringing near to a movable conductor (Fig. 158) another conductor consisting of a wire doubled back upon itself, having one of its portions straight, and the other sinuous, but very near the first. A current sent through this double wire traverses the straight and the sinuous

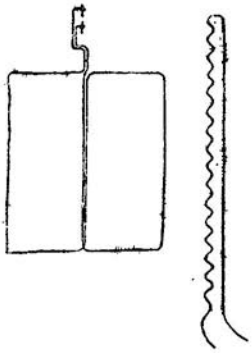


Fig. 158.

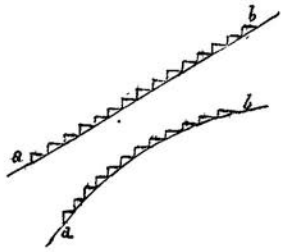


Fig. 159.

Sinuous Currents.

portions in opposite directions, and it will be found that their joint effect upon the movable conductor is inappreciable.

This principle holds not only for rectilinear currents but for currents of any form, and is very extensively employed in the analytical investigations of electrodynamics. In computing the action exercised by or upon a conductor of any form, it is generally convenient to substitute for the conductor itself an imaginary conductor, nearly coincident with it, and consisting of a succession of short straight portions at right angles to one another (Fig. 159).

246. Mutual Action of Two Elements of Currents.—Ampère based his analytical investigations on the assumption that the action exercised by an

element (*i.e.* a very short portion) of one current upon an element of another, consists of a single force directed along the joining line. This assumption conducted him to a formula for the amount of this force, which has been found to give true results in every case capable of being tested by experiment. Nevertheless, it is by no means certain that either Ampère's formula or his fundamental assumption is true. Other assumptions have been made, leading to other formulæ in contradiction to that of Ampère, which also give true results in every case capable of being experimentally tested.[1] The fact is that experiments can only be performed with complete circuits, and the contradictions which subsist between the different assumptions, in the case of the several parts of a circuit, vanish when the circuit is considered as a whole. All the formulæ, however, agree in making the mutual force or forces between two elements vary inversely as the square of their distance, and directly as the products of the currents which pass through them. Professor Clerk Maxwell[2] discards all assumptions as to mutual actions between elements at a distance, and employs the principle that a circuit conveying a current always tends to move in such a manner as to increase the number of magnetic force-tubes (§§ 53, 109) which pass through it. The work done in any displacement is measured by the number of tubes thus added; but tubes which cross the circuit in the opposite direction to those due to the current in the circuit are to be regarded as negative.

We have seen (§ 164, 165) that the lines of magnetic force due to a current are circles surrounding it; and also that, when a line of magnetic force cuts a current, the latter experiences a force tending to move it at right angles to the plane of itself and the line of force. In the case of two parallel currents, each is cut at right angles by the lines of magnetic force due to the other; the direction of the force experienced by either current is therefore directly to or from the other current; and the criterion of § 165 will be found to indicate attraction when the directions of the currents are the same, and repulsion when they are opposite.

In Fig. 152 the lines of magnetic force cut O A in a direction perpendicular to the plane of the diagram, O A accordingly experiences a force perpendicular to its own length in the plane of the diagram; and the same remarks apply to A B in Fig. 155. All the mutual mechanical actions of currents are in fact thus explicable.

247. Action of the Earth on Currents.—In virtue of terrestrial magnetism, movable circuits, when left to themselves, take up definite positions having well-marked relations to the lines of terrestrial magnetic force. For example, in the apparatus of Fig. 157, the vertical wire *o p* will place itself to the west or east (magnetic) of the pillar A B, according as the current in *o p* is ascending or descending. This effect is due to the horizontal component of terrestrial magnetism.

In the apparatus of Fig. 152, if the current be sent only through the movable portion, continuous rotation will be produced, which will be with or against the hands of a watch according as the current in the top wires is inwards or outwards. This effect is due to the vertical component of the earth's magnetism, acting on the currents in the horizontal wires. Vertical lines of magnetic force falling on

[1] Maxwell, § 526, second edition.

[2] Maxwell "On Faraday's Lines of Force." *Camb. Trans.* 1858, p. 50.

a horizontal current give the latter a tendency to move perpendicular to its own length in a horizontal plane.

248. Solenoids.—If we suspend from Ampère's stand (Fig. 145) a plane circuit, whether rectangular or circular, it will place itself perpendicular to the magnetic meridian, in such a manner that the current in its lower side is from east to west; or, in other words, so that the ascending current is in its western and the descending current in its eastern side; this effect being due to the action of the horizontal component of terrestrial magnetism upon the ascending and descending parts of the current. If, then, we have a number of such circuits, rigidly connected together at right angles to a common axis, and with their currents all circulating the same way, their common axis will tend to place itself in the magnetic meridian, like the axis of a magnet. Such a system was called by Ampère a solenoid (σωλην, a tube), and was realized by him in the following manner.

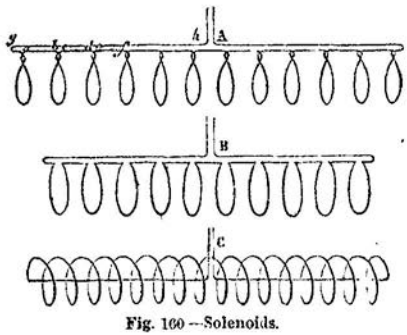


Fig. 160.—Solenoids.

Imagine a wire bent into such a shape as to consist of a number of rings united to each other by straight portions. It will differ from a theoretical solenoid only by having currents in these straight portions; but if the two ends of the wire be carried back till they nearly meet in the middle of the length, as shown at A and B (Fig. 160), the currents in these returning portions, being opposite to those in the other straight portions, will destroy their effect, and the resultant electro-dynamic action of the system will be simply due to the currents in the rings. The same effect is more conveniently obtained by substituting for the rings and intermediate straight portions, a helix, which, by the principles of sinuous currents, is equivalent to them. Each spire of the helix represents a circle perpendicular to the axis, together with a straight portion parallel to the axis and equal to the distance between two spires. The effect of all the straight portions is exactly destroyed by the wires which return from the ends of the helix and meet in the middle. This arrangement, which is represented at

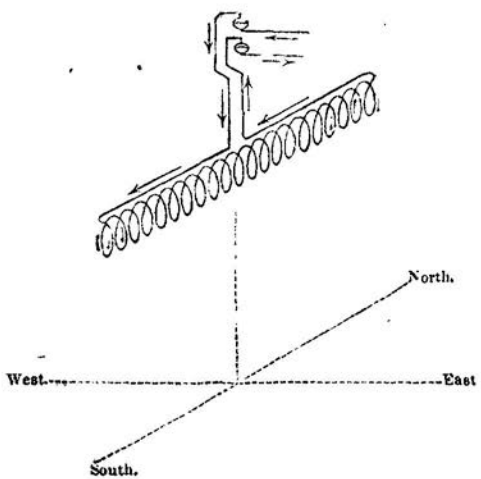


Fig. 161.—Orientation of Solenoid.

C, is that which is universally adopted, the returning wires being sometimes in the axis, and sometimes on the outside of the helix.

If a solenoid, thus constructed, be suspended on an Ampère's stand, as in Fig. 161, and a current sent through it, it will immediately place its axis parallel to a declination needle. It may accordingly be said to have poles. In Fig. 162, A represents the austral or north-seeking, B the boreal or south-seeking pole of the solenoid; that is to say, the direction of the current is against or with the hands of a watch according as the austral or boreal pole is presented to the observer. The same difference is illustrated by Fig. 161.

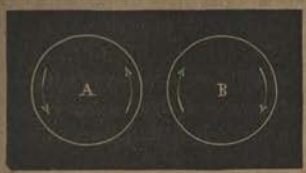


Fig. 162.—Poles of Solenoid.

249. Dip of Solenoid.—If a solenoid could be balanced so as to be perfectly free to move about its centre of gravity, it would place its axis parallel to the dipping-needle. The experiment would be scarcely practicable with a solenoid properly so called, on account of its weight; but it can be performed with a single plane circuit, such as that shown in Fig. 163. If such a circuit is nicely balanced

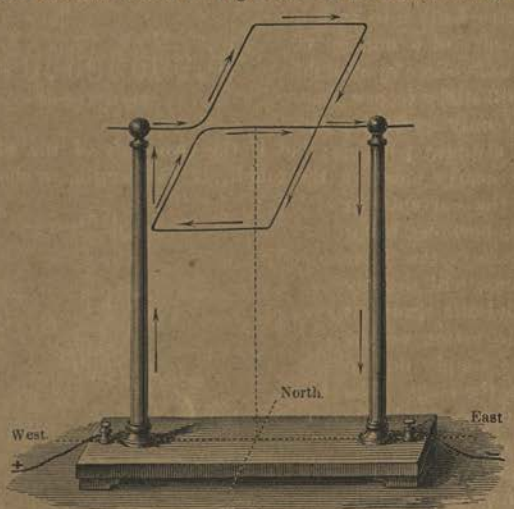


Fig. 163.—Dip of Element of Solenoid.

about an axis through its centre of gravity, and placed so that it can turn freely in the plane of the magnetic meridian, the passing of a current through it will cause it to set its plane perpendicular to the direction of a dipping-needle. This effect is due to the action of terrestrial magnetism on the upper and lower sides of the rectangle. The plane of the rectangle is represented in the figure as coinciding with the direction of dip. In this position the action of terrestrial magnetism urges the upper side backwards and the lower side forwards, and stable equilibrium will be attained when the rectangle has turned through 90°.

250. Mutual Actions of Solenoids.—Solenoids behave like magnets not only as regards the forces which they experience from terrestrial magnetism, but also as regards the actions which they exert upon one another. The similar poles of two solenoids repel, and the unlike poles attract each other, as we may easily prove by suspending one solenoid from an Ampère's stand and bringing another near it.

The reason of these attractions and repulsions is illustrated by Fig. 165. If two austral poles are placed opposite each other, as in the upper part of the figure, the currents are circulating round them in opposite directions, and, by the

Fig. 164.—Mutual Action of Solenoids.

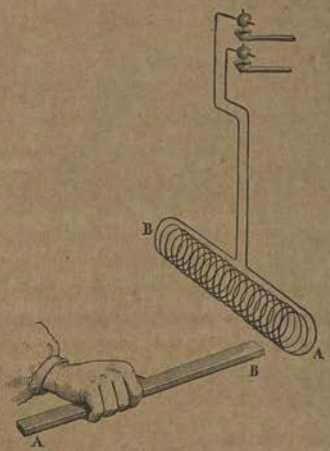


Fig. 165.—Action of Magnet on Solenoid.

laws of parallel currents, should therefore repel each other; whereas if two dissimilar poles be placed face to face, the currents which circulate round them are in the same direction, and attraction should therefore ensue.

Lastly, if one pole of an ordinary magnet be brought near one pole of a suspended solenoid, as in Fig. 165, repulsion or attraction will be exhibited according as the poles in question are similar or dissimilar. In the position represented in the figure, this action is mainly due to the action of the boreal pole of the magnet upon the descending currents in the near side of the solenoid. This action consists in a force to the left hand, nearly parallel to the axis of the solenoid, which tends to make the solenoid rotate about its supports, and thus to bring the end A of the solenoid into contact with the end B of the magnet.

It may be shown, by the aid of Ampère's formula for the mutual force between two elements, that the mutual action of two solenoids is equivalent to four forces, directed along lines joining the poles of the solenoids, and varying inversely as the squares of the distances between the poles; the forces between similar poles being repulsive, and the other two attractive. The analogy between solenoids and magnets is thus complete.

251. Astatic Circuits.—When it is desired to eliminate the influence of terrestrial magnetism in electro-dynamic experiments, an astatic circuit may be

employed as the movable conductor. Two such circuits are represented in the accompanying figures (Figs. 166, 167). In each of them the current in one half of the circuit circulates with, and in the other against the hands of a watch, thus producing equal and opposite tendencies to orientation, which destroy one another.

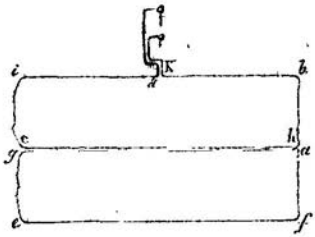


Fig. 166.

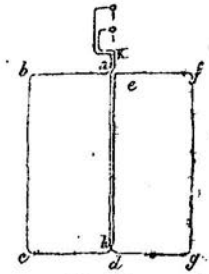


Fig. 167.

Astatic Circuits.

252. Ampere's Theory of Magnetism.—In accordance with the preceding facts, Ampère propounded the hypothesis that what is called magnetism consists in the existence of electric currents circulating round the particles of magnetic bodies. In iron or steel, when unmagnetized, according to this theory, the currents around different particles have different directions; but when it is magnetized, the directions of all are the same. Fig. 168 represents an ideal section of a

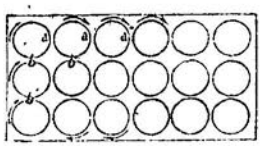


Fig. 168.

Amperian Currents in Magnets.

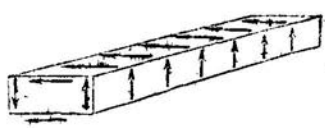


Fig. 169.

magnetized bar at right angles to the direction of its magnetization. On the neighbouring faces of any two particles, the currents are in opposite directions, hence, by the laws of sinuous currents, there is a mutual destruction of force through the whole interior, and the resultant effect is the same as if there were currents circulating round the exterior of the magnet, as represented in Fig. 169.

Magnetization by influence depends, according to this theory, on the tendency of currents to set themselves parallel and in similar directions; and if the substance magnetized possesses coercive force, the direction thus impressed on its currents persists after the influence is removed. In soft iron, on the contrary, they resume their former irregularity.

Ampère's theory of magnetism is in complete accordance with all known facts. But it admits of question whether it is simpler to deduce the laws of magnetism and electro-magnetism from those of electro-dynamics; or to adopt the reverse order, and deduce the laws of electro-dynamics from those of electro-magnetism.

253. Rotations of Magnets.—The following experiment is due to Ampère.

A magnet, loaded with platinum at its lower end, floats upright in mercury contained in a glass vessel (Fig. 170). A cavity is hollowed out in the top of the magnet. This contains mercury, in which a point dips. On connecting one of

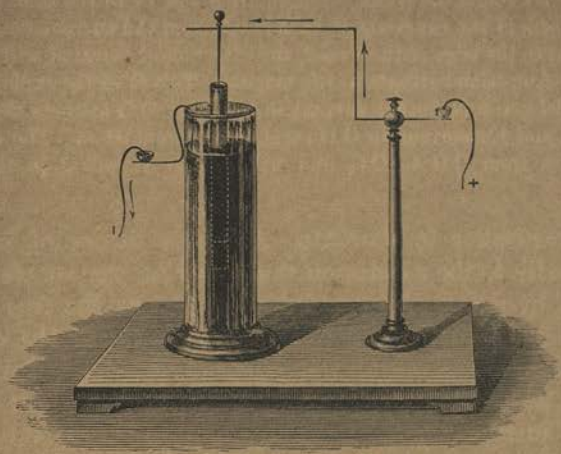


Fig. 170.—Rotation of Magnet.

the terminals of a battery with this point, and the other with the outer edge of the mercury in the vessel, the magnet is seen to rotate on its axis. If the north-seeking pole is uppermost, and the positive pole of the battery is connected with the point, the direction of rotation is N.E.S.W.

The Amperian explanation of this phenomenon is, that it is due to the action

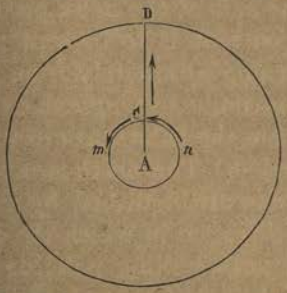


Fig. 171.—Explanation of Rotations.

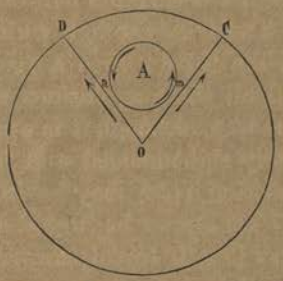


Fig. 172.—Explanation of Rotations.

between the outward-flowing current in the mercury and the Amperian currents which circulate round the magnet. The latter, as represented by the arrows n C, C m in Fig. 171, are opposite to watch-hands. The outward-flowing current in C D attracts the current in C m, since they are both directed away from the angular point C, and repels the current in n C. Hence the magnet is made to rotate in the direction m C n, opposite to that of the Amperian current.

The experiment is sometimes varied by making the point dip in the mercury in

the vessel, the magnet being allowed to float freely near it, and a metallic ring being immersed at the outer edge of the mercury, to which the current flows out in all directions from the point. As soon as the circuit is completed, the magnet begins to revolve round the point. The rotation will be in the same direction as in the other form of the experiment; that is to say, if the current flows outwards from the point to the edge of the vessel, the direction of rotation will be opposite to that of the Amperian currents in the magnet. This is easily explained by the laws of parallel currents, for the current in O C (Fig. 172) attracts the Amperian current at *m*, and the current in O D repels the current at *n*. The magnet will therefore move from O D to O C, and will revolve round O in the direction N.E.S.W.

254. Magnetization by Currents.—Ampère's theory of magnetism leads naturally to the conclusion that a bar of iron or steel may be magnetized by means of a current. Arago was the first to establish this fact, but without a clear apprehension of the conditions necessary for success, or of the criterion for determining which will be the austral, and which the boreal pole. Ampère conceived the idea of introducing the needle to be magnetized into the axis of a solenoid, and the result confirmed his prediction that the poles of the needle would be turned the same way as those of the magnetizing helix. This is what must happen if the currents in the helix force the Amperian currents in the bar into parallelism with themselves, so that all rotate the same way.

It is to be remarked that, in this process of magnetization, the portions of the currents parallel to the axis of the helix produce no effect. The wire through which the current is to be sent may be wound like thread upon a reel, returning alternately from end to end, and all the convolutions will contribute to magnetize the bar the same way, although it is evident that the helices are in this case alternately right-handed and left-handed. The north-seeking and south-seeking poles may be in all cases distinguished by the rule that if a corkscrew be turned in the direction in which the current circulates it will travel towards the north-seeking pole; or it may be remembered by the rule, that if I identify my own body in imagination with a portion of the wire, and suppose the current to enter at my feet, while my face is towards the needle, the north-seeking pole will be to my left. In 1 and 2 (Fig. 173), *a* will be the austral (or north-seeking), and *b* the boreal pole of the inclosed needle, when the current in the helix has the direction indicated by the arrows.

If the direction of winding is changed, in the manner represented

in 3, so that, as seen from one end, the direction in which the current circulates is in one part with and in another against the hands of a watch, consequent points (§ 126) will be formed at the points

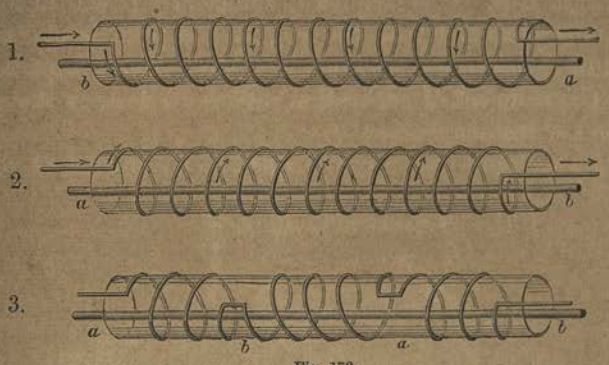


Fig. 173.

1. Right-handed Helix. 2. Left-handed Helix. 3. Arrangement for Consequent Points.

of change. Thus, if the current enters at the left-hand end of the coil, the points $a\,a$ will be austral, and the points $b\,b$ boreal poles.

255. Electro-magnets.—A cylindrical bar of iron can be powerfully magnetized by wrapping round it a coil of insulated wire and sending a current through this coil. Stout copper wire is generally employed for this purpose. Such an arrangement is called an *electro-magnet.*

The bar has often the horse-shoe form, as in Fig. 174, and in this case the central part is usually left bare. The direction of winding on the ends must be such that, if the bar were straightened out, the current would circulate in the same direction round every part. This is clearly shown in the figure. Electro-magnets have been constructed capable of sustaining a load of many tons.

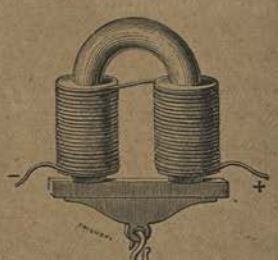


Fig. 174.—Horse-shoe Electro-magnet.

Besides the enormous power that can be given them, electro-magnets have the advantage of being readily made or unmade instantaneously, by completing or interrupting the circuit to which the coil belongs. This principle has received very numerous and varied applications, some of which will be mentioned in later chapters.

256. Residual Magnetism.—When the current round an electro-magnet is interrupted, the destruction of the magnetization is not complete. The small remaining magnetization is called *remanent* or *residual* magnetism. It is frequently sufficiently powerful to retain the armatures in contact with the magnet, and thus necessitates the employment of *opposing springs* if instantaneous separation is desired. The mere act of separation suffices to destroy the greater part of the residual magnetism.

Fig. 175 represents an electro-magnet E E′, furnished with an

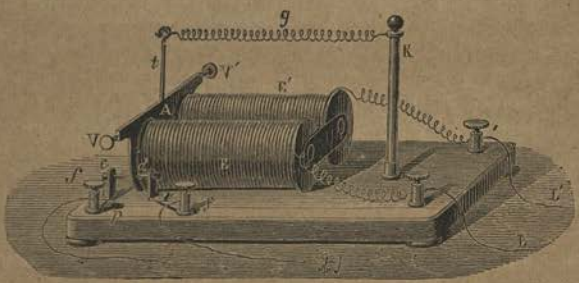


Fig. 175.—Electro-magnet with Opposing Spring.

opposing spring *g*. The armature A, with its lever *t*, turns about the axis V V′. The opposing spring *g* has one end fixed at K, and the other attached to the end of the lever *t*. It therefore tends to remove the armature from the magnet. *c* and *d* are two points whose distance can be regulated, which serve to limit the movements of the armature.

CHAPTER XIX.

INDUCTION OF CURRENTS, OTHERWISE CALLED MAGNETO-ELECTRIC INDUCTION.

257. Induced Currents.—Induced currents may be described as currents produced in conductors by the influence of neighbouring currents or magnets. Their discovery by Faraday in 1831 constitutes an epoch in the history of electrical science. We shall first describe some modes of producing them; and then state their general laws.

258. Currents induced by Commencement and Cessation of Currents.—Let two coils be wound upon the same frame B, one of them, called

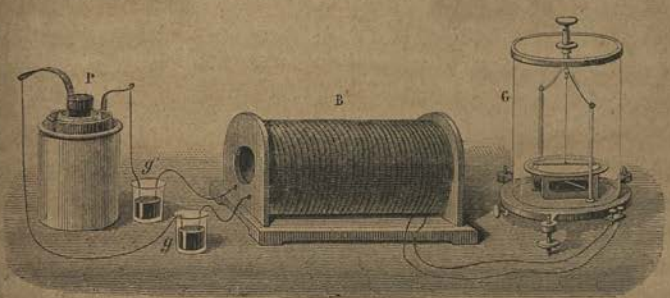


Fig. 176.—Current induced by Commencement or Cessation.

the secondary coil, having its ends connected with the binding-screws of the galvanometer G, while the ends of the other, which is called the primary coil, dip in two cups of mercury *gg'* connected with the two plates of the voltaic element P. As long as the current is passing steadily in the primary coil, the needle of the galvanometer remains undeflected; but if the current be stopped, by lifting a wire out of one of the mercury cups, the needle is immediately deflected, indicating the existence of a current in the same direction as that which

was previously circulating in the primary coil. This effect is very transitory. The needle appears to receive a sudden impulse which immediately passes away. If the current be then re-established, there is a deviation to the other side, indicating a current in the opposite direction to that in the primary coil; and this deviation, like that which occurred before, is merely the effect of an instantaneous impulse, the needle making a few oscillations from side to side, and then remaining steadily at zero. This experiment, which is substantially the same as that by which Faraday first made the discovery, establishes the following proposition:—*When a current begins to flow, it induces an inverse current in a neighbouring conductor; when it ceases, it induces a direct current; and both the currents thus induced are merely instantaneous.*

259. Currents induced by Variations of Strength of Primary Current.—Employing the same apparatus, let us, while the primary current

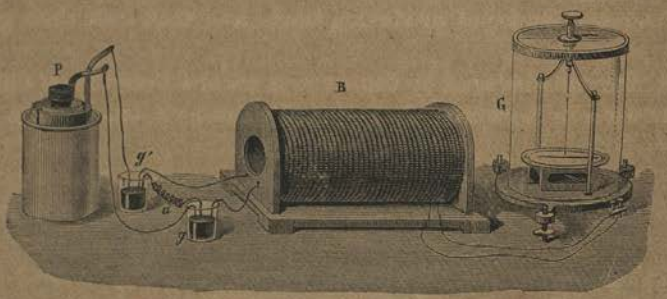


Fig. 177.—Current induced by Change of Strength.

is passing, connect the two mercury cups by the wire *d* (Fig. 177), thus dividing the circuit (§ 200), and causing a great diminution of the current in the primary coil. At the instant of making this connection, the needle of the galvanometer is affected, moving in the same direction as if the primary current were stopped; and on lifting the connecting wire out of one of the cups, so as to produce a sudden increase in the current in the primary coil, the needle moves in the opposite direction. *When a current receives a sudden increase, this produces an inverse current in a neighbouring conductor; and when it is suddenly decreased, a direct current is induced.*

260. Currents induced by Variations of Distance.—Currents may also be induced by change of distance between the primary and secondary conductors. Let the secondary coil, for example, be hollow,

as in Fig. 178, and let the primary coil, with the current passing in it, be suddenly introduced into its interior. The galvanometer will indicate the production of an inverse current in the secondary coil.

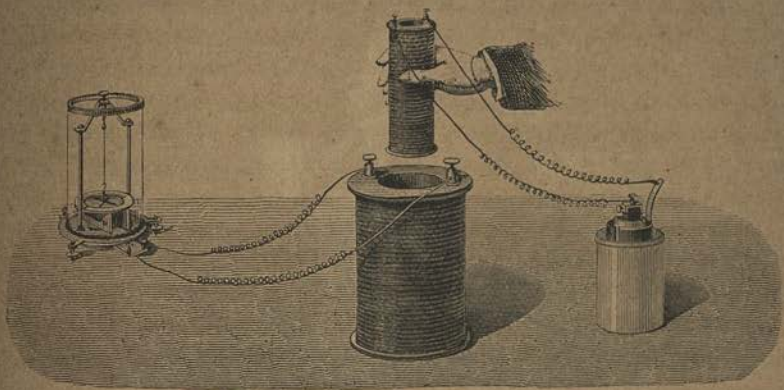


Fig. 178.—Current induced by Change of Distance.

When the needle has come to rest, let the primary coil be withdrawn, and a direct current will be indicated by the galvanometer. These currents differ from those previously mentioned in being less sudden. They last as long as the relative motion of the two coils continues. *When a conductor conveying a current approaches or is approached by a neighbouring conductor, an inverse current is induced in the latter; and when one of these conductors moves away from the other, a direct current is induced.*

Fig. 179.—Current induced by Motion of Magnet.

261. Induction by Magnet.—Let a hollow coil be connected with a galvanometer, and a magnet held over it, as in Fig. 179. As long as the magnet remains stationary, no current is indicated; but when one pole of the magnet is thrust into the interior of the coil, the needle is deflected by an impulse which lasts only as long as the motion of the magnet. If the magnet is allowed to remain at rest in this position, the needle, as soon as it has time to recover from its oscillations, stands at zero; but on withdrawing the magnet, another current will be indicated in the opposite direction to the former.

Currents may also be induced, with even more striking effect, by moving one pole of a magnet towards or from one end of a soft-iron bar previously placed in the interior of the coil (Fig. 180). These currents are due to the magnetism produced and destroyed in the

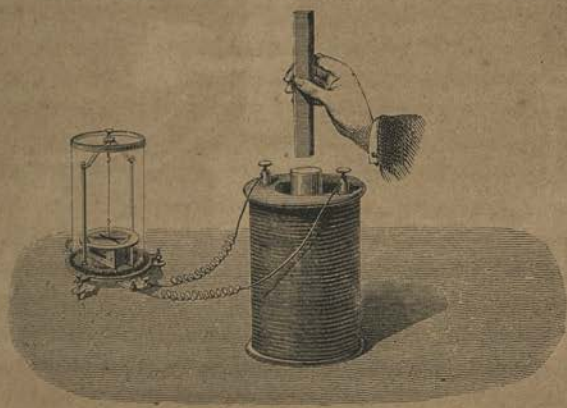


Fig. 180.—Current induced by Magnetization of Soft Iron.

soft iron. *When the intensity of magnetization of a piece of iron or steel undergoes changes, currents are induced in neighbouring conductors.* The directions of these currents can be inferred from the preceding rules by supposing a solenoid to be substituted for the magnet.

262. Lenz's Law.—*The currents induced by the relative movement either of two circuits or of a circuit and a magnet are always in such directions as to produce mechanical forces tending to oppose the movement.* For example, when two parallel wires, through one of which a current is passing, are made to approach, an opposite current is induced in the other; and opposite currents by their mutual repulsion resist approach. This general law as to the direction of induced currents was first distinctly enunciated by Lenz, a Russian philosopher.

263. Arago's Rotations.—Faraday successfully applied his discovery of magneto-electric induction to account for a phenomenon first observed by Arago in 1824, and subsequently investigated by Babbage and Sir John Herschel. A horizontal disc of copper *b b* (Fig. 181), placed in the interior of a box, is set in rapid rotation by turning a handle. Just over the copper disc, but above the thin glass plate which forms the top of the box, a magnetized needle *a a*

is balanced horizontally. When the disc is made to rotate, the needle is observed to deviate from the meridian in the direction of the rotation. When the speed of rotation exceeds a certain limit, the needle is not only deflected, but carried round in continuous rotation in the same direction as the disc.

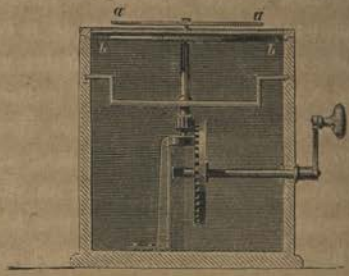


Fig. 181.—Arago's Rotations.

The explanation is to be found in the currents which are induced in the disc by its motion in the vicinity of the magnetized needle. The forces between these currents and the needle are (by Lenz's law) such as to urge the disc backwards; and, from the universal relation which subsists between action and reaction, they must be such as to urge the needle forwards, hence the motion. The direction of the induced current through the centre of the disc at any instant is along that diameter which is directly under the needle, the circuit being completed through the lateral portions of the disc; and it is evident that a current thus flowing parallel to the needle underneath it tends to produce deflection. If the continuity of the disc is interrupted by radial slits, the observed effect is considerably weakened inasmuch as the return circuit is broken. Faraday succeeded in directly demonstrating the existence of currents in a disc rotating near a fixed magnet, by exploring its surface with the amalgamated ends of two wires connected with a galvanometer.

The experiment performed by Arago may be reversed by setting the magnet in rotation, and observing the effect produced on the disc. The latter, if delicately suspended, will be found to rotate in the same direction as the magnet. This experiment was first performed by Babbage and Herschel. Its explanation is identical with that just given. In both cases the induced rotation must be slower than that of the body turned by hand, as the existence of the induced currents depends upon the motion of the one body relative to the other.

When an iron disc is used instead of a copper one, magnetism is induced in the portions which pass under the poles of the magnet; and as this requires a sensible time for its disappearance, there is always attraction between the poles of the needle and the portions

of the disc which have just moved past. The needle is thus drawn forwards by magnetic attraction, and the observed effect is similar to that obtained with the copper disc, though the cause[1] is altogether different.

264. Copper Dampers.—Precisely similar to the above is the explanation of the utility of a copper disc in checking the vibrations of a magnetized needle under which it is fixed. As the needle swings to either side, its motion induces currents in the copper which urge the needle in the opposite direction to that in which it is moving. When it rests for an instant at the extremity of its swing, the currents cease; and as soon as it begins to return, the currents again resist its motion. A copper plate thus used is called a *damper*, and the vibrations thus resisted and destroyed are said to be *damped*. The name is applied to any other means for gradually destroying vibrations.

The resistance which induced currents oppose to the motion producing them is well illustrated by Faraday's experiment of the *copper cube*. A cube of copper is suspended by a thread, and set spinning by twisting the thread and then allowing it to untwist. If, while spinning, it is held between the poles of a powerful magnet, like that represented in Fig. 88, it is instantly brought almost to rest. If the poles are brought very near together, so as to heighten the intensity of the field, and a thin sheet of copper is inserted between them and moved rapidly in its own plane, the operator feels its motion resisted by some invisible influence. The sensation has been compared to that of cutting cheese. Foucault's apparatus for the heating of a copper disc by rotating it between the poles of a magnet (see Part II.), is another illustration of the same principle. In all cases where induced currents are generated, they yield their full equivalent of heat, in accordance with the principles of § 221.

The advantage of employing copper in experiments of this kind arises from its superior conductivity, to which the induced currents are proportional.

265. Direction of Induced Currents specified by reference to Lines of Magnetic Force.—We have already mentioned, in connection with the mutual forces between magnets and currents (§ 165), that a

[1] That is to say, the *main cause;* for there must be induced currents in the iron as well as in the copper, though inferior in strength, on account of the inferior conductivity of the former metal.

wire conveying a current experiences force perpendicular to its length, and at the same time perpendicular to the lines of magnetic force, when placed in a magnetic field. We have seen that, if the current is from foot to head, and the lines of force (for an austral pole) run from front to back, the force experienced by the wire is a force to the right. Motion of the wire to the right will diminish this force by diminishing the current, motion to the left will increase it by increasing the current. Let the direction of the lines of magnetic force be called *from front to back;* then the motion of a conductor *to the right* generates a current in it *from head to foot,* and motion in the opposite direction generates an opposite current. We shall have frequent occasion to recur to this criterion of direction, which applies to every case of induced currents.

As the generation of currents by induction depends not on absolute but on relative motion, namely the relative motion of the conductor and the lines of magnetic force, the criterion of direction will take the following form when the conductor is supposed to be stationary, and the lines of force to move. Let the direction of the lines of magnetic force be called *from front to back,* then *if the lines of force move so as to cut through the conductor from right to left, a current will be induced in the conductor from head to foot.*

If the conductor forms part of a closed circuit, we shall have a continuous current flowing through it as long as the motion lasts. If the circuit is open, there will merely be an incipient current, which, if its direction be from head to foot, will reduce the end of the conductor which we are regarding as its foot to a higher electrical potential than the other, and this difference of potential will be maintained as long as the motion lasts.

266. Quantitative Statements.—In order to state the quantitative laws of induced currents in the simplest and most general manner, we must employ the conception of tubes of force as explained in § 52 (but they will now be tubes not of electrical but of magnetic force), and we must suppose them to be arranged in the equable manner described in § 53. That is, to say, we must suppose the whole field cut up into tubes of force in such a manner that, if a cross section (an equipotential surface as regards magnetic potential) be made in any part of the field, the number of tubes per unit of sectional area is equal to the intensity in that part of the field. It is more usual to speak of *number of lines of force* than of *number*

of tubes, the convention being that each tube contains one line; but the counting of tubes rather than lines has the advantage of naturally allowing fractional parts to be reckoned, and not suggesting the idea of discontinuity.

The tubes of force due to a magnet are to be regarded as rigidly attached to the magnet, and carried with it in all its movements, whether of translation or rotation.[1] They undergo no change of size or form unless the magnet itself undergoes changes in its magnetization.

These conceptions being premised, the quantitative laws of induced currents can be stated with great simplicity and complete generality.

1. When a conductor is moved in a magnetic field, the ELECTROMOTIVE FORCE generated by the motion is equal to the number of tubes which the conductor cuts through per unit time.

2. If the conductor forms part of a closed circuit, the CURRENT generated in the circuit is the quotient of the number of tubes cut through per unit time, by the resistance of the circuit; and lastly,

3. The whole QUANTITY of electricity conveyed by the current is the quotient of the number of tubes cut through, by the resistance of the circuit. The quantity of electricity conveyed by a current of brief duration is measured by observing the swing of a galvanometer needle (§ 175). It is proportional to the greatest deviation of the needle from zero, provided that this deviation is small, and that the duration of the current is less than that of the swing. When experiments on induced currents are made under these conditions, it is found that the deviation of the needle depends only on the initial and final positions of the body which is moved, being independent both of the path taken and of the velocity.

The dependence of the quantity of electricity induced upon the number of tubes cut through, was discovered by Faraday, who established it experimentally by moving a loop of wire in various ways in the vicinity of a magnet. The three foregoing laws were all, in fact, substantially established by the series of researches in which these experiments occur.[2]

[1] It may be noted that when a cylindrical magnet is rotated on its axis, it induces no current in a neighbouring wire, inasmuch as its lines of force cut such a wire once positively and once negatively.

[2] *Researches*, vol. iii. series xxviii.

In counting the tubes cut through, it is necessary to attend to the direction of the current due to the cutting of each tube. Those tubes which are so cut as to give currents in one direction round the circuit (when tested by the criterion of § 265) must have one sign given them, and those which give currents in the opposite direction must be reckoned as of the opposite sign. It is in every case the algebraic sum that is to be taken; and if a tube is cut once positively and once negatively, it may be left out of the reckoning.

267. **Deduction of the Laws from Maxwell's Rule.**—The laws in the preceding section are deducible by the principle of energy from Maxwell's rule (§ 246), which asserts that the mechanical work requisite to produce a given displacement of a circuit in a magnetic field is equal to the strength of the current multiplied by the number of tubes of force cut through. Call this number n and the current C. Then, if there is no other electro-motive force in the circuit beside that which is due to the motion, the work Cn must be equal to the energy of the current, which is ECt, t denoting the time. Hence we have

$$E=\frac{n}{t}$$

that is, the electro-motive force is equal to the number of tubes cut through per unit time.

If the circuit contains a battery of electro-motive force E_0, the energy supplied by this battery is $E_0 Ct$; hence we have

$$ECt=E_0Ct+Cn,$$

$$\text{Or} \qquad E=E_0+\frac{n}{t},$$

that is, the electro-motive force $E-E_0$ produced by the motion is equal to the number of tubes cut through per unit time; and these are to be counted as positive when the direction of the motion is opposed to the forces of the field, since we have supposed positive work to be supplied. The induced current is therefore *with* or *against* the original current according as the motion is *against* or *with* the mutual forces between the current and the field.

268. **Tubes of Force for Resultant and Components.**—The principle of superposition can be applied to tubes of force. For if we resolve the whole force into any two components, these three forces will be represented by the three sides of a triangle as in Fig. 182, where BC represents the resultant, and BA, AC the two components.

By producing each side, and drawing a parallel to it through the opposite angle, we obtain longitudinal sections of three tubes, whose cross sections are proportional to the perpendiculars on the sides of the triangle A B C from the opposite angles. These perpendiculars are inversely as the sides (since side × perpendicular = double area of triangle). Hence the cross sections of the three tubes are inversely as the forces, and the number of tubes per unit area of cross section will be directly as the forces.

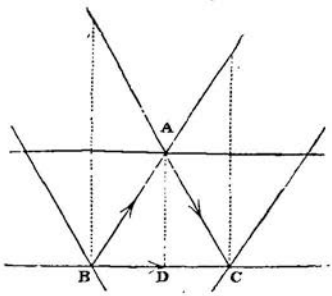


Fig. 182.—Superposition of Tubes.

In moving parallel to B C, no resultant tubes are cut, and as many component tubes are cut from the right to the left as from the left to the right of a person travelling with the component forces. In moving perpendicular to B C, through a distance equal to A D the perpendicular from A on B C, the fraction $\frac{B D}{B C}$ of one component tube is cut, and the fraction $\frac{D C}{B C}$ of another, the sum of these fractions being unity. Hence, in every motion, *the number of component tubes cut* (reckoned algebraically) *is equal to the number of resultant tubes.*

In dealing with terrestrial magnetism, it is often convenient to consider separately the tubes of vertical and of horizontal force; for example, effects depending only on horizontal force can be determined by considering the horizontal tubes alone.

269. **Uniform Field.**—In a field of uniform intensity the lines of force cannot be curved, for if they were, the equipotential surfaces, which cut them at right angles, would not be equidistant, but would be nearer together on the side next the centres of curvature than on the opposite side, and the force would be greatest where these surfaces were nearest (§ 49). The lines of force in a field of uniform intensity are therefore straight; and, since the tubes of force must have a constant cross section (§ 52), these tubes must be cylinders or prisms, and the lines of force parallel. A field of uniform intensity of force is therefore also uniform as regards direction of force.

The electro-motive force generated by the motion of a straight wire of length L in a field of uniform intensity I, with a velocity of translation V, being equal to the number of tubes cut through in unit time, will be L V I, if the length of the wire and the direction of

motion are perpendicular to each other and to the lines of force. For any other position of the wire, and for any other direction of motion, the number of tubes cut through will evidently be less. If the length of the wire is parallel to the lines of force, no tubes will be cut through, whatever be the direction of motion; and if the direction of motion be parallel to the lines of force, no tubes will be cut through, whatever be the position of the wire. In these two cases, then, there is no generation of electro-motive force tending to produce a current along the wire.

Terrestrial magnetism furnishes us with an example of a uniform field, so long as we confine our attention to a space of moderate dimensions, such as the interior of a room.

270. **Movement of Lines of Force with Change of Magnetization.**—As long as a piece of iron or steel remains unchanged in its magnetization, its tubes of force are to be conceived of as a rigid system rigidly connected with it. When the intensity of magnetization is increased, new tubes are added and the old ones are crushed together. The new tubes are to be regarded as coming into existence at the magnetic axis of the magnet, and pushing the old ones further away from the axis. When the intensity of magnetization falls off, a reverse motion occurs, and the axis absorbs those tubes which lie next it.

Similar remarks apply to changes of strength in a current. The lines of magnetic force due to a current in a straight wire are circles, and the tubes of force are rings, having the wire for their common axis. When the current receives an increase of strength, the new rings must all be conceived of as starting from the wire, and pushing out the old rings before them, and on the diminution or cessation of the current a reverse movement occurs.

When a current suddenly commences in a wire, or a piece of soft iron is suddenly magnetized, the effect upon a neighbouring conductor is the same (so far as this source of magnetic force is concerned) as if the conductor were suddenly moved up from a great distance into its actual position. The experimental results described in §§ 258–261 are thus only particular cases of the general principles of §§ 265, 266.

271. **Unit of Resistance.**—Units of *length*, *mass*, and *time*, having been selected, unit *force* is defined as that which, acting on unit mass for unit time, generates unit velocity.

A magnetic pole of unit strength, or a unit *pole*, is defined as that

which attracts or repels an equal pole at unit distance with unit force.

Unit *intensity of field* is defined as the intensity at a place where a unit pole experiences unit force.

A unit *current*, or a current of unit strength, is one which, for each unit of its length, affects a unit pole at unit distance with unit force. In passing through a circular coil of unit radius and length l, the force which it exerts on a unit pole at the centre is l.

Unit *electro-motive force* is the electro-motive force existing in a circuit in which unit current does unit work in each unit of time; and unit *resistance* is the resistance of a circuit in which unit electro-motive force would produce unit current.

The course of the above investigation shows that the units of length, mass, and time are sufficient to determine all the other units mentioned. It can be further shown[1] that the unit of resistance is independent of the unit of mass, and depends only on the units of length and time, being directly as the unit of length, and inversely as the unit of time—a property which is also characteristic of the unit of velocity. Hence a resistance, like a velocity, can be adequately expressed in *metres per second*. The unit of resistance now commonly employed is the *ohm*, which is defined as *ten million metres per second*. It is the resistance of a column of pure mercury at 0° C. 1·063 metre long and a millimetre in diameter, or of about 50 metres of pure copper wire a millimetre in diameter.

272. Induction by Means of Terrestrial Magnetism.—If a wire ring, or any other form of closed circuit, receives a movement of translation in a uniform field, no current is generated, because the same number of force-tubes are cut negatively as positively. Whatever currents are generated by the motion of a closed circuit in the terrestrial magnetic field, must therefore be due solely to rotational movements. Suppose the circuit to consist of a single circle of wire, and let it be initially placed so that its plane is perpendicular to the dipping-needle, and therefore perpendicular to the lines of magnetic force. In this position, the number of force-tubes which it incloses is equal to the product of the inclosed area by the total intensity of terrestrial magnetic force, that is to $\pi r^2 I$, I denoting this intensity, and r the radius of the circle. Now let the ring rotate through 180° about any diameter, so that it comes back into its original place, but facing the opposite way. During this semi-revolution, each half of

[1] See Appendix at the end of this Part.

the ring has cut through all the tubes which passed through the ring, and though in one sense the two halves have been cutting the tubes in opposite directions, the application of the criterion of § 265 shows that the resulting currents are in the same direction round the circuit. The number of tubes cut through is therefore to be reckoned as $2\pi r^2 I$, and the quotient of this by the time occupied in a semi-revolution is the average electro-motive force (§ 266). If the rotation be uniform, the actual electro-motive force is greatest in the middle of the semi-revolution, and is zero at its commencement and termination. During the other half-revolution the circumstances are precisely the same, except that the two halves of the ring have changed places. If we compare the currents in two positions of the ring which differ by 180°, we see that the current round the ring has the same direction in space, but opposite directions as regards the ring itself.

If, instead of a single ring of wire, we have a circular coil consisting of any number of convolutions, with its two ends united, the same principles apply. If there are n convolutions, the electro-motive force will be n times greater than with one, but as the resistance is also n times greater the strength of current is the same.

In the apparatus called *Delezenne's Circle*, a coil of wire revolves about a diameter, but the two ends of the coil, instead of being directly united, are so connected with the two ends of the axis of rotation that the circuit is completed through a galvanometer. On rotating the coil rapidly by means of a handle provided for the purpose, a current is indicated by the galvanometer, and this current is found to be strongest (for a given rate of rotation) when the axis is perpendicular to the dipping-needle. If the axis is inclined at an angle θ to the dipping-needle, the current is proportional to $\sin\theta$; and if the axis is parallel to the dipping-needle there is no current at all. For a given position of the axis, the current varies directly as the speed of rotation. When the time of a revolution is only a small fraction of the time in which the needle would oscillate, the variations of electro-motive force, and consequently of current, which take place during a revolution, have not time to manifest themselves, and the deflection of the needle is that due to the average current. It is necessary, however, that a commutator be employed to prevent the reversal of the current at each half-revolution. The proportionality of the current to $\sin\theta$ is easily inferred from the principles of the foregoing sections; for if the plane of a circle, instead of being per-

pendicular to the lines of force, is inclined to them at an angle θ, the number of force-tubes which it incloses will be not $\pi r^2 I$, but $\pi r^2 I \sin \theta$.

273. **British Association Experiment.**—The first experiments for constructing a coil whose resistance should be a known number of metres per second, were conducted by a committee of the British Association in 1862. A circular coil of wire, with its ends joined, was made to revolve rapidly, at a measured rate, about a vertical axis; and the current induced was measured by the deflection of a magnetized needle suspended, within a glass case, in the centre of the coil. The part of the earth's magnetic force which comes into play in this arrangement, is only the horizontal component, and it is worthy of remark that variations in the horizontal intensity do not alter the deflection of the needle, since they affect to the same extent the amount of the induced current, and the terrestrial couple on the needle tending to resist deflection.

All the other elements involved were determined by observation, and hence the value of R in metres per second was calculated. By comparing the resistances of other coils with that of the coil used in this experiment (a comparison which can be made with great accuracy by means of Wheatstone's bridge), their values in metres per second were at once determined; and it was easy to construct a resistance-coil of ten million metres per second, or any other desired amount of resistance. As the resistances of metals are increased by heat, a standard coil can only be correct at one particular temperature.

274. **Induction of a Current on Itself: Extra Current.**—If two portions of the same wire are side by side, the sudden commencement or cessation of a current in one, induces a current in the other, just as if they were portions of two unconnected circuits. An action of this kind occurs whenever a current commences or ceases in a coil, each convolution exercising an inductive influence on the rest. This action is called the *induction of a current upon itself*, and the current due to it is called an *extra current*.

The extra current on the commencement of the primary current is inverse, and merely acts as a hindrance to commencement; but the extra current on the stoppage of the primary current is direct, and is often a strongly-marked phenomenon. Hence it is that, with batteries of ordinary power, a spark is obtained on breaking, but not on making connection. The spark is particularly brilliant when a coil of many convolutions is included in the circuit, and especially if

this coil incloses a core of soft iron. If an observer holds in his hands two metallic handles permanently connected with the two ends of such a coil, and if the circuit of the battery is alternately made and broken, he will receive a shock from the extra current at each interruption. If the interruptions succeed each other rapidly, the physiological effect may become very intense. Many of the machines employed for medical purposes are constructed on this plan.

Special contrivances are provided for producing a rapid succession of interruptions at regular intervals. They are called *rheotomes* or *contact-breakers*. Sometimes they consist of toothed wheels turned by hand,—sometimes of vibrating armatures moved automatically.

275. Ruhmkorff's Induction-coil.—Induced currents capable of producing very striking effects are furnished by the apparatus first successfully constructed by Ruhmkorff, and hence known as Ruhmkorff's coil.

It contains two coils of wire, one of them forming part of the circuit of a battery, and called the primary coil; while in the other, called the secondary coil, the induced currents are generated. In the axis of the coils is a bundle of stout straight wires of soft iron, with a disc of the same material at each end, to which the wires are united. Around this core is wound the primary coil, consisting of a copper wire about two millimetres in diameter. The ends of this wire are shown at f and f'. The secondary coil consists of much finer wire (about a quarter of a millimetre in diameter) and of much greater length. In large instruments the primary coil may have a length of 80 metres, and the secondary a length of 150 kilometres (94 miles). Special precautions must be taken to insulate the different convolutions of the secondary coil from one another, and from the primary coil. The two ends of the secondary wire are at the binding-screws A, B, which are supported on glass pillars. It is obvious that if currents are alternately passed and stopped in the primary coil, there will be an alternate generation of currents (or at all events of electro-motive forces) in opposite directions in the secondary coil. The action of the core is similar to that of the soft-iron bar in Fig. 180, and its inductive effect is always in the same direction as that of the primary coil, for the primary coil may itself be regarded as a temporary magnet with its poles turned the same way as those of the core.

The wooden stand which forms the base of the instrument is

hollow, and contains a condenser of the kind described in § 73, its two coatings being connected with the primary coil at the two sides of the break. It mitigates the spark which occurs at the break, and the two opposite electricities, after rushing into its two coatings, rebound, so as to produce a reverse current in the primary, thus doubling the inductive effect of a mere stoppage. When the primary current is supplied by an alternating dynamo (without any breaking of contact) the condenser is not required.

The successive makes and breaks are effected automatically in

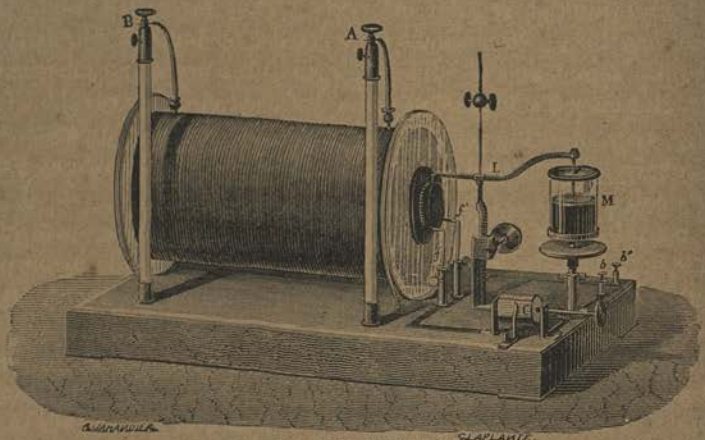


Fig. 183.—Ruhmkorff's Coil.

various ways. In small instruments the arrangement adopted is usually the same as that of the vibrating alarum described in § 300, but for large instruments Foucault's contact-breaker is preferred. It is represented in its place in Fig. 183.

The wires from the battery are attached at b and b'. The current entering for example at b, passes to the commutator C, and thence, through a brass bar let into the table, to the end f of the primary coil. Having traversed this coil, it comes out at f', and is conducted to a vertical pillar, carrying at its upper end a spring, to which the transverse lever L is attached. One end of the lever carries a point which just dips in the mercury of the vessel M, the bottom of which is metallic, and is in communication with b'. The other end of the lever carries a small armature of soft iron just above the end of the core.

When the current passes, the core becomes magnetized and attracts this armature, thus lifting the point at the other end of the lever out of the mercury and breaking circuit. The core being thus

demagnetized, the elasticity of the spring releases the armature, and the point again dips in the mercury, and completes the circuit. A thin layer of absolute alcohol is usually poured on the surface of the mercury, and serves, by its eminent non-conducting power, to make the interruptions and renewals of the current more sudden.

The *commutator* C serves to stop the current from passing or to make it pass in either direction, at pleasure. It is represented in end view and bird's-eye view in the two parts of Fig. 184. There is a cylinder of insulating material, turning by means of metallic axle-ends on metallic supports connected with the two ends of the primary coil. One axle-end is permanently connected by means of the screw g with the brass plate C on the surface of the cylinder, and the other axle-end is in like manner connected with the plate C′ diametrically opposite to C. The contact-springs ff are in permanent connection with the two binding-screws A A′ which receive the wires from the battery. When f presses against C, and f' against C′, as shown in the figure, A is connected with one end of the primary coil and A′ with the other; and when the commutator is turned (by its milled head) through 180°, these connections will be reversed. If it is turned through 90°, the connections will be interrupted, as the contact springs will bear against insulating portions of the cylinder. The milled head is, of course, insulated from the axle-ends so as to protect the operator.

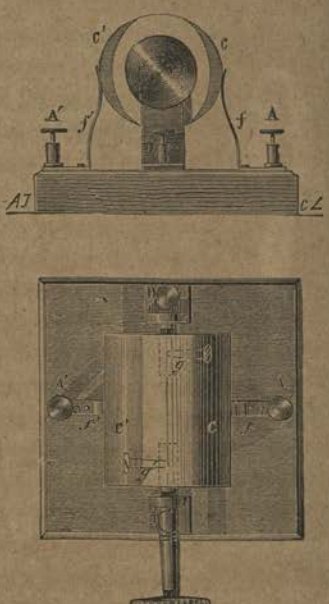


Fig. 184.—Commutator.

276. Spark from Induction-coil.—When the ends of the secondary coil are connected, currents traverse it alternately in opposite directions, as the primary circuit is made and broken. These opposite currents convey equal quantities of electricity, and if they are employed for decomposing water in a voltameter, the same proportions of oxygen and hydrogen are collected at both electrodes. If, however, the ends are disconnected, so that only disruptive discharge can

occur between them, the inverse current, on account of its lower electro-motive force, is unable to overcome the intervening resistance, and only the direct current passes (that is, the current produced by breaking the primary circuit). The sparks are usually from 1 inch to about 18 inches long, according to the size and power of the apparatus, and exhibit effects comparable to those obtained by electrical machines. A Leyden battery may be charged, glass pierced, or combustible bodies inflamed.

The great electro-motive force of the induced current, which enables it to produce these striking effects, depends on the great number of convolutions of the secondary coil, and on the suddenness of the interruptions of the primary current. The average electro-motive force is the product of the number of convolutions by the number of tubes of force which cut through them, divided by the time occupied (§ 263).

Fig. 185. Statham's Fuse.

The discharges from a Ruhmkorff's coil become more violent and detonating if the two electrodes of the secondary coil are connected respectively with the two coatings of a Leyden jar, but the length of the spark is very much diminished.

Induction-coils are often used for firing mines, by means of Statham's fuse, which is represented in the annexed figure (Fig. 185). Two copper wires covered with gutta-percha have their ends separated by a space of a few millimetres, and inclosed in a little cylinder of gutta-percha containing sulphuret of copper. This, again, is inclosed in a cartridge, C D, which is filled up with gunpowder. The two wires are connected with the two ends of the secondary coil, and when the instrument is set in action, sparks pass between the ends A, B, heating the sulphuret of copper to redness, and exploding the powder.

277. **Discharge in Rarefied Gases.**—When the ends of the secondary coil are connected with the electrodes of the electric egg (Fig. 186), which has first been exhausted as completely as possible by the air-pump, a luminous sheaf, of purple colour, is seen extending from the positive ball to within a little distance of the negative ball. The latter is surrounded by a bluish glow. The blue and purple lights are separated by a small interval of darkness. If other gases are used instead of air, the tints change, but there is always a decided

difference of tint between the positive and negative extremities. By the aid of the commutator it is easy to reverse the current, and thus produce at pleasure an interchange of the appearances presented by the two terminals.

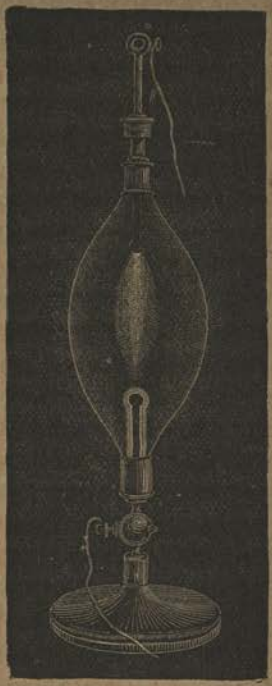


Fig. 184.—Electric Egg.

If, before exhausting, we introduce into the egg a little alcohol, turpentine, or other volatile liquid, the light presents a series of bright bands alternating with dark spaces. Plate II. fig. 1 represents these stratifications as seen in vapour of alcohol.

The phenomenon of stratification is seen to more advantage in long tubes than in the electric egg; and the presence of alcoholic or other vapour may be dispensed with if the exhaustion be carried sufficiently far, as in the tubes constructed by Geissler of Bonn, which contain various gases very highly rarefied, and have platinum wires sealed into their extremities to serve as electrodes. Four such tubes are represented in Plate II. Certain substances, such as uranium glass, and solution of sulphate of quinine, become luminous in the presence of the electric light, and are called *fluorescent*. Such substances are often introduced into Geissler's tubes, for the sake of the brilliant effects which they produce.

If a Geissler's tube is fastened to the front of a disc which can be rapidly rotated, the instantaneousness of the discharges, combined with the persistence of visual impressions, cause it to present the appearance of the rays of a star. The star appears stationary if an exact number of discharges occur in the time of one revolution. A slight increase or diminution in the speed of rotation will then make the star appear to revolve forwards or backwards.

The discharge in a Geissler's tube is capable of deflecting a magnetic needle, and can itself be deflected to one side of the tube by the action of a magnet.

278. Experiments of Gassiot and De La Rue.—By means of a battery of some thousands of cells, discharge in rarefied gases can be obtained

without the use of an induction-coil, and with the advantage of greater steadiness. This was done by Mr. Gassiot, and on a larger scale by Dr. De La Rue. The stratifications were still observed, and appeared absolutely fixed in position to the naked eye. When examined by a revolving mirror they were found to exhibit the appearance of a rapid succession of discharges.

Dr. De La Rue's battery consisted of 11,000 small chloride of silver cells; and when all the cells were used, a steady stream of fire passed between the terminals as soon as they were brought within about half an inch of each other, in air at atmospheric pressure.

279. **Spectra.**—The spectrum of an electric discharge usually consists of the spectrum of the gas in which it occurs, superposed on the spectrum of the metallic terminals, a small portion of the latter being vaporized. The source of electricity usually employed is the Ruhmkorff coil. As the discharge is rendered more violent, whether by increasing the strength of the primary current, or by connecting the terminals of the secondary with a Leyden jar, the number of lines in the spectrum is usually increased. The spectra of gases are usually observed by means of Geissler's tubes; which, for this purpose, are sometimes constructed of such a shape as to admit of directing the spectroscope along the axis of the tube, so as to obtain greater concentration of light.

280. **Change of Resistance with Diminution of Pressure.**—The resistance of a gas to electric discharge diminishes as the pressure diminishes. A coil which can only give a half-inch spark in air at atmospheric pressure may give a discharge between terminals more than a foot apart in highly rarefied air. But this diminution does not go on indefinitely. At a certain very small pressure (different for different gases) the resistance is a minimum, and at the lowest pressures attainable by modern methods the resistance is many times greater than at atmospheric pressure. This fact was for a long time disputed, but the earlier experiments of Gassiot and Alvergniat have been confirmed by the later ones of De La Rue and Crookes, and the result is now conclusively established.

De La Rue employed a vacuum tube connected with a bulb containing caustic potash which had absorbed the last traces of carbonic acid gas, after the tube, originally filled with this gas, had been exhausted by a Sprengel pump. No discharge passed until heat was applied to the bulb, so as to drive off some of the gas, and thus diminish the perfection of the vacuum. The discharge ceased as the

potash cooled down, and could be renewed any number of times at pleasure.

Crookes, who has given increased command of vacua by his improvements in the Sprengel pump, has succeeded in measuring the pressures at which the resistances of air and some other gases are a minimum, and has arrived at the conclusion that across an absolute vacuum discharge would be impossible. The explanation stated roughly seems to be that the particles of gas act as carriers of the discharge. When there are too many carriers they hamper one another, and when there are no carriers at all nothing is carried.

281. Crookes' "Radiant Matter."—Crookes, by carrying exhaustion to a higher point than had before been reached, has obtained a new set of phenomena. An influence (to which he gives the name of *radiant matter*) emanates from the negative terminal, and proceeds only in straight lines. Glass and precious stones exposed to this influence become strongly phosphorescent, and a delicately-balanced glass wheel with blades like a paddle-wheel is set in rotation by the successive impacts of the "radiant matter" upon its blades. The influence is only exerted on surfaces which are directly visible from the negative terminal, and is stopped by glass, so that a piece of glass interposed casts a shadow. By employing as the negative terminal a platinum saucer shaped like a concave mirror the influence can be made to converge on an object placed at its centre of curvature, and platinum foil can thus be heated to incandescence. No special effect is exhibited by the positive terminal. These phenomena are at their best when the vacuum is such as to give minimum resistance.

282. Tesla's Experiments with High Frequencies.—Tesla has shown that the character of the discharge is greatly altered by raising the frequency of alternation in the primary to a high pitch. This he has done sometimes by employing a specially constructed alternating dynamo giving many thousand reversals per second, but more usually and with more strongly marked effects by employing the oscillatory discharge of a jar or battery of jars, the oscillations in this case ranging from hundreds of thousands to some millions per second. It is necessary so to regulate the dimensions of the primary and secondary conductors that the electric oscillations in the one correspond in natural period with those of the other, so that the effects are exalted by means of resonance, as in Hertz's experiments.

A discharge, in the form of powerful brushes and luminous streams, can thus be obtained, covering an area of several square feet, and this not only in exhausted spaces but in ordinary air.

To produce discharge in a Geissler tube by such means, it is sufficient to connect one terminal of the tube with a terminal of the coil, and to bring within a moderate distance of the other terminal of the tube a large conductor, such as the human body.

When the frequency is sufficiently high, the effects are mainly due to the electrostatic induction of the alternating charges at the terminals of the secondary; and as this action can take place through glass, there is no necessity for wires sealed through the glass. It is only necessary to have two small pieces of metal inside the tube, one at each end, and to have two conductors of considerable size serving as terminals of the secondary coil and brought near the ends of the tube.

Either by means of large luminous brushes, or of Crookes' phosphorescence, it is hoped that these excessively rapid alternations will furnish, when fully worked out, a more economical system of electric lighting than any at present in use. Tesla expresses the conviction that, "to whatever kind of motion light may be due, it is produced by tremendous electrostatic stresses vibrating with extreme rapidity."[1]

It is noteworthy that the currents which produce the brilliant discharges above mentioned can pass through the human body without producing injury or inconvenience. The Geissler tube can conveniently be excited by placing one hand on a terminal of the coil and the other on a terminal of the tube.

[1] For an account of Tesla's experiments see No. 97 of *Journal of Inst. of Elec. Eng.*, issued April, 1892. (E. & F. N. Spon.)

CHAPTER XX.

MAGNETO AND DYNAMO MACHINES.

283. Magneto-electric Machines.—Faraday's discovery of the induction of currents by magnets, was speedily utilized in the construction of magneto-electric machines, which, without a battery, and with no other stimulus than that afforded by the presence of a permanent magnet, enable the operator, by the expenditure of mechanical work, to obtain powerful electrical effects. The first machine of this kind was constructed in 1833 by Pixii, and had a magnet revolving close to a double coil, in which a current was thus generated. The construction was improved by Saxton, and afterwards by Clarke, who made the magnet fixed, and caused the coil, which is much lighter, to rotate in front of it.

284. Clarke's Machine.—In this machine there is a compound horse-shoe magnet fixed to a vertical support. Close in front of the magnet, near its poles, are two connected coils t, t', each containing a soft-iron core. The two cores are united by a plate of copper on the side next the magnet, and by a plate of soft iron on the remote side. The direction of winding in the two coils is the same as for an ordinary horse-shoe electro-magnet. The coils are mounted on an axis f, which passes through the support of the steel magnet, and carries a pinion. By means of an endless chain passing over this pinion, and over a large wheel to which a handle is attached, the pinion, and with it the coils, can be made to revolve rapidly. The ends of the wire which forms the two coils are connected respectively with the two metallic pieces E, E′ (Fig. 188), which are mounted on the axis, but insulated from it and from each other.

Let us now examine the formation of the currents. The two iron cores, with their connecting iron plate, may be regarded as a temporary horse-shoe magnet, whose poles are always of opposite name

to those of the steel magnet which are respectively nearest to them. The intensity of magnetization is greatest when the soft-iron magnet is horizontal, vanishes when it is vertical, and in passing through the vertical position undergoes reversal. If we call one direction of magnetization positive and the opposite direction negative, the

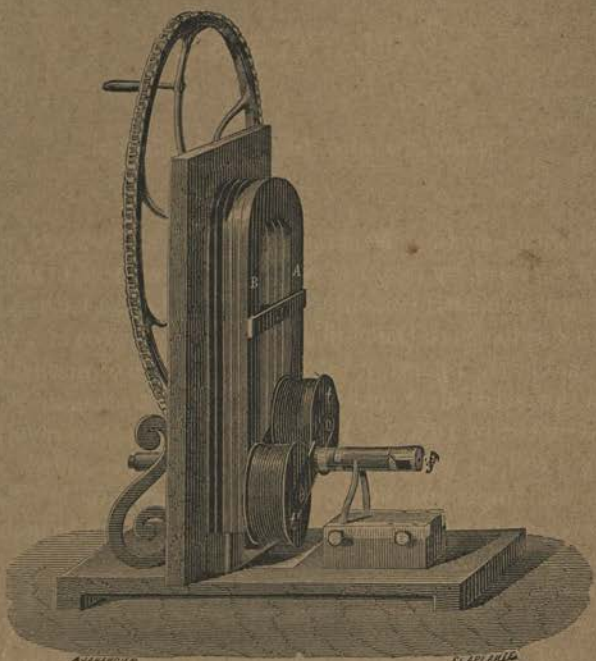


Fig. 187.—Clarke's Machine.

strongest positive magnetization corresponds to one of the two horizontal positions, and the strongest negative to the other, the two positions differing by 180°. While the magnet, then, is revolving from one horizontal position to the other, its magnetization is changing from the strongest positive to the strongest negative, and this change produces a current in one definite direction in the surrounding coil. During the next half-revolution the magnetization is again gradually reversed, and an opposite current is generated in the coil. If we examine the direction of the currents due to the cutting across of the lines of force of the permanent magnet by the convolutions of the coil, we shall find that they concur with those due to the action of the cores. The current in the coils circulates

in one direction as long as the electro-magnet is moving from one horizontal position to the other, and changes its direction at the instant when the cores come opposite the poles of the steel magnet.

By the aid of the commutator represented in Fig. 188, the currents may be made to pass always in the same direction through an external circuit. r and r' are two contact-springs bearing against the two metal pieces E, E′, which are the terminals of the coil. At the instant when the current in the coil is reversed, these springs are in contact with intermediate insulating pieces which separate the metallic pieces E, E′. When the current in the coil is in one direction (say from E to E′), r is in contact with E, and r' with E′. When the current in the coil is in the opposite direction (E′ to E), r is in contact with E′, and r' with E; thus in each case r is the positive and r' the negative spring, and the current will be from r to r' in an external connecting wire. O O, O′ O′, are metallic pieces insulated from each other, and connected with the springs r r' respectively. Binding-screws are provided for attaching wires through which the current is to be passed.

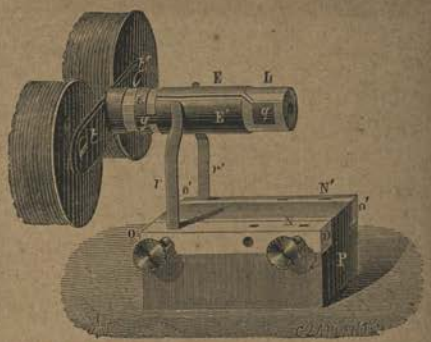


Fig. 188.—Commutator of Clarke's Machine.

With this machine water can be decomposed, wire heated to redness, or soft iron magnetized; but these effects are usually on a small scale on account of the small dimensions of the machine.

For giving shocks, two wires furnished with metallic handles are attached to the binding-screws, and a third spring is employed which puts the terminals E E′ in direct connection with each other twice in each revolution, by making contact with two plates q. When these contacts cease, the current is greatly diminished by having to pass through the body of the person holding the handles, and the extra-current thus induced gives the shock. To obtain the strongest effect, the hands should be moistened with acidulated water before grasping the handles.

285. Magneto-electric Machines for Lighthouses.—Very powerful effects can be obtained from magneto-electric machines of large size driven rapidly. Such machines were first suggested by Professor

Nollet of Brussels; and they have been constructed by Holmes of London and the Compagnie l'Alliance of Paris. It is by means of these machines that the electric light is maintained in several lighthouses; they have also been employed to some extent in electro-metallurgy. Fig. 189 represents the pattern adopted by the French company.

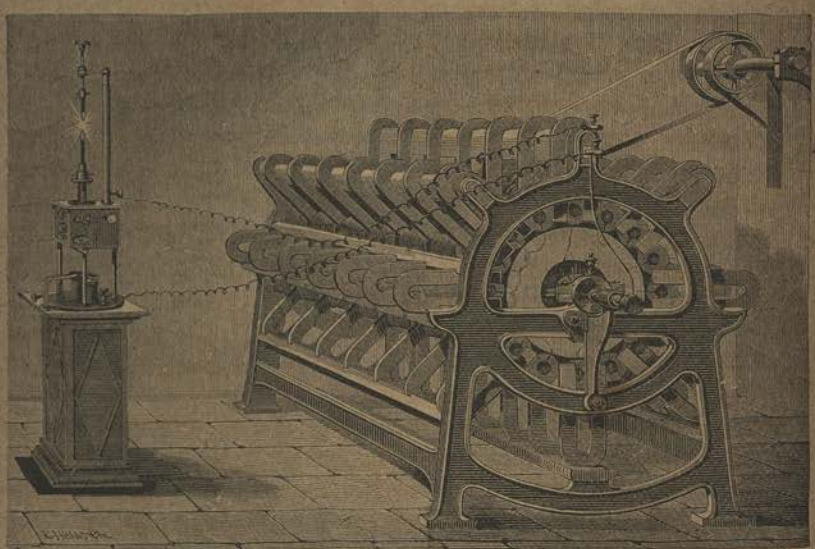


Fig. 189.—Lighthouse Machine.

It has eight rows of compound horse-shoe magnets fixed symmetrically round a cast-iron frame. They are so arranged that opposite poles always succeed each other, both in each row and in each circular set. There are seven of these circular sets, with of course six intervening spaces. Six bronze wheels, mounted on one central axle, revolve in these intervals, the axle being driven by steam-power transmitted by a pulley and belt. The speed of rotation is usually about 350 revolutions of the axle per minute. Each of the six bronze wheels carries at its circumference sixteen coils, corresponding to the number of poles in each circular set. The core of each coil is a cleft tube of soft iron, this form having been found peculiarly favourable to rapid demagnetization.

Each core has its magnetism reversed sixteen times in each revolution, by the influence of the sixteen successive pairs of poles between which it passes, and the same number of currents in alternately

opposite directions are generated in the coils. The coils can be connected in different ways, according as great electromotive force or small resistance is required. The positive ends are connected with the axle of the machine, which thus serves as the positive electrode, and a concentric cylinder, well insulated from it, is employed as the negative electrode.

When the machine is employed for the production of the electric light, the currents may be transmitted to the carbon points in alternate directions, as they are produced. For electro-metallurgical purposes they are brought into one constant direction by a commutator, as in Clarke's machine above described. The driving-power required for lighthouse purposes is about three horse-power.

Machines of this class are seldom constructed now, as the same power is obtained with only a fifth of the weight by employing electro-magnets instead of permanent magnets.

Fig. 190. Siemens' Armature.

286. Siemens' Armature.—An important improvement in Clarke's machine was introduced by Siemens of Berlin in 1854. It consists in the adoption of a peculiar form of electro-magnet, which is represented in Fig. 190. The iron portion is a cylinder with a very deep and wide groove cut along a pair of opposite sides, and continued round the ends. The coil is wound in this

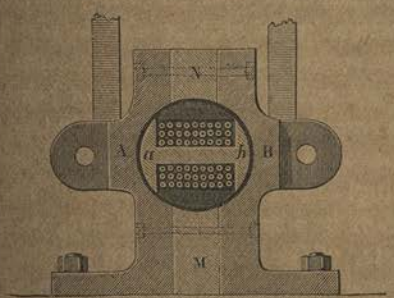


Fig. 191.—Section of Siemens' Armature.

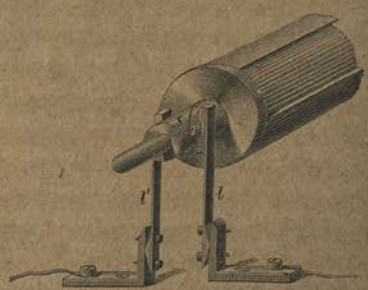


Fig. 192.—Commutator.

groove like thread upon a shuttle. Regarded as an electro-magnet, the poles are not the ends of the cylinder, but are the two cylindrical

faces which have not been cut away. In Fig. 191, *a b* is a section of the armature with the coil wound upon it. A B M N is a socket within which the armature revolves, the portions A B being of iron, and M N of brass.

The advantage of Siemens' armature is that, on account of the small space required for its rotation, it can be kept in a region of very intense magnetic force by the use of comparatively small magnets. Its form is also eminently favourable to rapid rotation. It is placed between the opposite poles of a row of horse-shoe magnets which bestride it along the whole of its length, as shown at the top of Fig. 193, and is rotated by means of a driving-band passing over the pulley shown at the lower end of Fig. 190.

The polarity of the electro-magnet is reversed at each half-revolution as in Clarke's arrangement, and the alternately opposite currents generated are reduced to a common direction by a commutator nearly identical with Clarke's, and represented in Figs. 190, 192. Siemens' machines are much more powerful than Clarke's when of the same size.

287. **Accumulation by Successive Action: Wilde's Machine.**—By employing the current from a Siemens' machine to magnetize soft iron, we can obtain an electro-magnet of much greater power than the steel magnets from whose induction the current was derived By causing a second coil to rotate between the poles of this electro-magnet, we can obtain a current of much greater power than the primary current. This is the principle of Wilde's machine, which is represented in Fig. 193. It consists of two Siemens' machines, one above the other. The upper machine derives its inductive action from a row of steel magnets M, whose poles rest on the soft-iron masses *m*, *n*, forming the sides of the socket within which a Siemens' armature *r* rotates. The currents generated in the coil, after being reduced to a uniform direction by a commutator, flow to the binding-screws *p*, *q*. These are the terminals of the coil of the large electro-magnet A B, through which accordingly the current circulates. The core of this electro-magnet consists of two large plates of iron, connected above by another iron plate, which supports the primary machine. Its lower extremities rest, like those of the primary magnets, on two iron masses T, T, separated by a mass of brass *i*; and a second Siemens' armature F, of large size, revolving within this system, furnishes the currents which are utilized

Wilde's principle can be carried further. The current of the second armature can be employed to animate a second electro-magnet of greater power than the first, with a third Siemens' armature revolving between its poles. This was actually done by Wilde. By

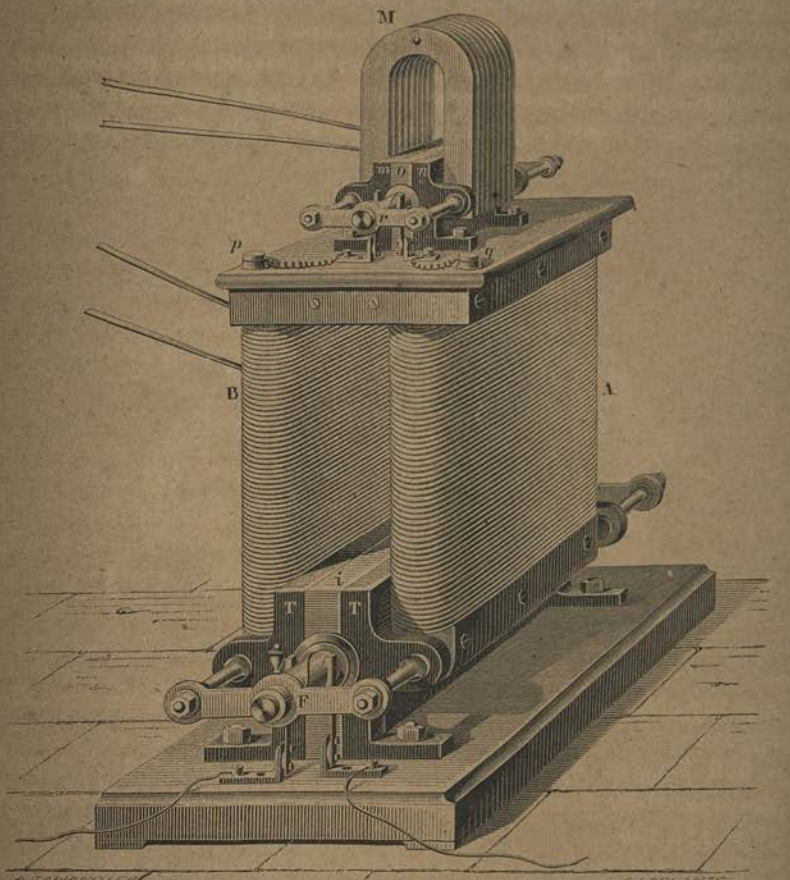


Fig. 193.—Wilde's Machine.

means of the current from this triple machine, driven by 15 horse-power, a bar of platinum 2 feet long and a quarter of an inch in diameter was quickly melted. This system of accumulation could probably be carried several steps further if desired.

288. Accumulation by Mutual Action; Dynamo Machines.—Siemens and Wheatstone nearly simultaneously proposed the construction

of a magneto-electric machine in which the induced current, or a portion of it, is made to circulate round the soft-iron magnet which produced it. Iron has usually some traces of permanent magnetism, especially if it has once been strongly magnetized. This magnetism serves to induce very feeble currents in a revolving armature. These currents are sent round the iron magnet, thus increasing its magnetization. This again produces a proportionate increase in the induced currents; and thus, by a succession of mutual actions, very intense magnetization and very powerful currents are

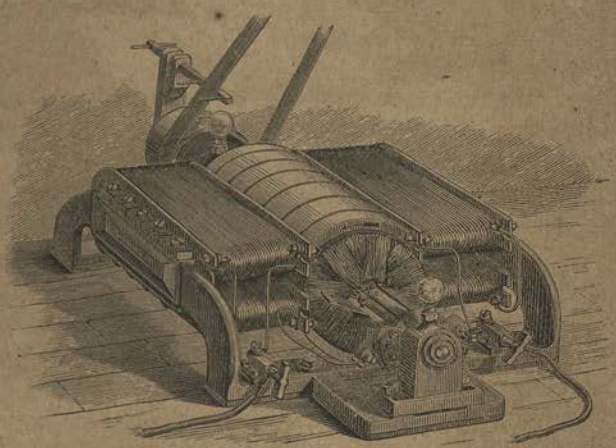


Fig. 194.—Siemens' Dynamo.

speedily obtained. Machines constructed on this principle are called *dynamos*.

289. Siemens' Dynamo.—One of the earliest patterns of dynamo constructed by Messrs. Siemens for commercial purposes is shown in Figs. 194, 195.

The armature consists of a hollow iron cylinder, on the outside of which the wire is wound lengthwise in from ten to twenty successive sections, each section being wound in the same manner as the whole coil of the original Siemens' armature (Fig. 190), and being like it connected at its ends to two opposite segments of a commutator, as well as to one end each of the two adjacent sections; so that the whole wire forms one continuous coil, connected at a number of equidistant points with the successive segments of a commutator. The commutator is on the same plan as that of

Clarke (Fig. 188), but instead of only two segments has a considerable number. Two contact springs, or flexible bundles of wire, called brushes, rub upon the commutator at two points fixed in space, as it revolves, one of them receiving from the armature positive and the other negative electricity. The armature is rapidly rotated between two sets of curved iron bars, one set above the armature, as shown in Fig. 194, and the other, precisely similar set below. These bars are prolonged at both ends through the four fixed coils shown in Figs. 194, 195, so that the upper bars, for example, form the core of the two upper fixed coils. The currents in these two coils are in

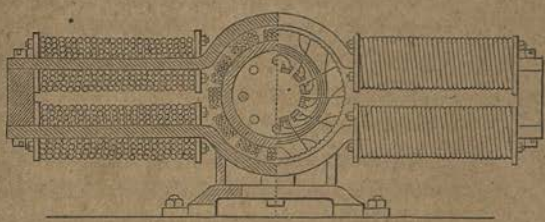


Fig. 195.—Section of Siemens' Machine.

opposite directions, so that the bar has a "consequent point" in the middle of its length. If these middle points are north poles, then the middle points of the lower bars are south poles, and each section of the coil by revolving between these two poles has currents generated in it which are reversed twice in each revolution, namely, in the two positions (differing by 180°) in which the maximum number of tubes of force pass through it. The two segments of the commutator which are in direct connection with the section in question make contact with the collecting brushes midway between these two reversals; and the current given off is due partly to this section and partly to the neighbouring sections on each side of it.

The fixed magnets are called the *field-magnets*, because their function is to produce a strong magnetic field for the armature to move in. Their coils are of stout wire, and the connections are such that the whole current generated in the armature passes through them; the armature coils, the field-magnet coils, and the external circuit being joined in series.

290. Gramme's Machine.—Another well-known type of magneto-electric machine is that invented by M. Gramme. Let C D E F (Fig. 196) be a ring of soft iron, wrapped round with insulated

copper wire, and revolving in its own plane between the poles P, P′ of a fixed magnet. The ring will, at any given instant, consist virtually of two semicircular magnets, F C D, F E D, having a pair of similar poles at F, and the other pair at D, these being the points directly opposite the poles of the fixed magnet. Since the poles of the ring remain fixed in space, the electric effect in the copper wire is the same as if the wire coil alone rotated, its core remaining stationary. The effect of this rotation would be, that in the portion C F E of the coil there would be electromotive force tending to produce a current in one direction,—say the direction C F E; while in the other half, C D E, there would be electromotive force tending to produce a current in the opposite direction—that is the direction C D E. The effects in the two halves are opposite as regards the current which they tend to produce in the coil as a whole; but they are the same as regards the electromotive force between the opposite points C and E; and if the two ends of an external conductor be maintained in rubbing contact with the coil at these two points, a permanent current will flow through it in virtue of this electromotive force.

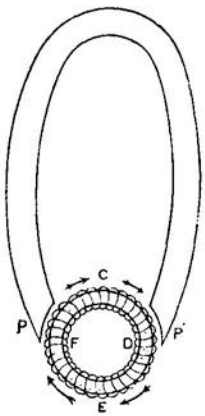


Fig. 196.—Magnet and Ring.

The above reasoning may be put in the following form. Nearly all the tubes of force which run from one pole to the other of the permanent magnet are concentrated in the substance of the iron ring, one half traversing the upper and the other the lower half-ring. Each convolution of the coil, in ascending from its lowest position E by way of F to its highest position C, cuts each of these tubes once, and all in the same direction, namely, from below to above. In descending on the other side by way of D to E, the same tubes are cut, each once, in the opposite direction, namely, from above to below. Hence the movement in E F C generates electromotive force in one direction through the wire composing the coil, and the movement in C D E generates electromotive force in the opposite direction; both parts of the motion conspiring to produce difference of potential between the convolution at C and that at E.

The details of the armature of Gramme's machine are shown in Fig. 197, in which different parts are represented in different stages of construction.

The ring or core consists of a bundle of iron wires, shown in section at A. The copper wire, covered as usual with an insulating material, is divided into a number of separate coils, as B B. The two ends of each coil are respectively connected to two thick pieces of copper (one of which is marked R R in the figure), which are the segments of the commutator, their number being equal to the number of separate coils. In passing the two points most remote from the poles, these coppers rub against two brushes, connected respectively with two binding-screws, one forming the positive and the other the negative electrode of the machine. As each brush makes

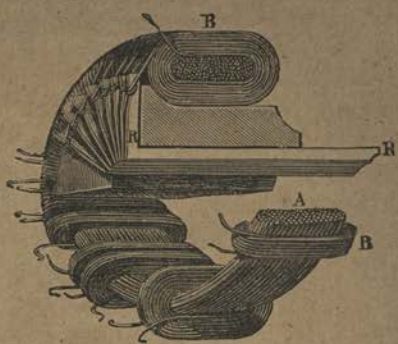


Fig. 197.

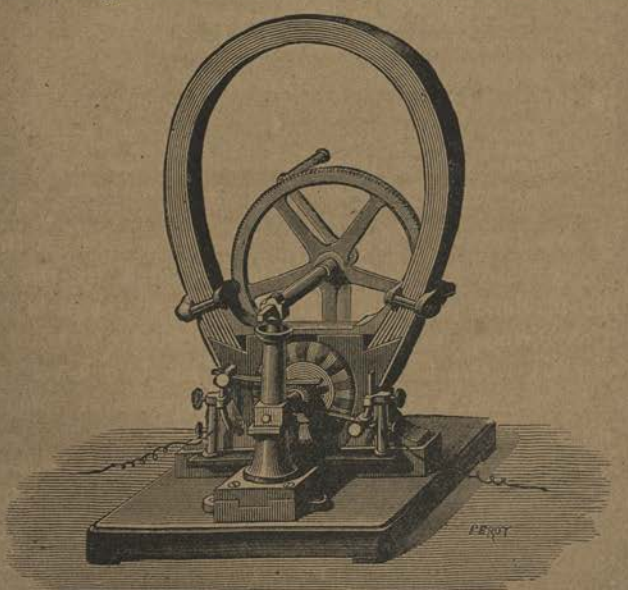


Fig. 198.—Gramme's Magneto-electro Machine, for hand-power.

contact with two or more coppers at the same time, the current is never interrupted, and undergoes but small fluctuations of strength, —a remark which also applies to Siemens' machine.

In consequence of the great steadiness of the current thus obtained, such machines can be used instead of galvanic batteries for nearly all purposes. The machine as constructed for hand use is shown in Fig. 198. Fig. 199 represents a larger pattern, intended to be

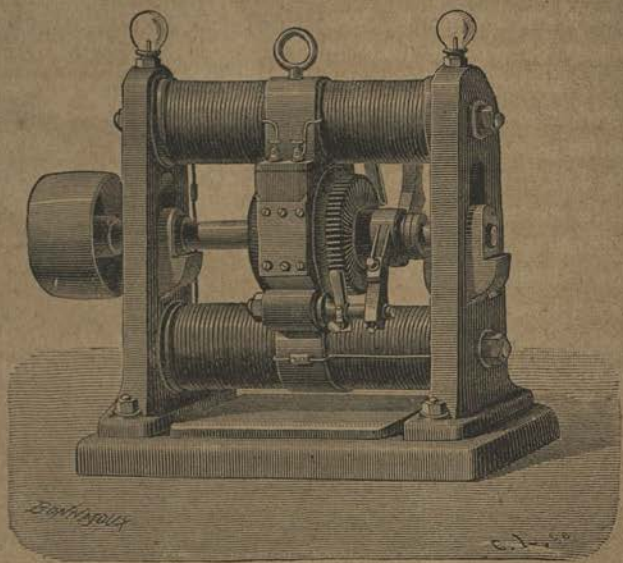


Fig. 199.—Gramme's Dynamo-electro Machine, for steam-power.

driven by steam power. The armature revolves between the poles of four very powerful electro-magnets actuated by the current which the machine produces. The two upper magnets may be regarded as forming one magnet, with a consequent point in the centre, directly above the armature; and the two lower magnets give, in like manner, a consequent point, opposite in name to the former, directly below the armature.

The first machine with a ring like that of Gramme was a small laboratory model constructed by Pacinotti at Pisa in 1860, and described by him in an Italian publication in 1864, but it did not become generally known, nor was any large machine of the kind made, till the reinvention of the ring by Gramme, who was the first to introduce the now usual arrangement of collecting currents by flexible bundles of wire rubbing on a commutator of many segments.

291.—The machines of Siemens and Gramme above described are

among the earliest types of commercial dynamos. Later patterns contain many improvements. One of the most important consists in making the cores, pole-pieces, and yokes (*i.e.* pieces connecting the cores) of the field-magnets more massive. This diminishes what is called the magnetic resistance, and thus makes the field stronger.

292. Classification of Dynamos.—Dynamos which give an external current constantly in one direction are called *direct-current* machines.

Two different arrangements are in use for supplying their field-magnets with current; thus we have the distinction of *series dynamos* and *shunt dynamos*.

In the *series dynamo*, the whole current from the armature passes through the field-magnet coils, which must be composed of stout wire, so as to have low resistance and not consume much of the energy of the current.

In the *shunt dynamo*, the current from the armature divides into two parts, the larger part going through the external circuit, and the smaller through the field-magnet coils, which must in this case consist of a great length of fine wire, so as to have much greater resistance than the external circuit.

In many modern installations, a combination of the two systems is adopted, the field-magnets being wound with two coils, one in parallel and the other in series with the external circuit. This is called *compound* winding. It is specially useful for keeping the e.m.f. of the machine nearly constant in spite of changes of resistance in the external circuit.

All these belong to the class of direct-current dynamos as distinguished from *alternate-current* dynamos. These latter give currents alternately in one direction and the opposite, the reversals succeeding each other usually some hundreds or thousands of times in a second. For this purpose no commutator is required, inasmuch as the current in the armature itself is alternating in all machines; but the two collecting springs rub without interruption on the surfaces of two revolving cylinders, to which the ends of the armature-coil or of its several sections are connected. Each cylinder gives off positive and negative electricity alternately, and when the one is giving off positive the other is giving off negative. This is the favourite plan for lighthouses, because the alternating currents make the two carbons of the electric lamp burn away equally, and thus facilitate the keeping of the light in the focus of the optical apparatus. The field-magnets of an alternate-current machine, if

they are electro-magnets, must be excited by a current distinct from that of the machine itself; as alternating currents will not serve for this purpose.

293.—In most of the alternate-current dynamos now constructed, the field-magnets are arranged in two circles facing each other with a narrow space between. In this intervening space the armature rotates. It consists of a series of flat coils equal in number and about equal in area to the ends of the field-magnets in either circle.

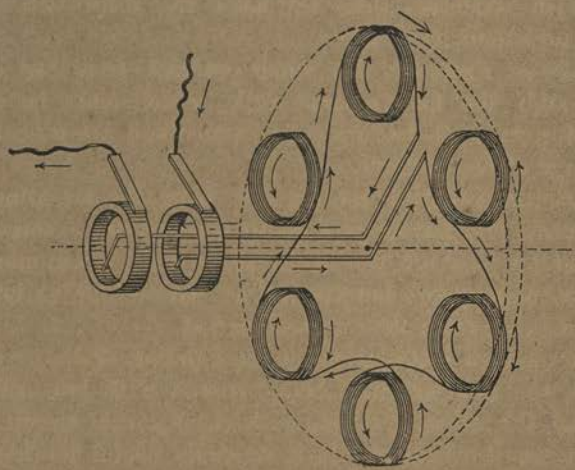


Fig. 200.—Armature of Alternator.

These coils have usually no iron core, but are wound upon an insulating material which holds them in their places. The field-magnets are so arranged that unlike poles face each other, and also unlike poles succeed each other. The arrangement of the armature-coils and collecting rings is shown in Fig. 200[1]. By the passage of one of the armature coils between a pair of poles two successive currents are produced, one by the passage of its forward and the other of its hinder half, these currents having opposite directions. While the forward half is passing between one pair of poles, the hinder half is passing between the next pair of poles, and the two currents due to these crossings are in the same direction. This action goes on in all the coils simultaneously, and if they are connected in series in the manner shown in the figure their electromotive forces will be com-

[1] This figure is borrowed with permission from Professor S. P. Thompson's *Dynamo-Electric Machinery*, to which we would refer our readers for further information.

bined.[1] The number of reversals of current in one revolution of the armature is equal to the number of pairs of poles between which it revolves. A dynamo of this kind is called *multipolar*.

The field-magnets are electro-magnets supplied with a constant current by a *separate exciter*, that is by a relatively small direct-current dynamo employed for the purpose.

294. Conditions of Self-excitation in a Dynamo.—It might be inferred from § 288 that the initial traces of magnetism in the field-magnets will be gradually augmented even when the armature is made to revolve slowly. Such an inference would however be erroneous. A dynamo does not work at all unless the speed of revolution exceeds a certain critical value, which is definite when the resistance in circuit is given. The reason may be stated as follows.

Let C stand for the current in the armature. We shall assume that either the whole of C or a definite fraction of it flows through the coils of the field-magnets. Let M denote the intensity of magnetization of these magnets, and n the number of revolutions of the armature per minute. Then in comparing the results obtained with the same machine at different speeds, C will be jointly proportional to n and the intensity of the field in which the armature revolves, which latter may be taken as proportional to M, that is, we may assume

$$k\mathrm{C} = n\mathrm{M}, \text{ or } n = k\frac{\mathrm{C}}{\mathrm{M}},$$

k denoting a constant multiplier.

Now the value of $\frac{\mathrm{C}}{\mathrm{M}}$ increases with the magnetization except when the magnetization is feeble, in which case it is nearly constant. Let this constant value of $\frac{\mathrm{C}}{\mathrm{M}}$ for weak magnetization be denoted by k'. Then the least possible value of n will be kk'. This is the critical speed. At lower speeds the field-magnets will not be excited, and at higher speeds they will be excited more and more as the speed is increased, since M increases with $\frac{\mathrm{C}}{\mathrm{M}}$.

295. An empirical formula which gives a fair approximation to the truth in ordinary cases can be obtained by assuming what

[1] In any two consecutive coils the currents are opposite ways round, because they are moving across fields in which the lines have opposite directions. Hence the direction of winding must be reversed from each coil to the next, as shown in the figure.

mathematicians call a "homographic relation" to exist between C and M. The most general expression of such a relation is,

$$CM + aC + bM + c = 0, \qquad (1)$$

$a\ b\ c$ denoting constants. In the present case c is zero, since C and M vanish together, the formula is thus reduced to

$$CM + aC + bM = 0. \qquad (2)$$

When C is very great the third term may be neglected, and we have therefore $M + a = 0$. But M in this case is the magnetization at saturation, which we will denote by m, hence the constant a denotes $-m$ and the formula may be written

$$CM - Cm + bM = 0. \qquad (3)$$

When C has the value b, we can divide out by it and we obtain

$$2M - m = 0, \text{ or } M = \tfrac{1}{2}m.$$

The constant b therefore denotes that value of C which produces half-saturation.

Dividing by Mm, we may write equation (3) in the form

$$\frac{C}{M} = \frac{C + b}{m}, \qquad (4)$$

which shows that the least value of $\frac{C}{M}$ is $\frac{b}{m}$. Hence, by the preceding section, the critical speed is

$$n = k\frac{b}{m}. \qquad (5)$$

At double the critical speed the value of $\frac{C}{M}$ will (by the preceding section) be doubled. It will therefore be $\frac{2b}{m}$, which when substituted in equation (4) gives $C = b$, showing that at double the critical speed the field-magnets will be half-saturated. This result is probably not far from the truth. Equation (3), when written in the form

$$M = \frac{mC}{C + b},$$

is known as the Lamont-Frölich formula.

CHAPTER XXI.

ELECTRIC TELEGRAPHS.

296. Electric Telegraph: History.—The discovery that electricity could be transmitted instantaneously to great distances, at once suggested the idea of employing it for signalling. Bishop Watson, already referred to in § 38, performed several experiments of this kind in the neighbourhood of London, the most remarkable being the transmission of the discharge of a Leyden-jar through 10,600 feet of wire suspended between wooden poles at Shooter's Hill. This was in 1747. A plan for an alphabetical telegraph to be worked by electricity is minutely described in the *Scots Magazine* for 1753, but appears to have been never experimentally realized. Lesage, in 1774, erected at Geneva a telegraph line, consisting of twenty-four wires connected with the same number of pith-ball electroscopes each representing a letter. Reusser, in Germany, proposed, in the same year, to replace the electroscopes by spangled panes exhibiting the letters themselves. The difficulty of managing frictional electricity was, however, sufficient to prevent these and other schemes founded on its employment from yielding any useful results. Volta's discoveries, by supplying electricity of a kind more easily retained on the conducting wires, afforded much greater facilities for transmitting signals to a distance.

Several suggestions were made for receiving-apparatus to exhibit the effects of the currents transmitted from a voltaic battery. Sömmering of Munich in 1811 proposed a telegraph, in which the signals were given by the decomposition of water in thirty-five vessels, each connected with a separate telegraph wire. Ampère, in 1820, proposed to utilize Œrsted's discovery, by employing twenty-four needles, to be deflected by currents sent through the same

number of wires; and Baron Schilling exhibited in Russia, in 1832, a telegraphic model in which the signals appear to have been given by the deflections of a single needle.[1]

Weber and Gauss carried out this plan in 1833, by leading two wires from the observatory of Göttingen to the Physical Cabinet, a distance of about 9000 feet. The signals consisted in small deflections of a bar-magnet, suspended horizontally with a mirror attached, on the plan since adopted in Thomson's mirror galvanometer.

At their request the subject was earnestly taken up by Professor Steinheil of Munich, whose inventions contributed more perhaps than those of any other single individual to render electric telegraphs commercially practicable. He was the first to ascertain that earth-connections might be made to supersede the use of a return wire. He also invented a convenient telegraphic alphabet, in which, as in most of the codes since employed, the different letters of the alphabet are represented by different combinations of two elementary signals. Two needles were employed, one or the other of which was deflected according as a positive or a negative current was sent, the deflections being always to the same side. Sometimes the needles were merely observed by eye, sometimes they were made to strike two bells, and sometimes to produce dots, by means of capillary tubes charged with ink, on an advancing strip of paper, thus leaving a permanent record on the strip in the shape of two rows of dots. His currents were magneto-electric, like those of Weber and Gauss.

The attraction of an electro-magnet on a movable armature furnishes another means of signalling. This was the foundation of

[1] The contributions of the late Sir Francis Ronalds to the art of telegraphy must not be altogether overlooked. According to an able notice in *Nature*, Nov. 23, 1871, "Sir Francis, before 1823, sent intelligible messages through more than eight miles of wire insulated and suspended in the air. His elementary signal was the divergence of the pith-balls of a Canton's electrometer produced by the communication of a statical charge to the wire. He used synchronous rotation of lettered dials at each end of the line, and charged the wire at the sending end whenever the letter to be indicated passed an opening provided in a cover; the electrometer at the far end then diverged, and thus informed the receiver of the message which letter was designated by the sender. The dials never stopped, and any slight want of synchronism was corrected by moving the cover. Hughes' printing instrument is the fully-developed form of this rudimentary instrument. A gas pistol was used to draw attention, just as now a bell is rung. The primary idea of reverse currents is to be found where Sir Francis suggests that the wire when charged with positive electricity should discharge not to earth but into a battery negatively charged. Equally interesting is the discussion on what we now call lateral induction, then known as compensation. The author clearly saw that in the underground wires which he suggests as substitutes for aerial lines, this induction would be or might be a cause of retardation."

Morse's telegraphic system, and was employed by Wheatstone for ringing a bell to call attention before transmitting a message.

About the year 1837 electric telegraphs were first established as commercial speculations, in three different countries. Steinheil's system was carried out at Munich, Morse's in America, and Wheatstone and Cooke's in England. The first telegraphs ever constructed for commercial use were laid down by Wheatstone and Cooke, on the London and Birmingham and Great Western Railways. The wires, which were buried in the earth, were five in number, each acting on a separate needle; but the expensiveness of this plan soon led to its being given up. The single-needle and double-needle telegraphs of the same inventors have been much more extensively used, the former requiring only one wire, and the latter two.

Wheatstone made several subsequent contributions to the art of telegraphy, some of which we shall have occasion to mention in later sections.

297. Needle Telegraph. Dial Telegraph.—In the needle telegraphs of Wheatstone and Cooke, which were the commonest instruments in this country before the taking over of the telegraphs by the Post Office, the deflections were produced in the same way as those of a galvanometer. By combining deflections to left and right an alphabet was constructed which, as devised by Wheatstone and Cooke, was framed with a view to being easily remembered. This has been superseded by the Morse alphabet, which is constructed on the principle of assigning the briefest signs to the letters of most frequent occurrence, and has now become international.

Dial telegraphs, in which the indications were given by a pointer directed towards any one of the letters of the alphabet arranged in a circle, were very common in counting-houses till they were superseded by the telephone. They were on the "step by step" plan. Every current that was sent advanced the pointer one step, and on the reversal (or in some instruments on the cessation) of the current the pointer advanced another step. The number of such steps was regulated by the advance of the handle of the sending instrument, which also travelled round a circle formed of the letters of the alphabet. When all was right the pointer of the receiving instrument was at the same letter as the handle of the sending instrument and in case of disarrangement, both could be brought to the intermediate space between Z and A. Descriptions of two such instruments will be found in previous editions of this work.

298. Batteries. Wires.—All the public telegraphs in this country have now for many years been worked by voltaic currents; the magneto-electric system, which was tried on some lines, having been found to involve a needless expenditure of labour. The modified Daniell's which was described in our earlier editions has been largely replaced by Fuller's bichromate battery, which we have described in § 161.

The wires for land telegraphs are commonly of what is called galvanized iron, that is, iron coated with zinc, supported on posts by means of glass or porcelain insulators, so contrived that some part of the porcelain surface is sheltered from rain, and insulates the wires from the posts even in wet weather. Wires thus suspended are called *air-lines*.

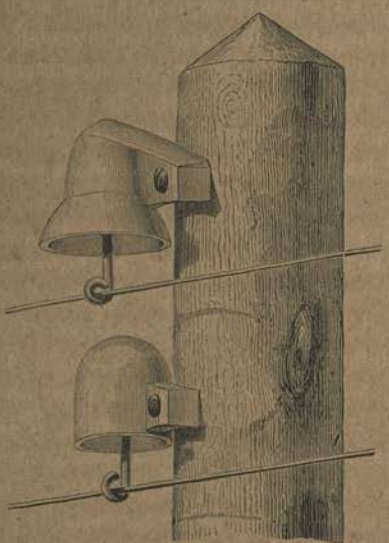


Fig. 201.—Insulators.

Underground wires are, however, sometimes employed. They are insulated by a coating of gutta percha, and are usually laid in pipes, an arrangement which admits of their being repaired or renewed, without opening the ground except at the drawing-in boxes. There is less leakage of electricity from subterranean than from air lines, but their cost is greater, and they are less suited for rapid signalling, on account of the retardation caused by the inductive action between the wire and the conducting earth, which is similar to that between the two coatings of a Leyden-jar.

299. Earth-Connections.—The early inventors of electric telegraphs supposed that a current could not be sent from one station to another without a return wire to complete the circuit. Steinheil, while conducting experiments on a railway, with the view of ascertaining whether the rails could be employed as lines of telegraph, made the discovery that the earth would serve instead of a return wire, and with the advantage of diminished resistance; the earth, in fact, behaving like a return wire of infinitely great cross-section, and therefore of no resistance. The earth-connection is usually made by

connecting one terminal of the sending battery and one terminal of the receiving instrument to large plates of metal buried in the soil.

We are not, however, to suppose that the current really returns from the receiving to the transmitting station through the earth. The duty actually performed by the earth consists in draining off the opposite electricities which would otherwise accumulate in the terminals. It keeps the two terminals at the same potential; and as long as this condition is fulfilled, the current will have the same strength as if the terminals were in actual contact.

300. Alarum.—In connection with all telegraphs an alarum is used for calling attention. There are several different kinds. Fig. 202 represents one of the simplest. It contains an electro-magnet *e*, with an armature *f* fixed to the end of an elastic plate. When no current is passing through the coil, the armature is held back by the elasticity of this plate, so as to press against a contact-spring *g* connected with the binding-screw *m*. The terminals of the coil are at the binding-screws *p*, *p'*, the former of which is in connection with the armature, and the latter with the earth. As long as the armature presses against the spring *g*, there is communication between the two binding-screws *m* and *p'* through the coil; but the passing of a current produces attraction of the armature, which draws it away from *g* and interrupts the current. The electro-magnet is thus demagnetized, and the armature springs back against *g*, so as to allow a fresh current to pass. The armature is thus kept in continual vibration; and a hammer K, which it carries above, produces repeated strokes on a bell T.

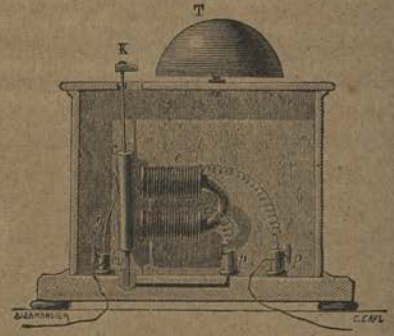


Fig. 202.—Vibrating Alarum.

301. Morse's Telegraph.—The basis of nearly all the telegraphs now used in all countries is Morse's system, which was first tried in America about 1837.

His receiving instrument, or *indicator*, in its primitive simplicity, consists (Fig. 203) of an electro-magnet, a lever movable about an axis, carrying a soft-iron armature at one end, and a pencil at the

other, and a strip of paper which is drawn past the pencil by a pair of rollers.

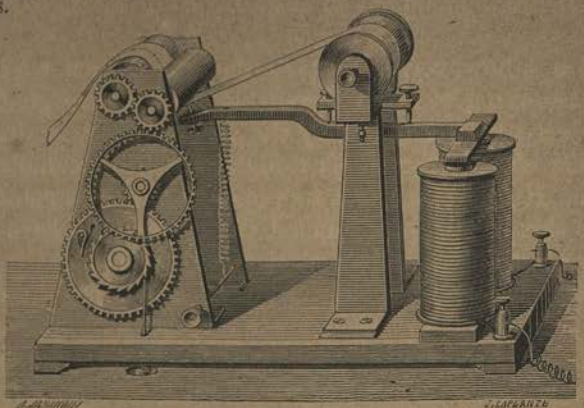


Fig. 203.—Morse's Telegraph.

As the pencil soon became blunt, and was uncertain in its marking, a point, which scratched the paper, was substituted. This has now

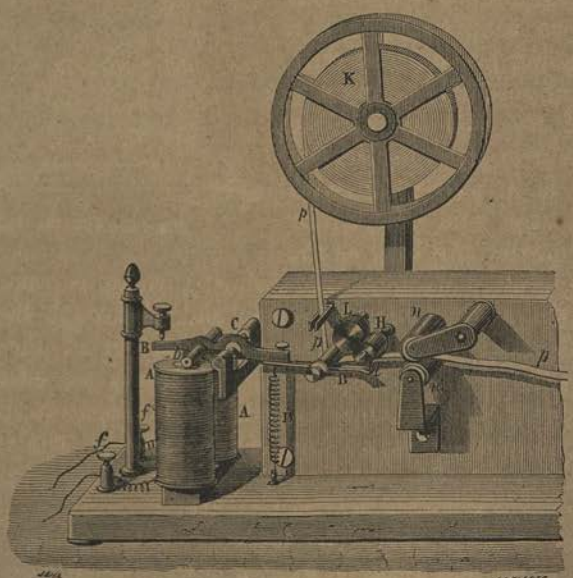


Fig. 204.—Modified Form.

to a great extent been superseded by an ink-writer, which requires the exertion of less force, and at the same time leaves a more visible trace.

Fig. 204 represents Morse's indicator as modified by Digney.

A train of clock-work, not shown in the figure, drives one of a pair of rollers *n m*, which draw forward a strip of paper *p p* forming part of a long roll K. The same train turns the printing-cylinder H, the surface of which is kept constantly charged with a thick greasy ink by rolling-contact with the ink-pad L. The armature B B′ of the electro-magnet A is mounted on an axis at C, and carries a style at its extremity just beneath the printing-cylinder. When a current passes, the armature is attracted, and the style presses the paper against the printing-cylinder, causing a line to be printed on it, the length of which depends on the duration of the current, as the paper continues to advance without interruption. The lines actually employed are of two lengths, one being made as short as possible (-) and called a *dot*, the other being about three times as long (—) and called a *dash*. The opposing spring D restores the armature to its original position the moment the current ceases.

Morse's key (Fig. 205) is simply a brass lever, mounted on a hinge at A, and pressed up by the spring *f*. When the operator puts down the key, by pressing on the button K with his finger, the projections *c d* are brought into contact, and a current passes from the battery-wire P to the line-wire L. When the key is up, the projections *a b* are in contact, and currents arriving by the line-wire pass by the wire R to the indicator or the relay. By keeping the key down for a longer or shorter time, a dash or a dot is produced at the station to which the signal is sent. The dash and dot are combined in different ways to indicate the different letters, as shown in the following scheme, which is now generally adopted both in Europe and America:—

MORSE'S ALPHABET.

A ·—	J ·———	T —	1 ·————
Ä ·—·—	K —·—	U ··—	2 ··———
B —···	L ·—··	Ü ··——	3 ···——
C —·—·	M ——	V ···—	4 ····—
D —··	N —·	W ·——	5 ·····
E ·	O ———	X —··—	6 —····
É ··—··	Ö ———·	Y —·——	7 ——···
F ··—·	P ·——·	Z ——··	8 ———··
G ——·	Q ——·—	Ch ————	9 ————·
H ····	R ·—·		0 —————
I ··	S ···	Understood ···—·	

A space about equal to the length of a dash is left between two letters, and a space of about twice this length between two words.

In needle-telegraphs, the dot is represented by a deflection to the left, and the dash by a deflection to the right.

302. Relay.—Fig. 206 represents Morse's indicator in connection with what is called a *relay;* that is to say, an apparatus which, on receiving a feeble current from a distance, sends on a much stronger current from a battery on the spot. The key B being up, a current arriving by the line-wire passes through the key from *c* to *a*, thence through another wire to the coil of the electro-magnet belonging to the relay, and through this coil to earth. The electro-magnet of the relay attracts an

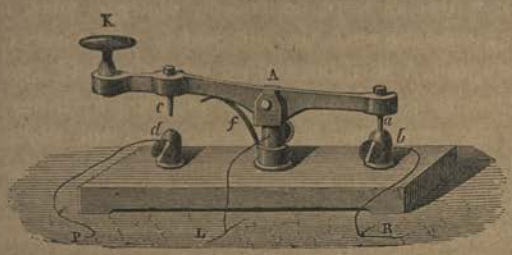


Fig. 205.—Morse's Key.

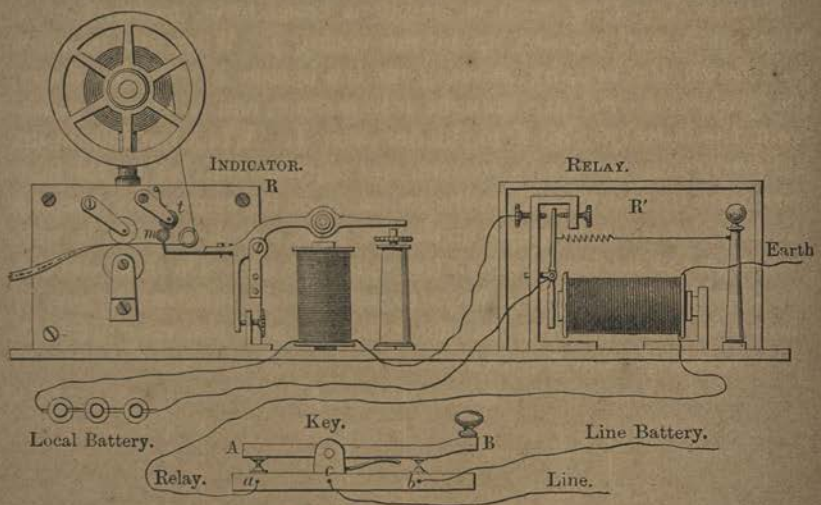


Fig. 206.—Morse's Apparatus, with Relay.

armature, the contact of which with the magnet completes the circuit of the local battery, in which circuit the coil belonging to the indicator is included. The armature of the indicator is thus compelled to follow the movements of the armature of the relay.

Relays are used when the currents which arrive are too much

enfeebled to give clear indications by direct action. They are also frequently introduced at intermediate points in long lines which could not otherwise be worked through from end to end. The analogy of this use to change of horses on a long journey is the origin of the name. Relays are also frequently used in connection with alarums when these are large and powerful.

303. Electro-chemical Telegraph.—Suppose a metallic cylinder, permanently connected with the earth, to be revolving, carrying with it on its surface a strip of paper freshly impregnated with cyanide of potassium. Also suppose a very light steel point permanently connected with the line-wire, and resting in contact with the paper. Every time that a current arrives by the line-wire, chemical action will take place at the point of contact, and the paper at this point will be discoloured by the formation of prussian blue. This is the principle of Bain's electro-chemical telegraph, which leaves a record in the shape of dots and dashes of prussian blue. The apparatus for sending signals is the same as in Morse's system. The paper must not be too wet, or the record will be blurred; neither must it be too dry, for then no record will be obtained.

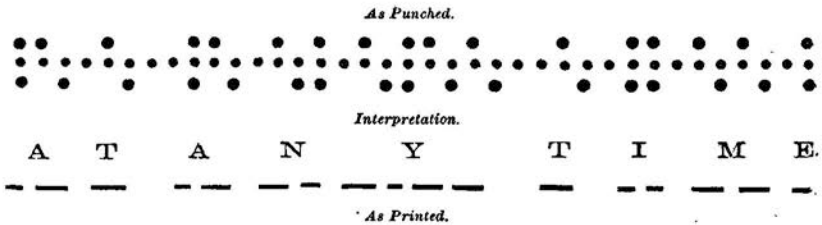
For a full account of Hughes' printing telegraph, which has done excellent service, and of some other telegraphs, which are interesting as ingenious curiosities, we must refer to previous editions.

304. Wheatstone's Automatic System.—A very effective contrivance for increasing the speed of signalling is Wheatstone's automatic apparatus, which is very extensively adopted by the authorities of the postal telegraphs. The first step towards sending a message by this system consists in punching the message in a ribbon of stiff paper. The punching is done by a special instrument, the operator having merely to put down three keys, one of which represents *dot*, another *dash*, and the third *blank*. The holes punched are in three rows. Those in the middle row are equi-distant, and are intended to perform the office of the teeth of a rack in guiding the paper uniformly forwards. Those in the two outside rows contain the message, a dot being represented by a pair of holes exactly opposite each other (:) one in each row, and a dash by two holes ranged obliquely (· .).

The punched strips are then put through the transmitting instrument, and, by regulating the movements of two pins, cause the transmission of the currents necessary for printing the message at

the receiving station. From 100 to 150 words are thus transmitted per minute and automatically printed.[1]

The following is a specimen of three consecutive words of a telegraphic message, as it appears on the punched strip at the sending station, and on the printed strip at the receiving station:



The practical limit to speed, in lines of considerable length, arises not so much from the difficulty of making quicker movements, as from the blending together of successive signals in travelling a great distance, especially if part of the distance be under ground or under water. This evil is partly remedied by making each signal consist, not of a single current, but of two; thus a dot will be produced by an instantaneous current, *immediately* succeeded by another of opposite sign; a dash by an equally short current followed *at a longer interval* by an opposite one. In this way, though a greater number of currents are required for each word, a greater number of words can be distinctly signalled in a given time; and, by sending three properly adjusted currents for each signal, a still greater speed of distinct transmission is possible. The transmitting instrument of Wheatstone's automatic system does in fact send three currents for each dot or dash.

305. Duplex Working.—Of late years methods of sending two messages in opposite directions along the same wire at the same time have been very extensively introduced. This mode of operating is called *duplex* telegraphy. It requires that the current sent from either station shall not affect the receiving instrument at that station.

One method of attaining this end depends upon the principle that, if the coil of an electro-magnet be tapped at any point in its length,

[1] We learn from Mr. Preece's Presidential Address to the Institution of Electrical Engineers, Jan. 1893, that the speed has been gradually increased till it now sometimes reaches 600 words per minute.

a current sent into it at this point will traverse the two portions of the coil in opposite directions; whereas a current sent in at either end of the coil traverses the whole of it in one direction. Suppose this coil to be the coil of the electro-magnet which actuates the receiving instrument at any station. The currents from the transmitter at this station are sent into the coil at a point near its middle, and, circulating in opposite directions in its two portions, annul each other's effect upon the core, the arrangements being such that one of these portions forms the inner and the other the outer portion of the whole coil, and that, with equal currents, their effects in magnetizing the core are equal. On the other hand, currents arriving from another station enter at one end of the coil, and circulate round its whole length in one direction.

In order to obtain an exact balance, the two currents which circulate in opposite directions must be equal, and as they are both produced by the same electromotive force, the resistances in the two partial circuits must be equal. One of these includes the line-wire. The other goes to earth, and on its way thither is made to pass through a resistance which must be adjusted to exact equality with the line-wire. This mode of duplex sending is called the *Differential* method, from a remote resemblance to the differential galvanometer.

Another mode of duplex working is known as the *Bridge* method, because it depends upon the principle of Wheatstone's Bridge.

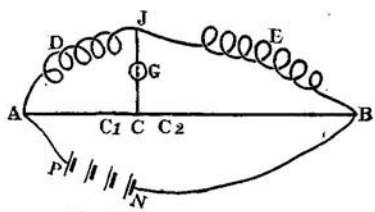


Fig. 207.—Wheatstone's Bridge.

In Fig. 207 (which we here reproduce from § 209) it is allowable to suppose the three conductors which meet at B to be disconnected from each other, and separately put to earth; for this is merely equivalent to keeping the point B at the potential of the earth, and will not affect the difference of potential of any two points.

Let P N be the battery at any station, its pole N being connected to earth by the wire N B, G the receiving instrument at the same station, D, A C, C B three resistances at this station, the point B of C B being to earth, and E the line-wire leading to a distant station, where it finally reaches earth at B. Then by the principle of the bridge, if the four resistances D, E, A C, C B are proportional, the

current of the battery will not affect the receiving instrument. On the other hand, the battery at the distant station will affect it, for this battery is in the branch J B, and will always produce a difference of potential between the points J and C, whatever be the relation of the resistances.

When both stations are sending to each other simultaneously, the actual current at any point will be the algebraic sum of the currents due to the two batteries.

It is practically necessary, in most cases, to connect the point C with a condenser, whose capacity should bear the same ratio to that of the line-wire E as the resistance in C B to the resistance in E.

306. Quadruplex Telegraphy.—Duplex telegraphy is converted into quadruplex by the following additional device, which is called *Diplex* working:—

At each station the current passes through two receiving instruments, one of which is only affected by strong currents, while the other is only affected by currents of a particular sign—say positive. A weak positive current will affect the second instrument and not the first, while a strong negative current will affect the first and not the second. There are, accordingly, two sending instruments employed at each station, one sending weak currents of a particular sign, and the other sending strong currents of the opposite sign. This arrangement, combined with one of the duplex arrangements above described, permits the sending of two currents each way at the same time, so that four currents are simultaneously passing through the same wire, producing signals in four separate instruments.

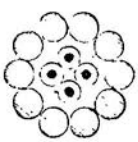


Fig. 208.
Submarine Cable.

307. Submarine Telegraphs.—The first submarine telegraph cable was laid between Dover and Calais in 1850; but, being insufficiently protected against the friction of the rocks, it only lasted a few hours. The two Atlantic cables which were laid in 1866 appear to be still in perfect order.

Submarine cables are now usually constructed by embedding a certain number of straight copper wires in gutta percha (Fig. 208), which insulates them from each other; this is surrounded with tarred hemp, and several strands of iron wire are wound outside of all. The copper wires in the interior

are the conductors for the transmission of the signals; the gutta percha is for insulation; the hemp and iron are for protection.

The Atlantic cables contain a central conductor, consisting of seven copper wires, twisted together and covered with three layers of gutta percha, forming altogether a cylinder $\frac{3}{5}$ of an inch in diameter. This is covered with a layer consisting of five strands of hemp, served with a composition consisting of 5 parts of Stockholm tar, 5 of pitch, 1 of linseed-oil, and 1 of bees'-wax. Lastly, the whole is covered by 18 strands of charcoal iron, each strand consisting of seven wires $\frac{7}{10}$ of a millimetre in diameter. On leaving the machine which put on the wire covering the cable was passed through a cauldron containing a mixture of pitch, tar, and linseed-oil. The difficulty of obtaining sufficiently good insulation has thus been completely surmounted.

A second difficulty attaching to submarine telegraphy depends upon the inductive action of the surrounding water, or of the iron sheath. This action, which is found quite sensible in subterranean lines of no great length, becomes of immense importance in long submarine cables. The cable forms one enormous condenser, the central conductor representing the inner coating, and the sea-water, or iron sheath, the outer coating of a Leyden-jar. In the Atlantic cables, the retardation of the signals due to this cause is so considerable that it would be barely possible to obtain a speed one-fifth of that usually attained on land-lines, if the same modes of sending and receiving signals were employed. The electrical capacity of the cable is in fact so enormous, that a long time is required to give it a full charge from a battery, or to discharge it again. The signals accordingly lose all their sharpness, and run into one another, unless special precautions are taken. After sending a current from one pole of the battery, the cable must be discharged, either by putting it to earth, or, still better, by connecting it for an instant with the other pole of the battery. The residual effects of the first current are thus quickly destroyed, and the line is left free for a second signal.

As the first effect received through such a cable is very slight, a very sensitive receiving instrument is necessary for quick working. Thomson's mirror galvanometer (§ 172) is the instrument which was first employed, the signals being read off by an attendant who watched the movements of the spot of light, dots and dashes being represented by deflections in opposite directions. The *siphon-*

recorder of the same inventor is now used, which writes the signals with ink discharged from a very light glass siphon, the siphon being moved by a very light coil of fine copper wire, suspended by a silk fibre between the poles of a very powerful permanent magnet.[1] The coil turns in one direction or the other according as the current transmitted is positive or negative, thus producing opposite sinuosities in the ink record which is traced upon an advancing strip of paper. The regular flow of the ink is assisted by electrical attraction, on the principle of the bucket or watering-pot described in § 44; but with this difference, that it is not the ink but the paper that is electrified. An electrical machine of peculiar and novel construction, bearing some resemblance to the replenisher of § 84, is employed for this purpose.

308. **Telephone.**—The articulating telephone invented by Professor

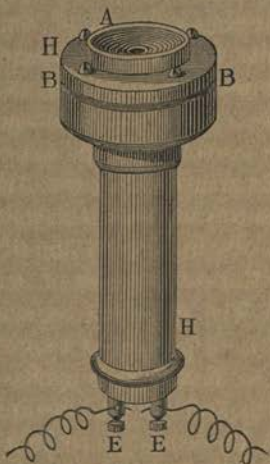


Fig. 209.—Telephone.

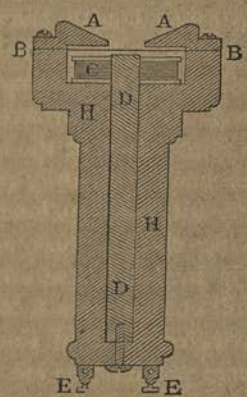


Fig. 210.—Section of Telephone.

Graham Bell is represented in Figs. 209, 210. D D is a steel magnet, C a coil of very fine silk-covered copper wire, surrounding the magnet close to one end, and having its terminals in permanent connection with the two binding-screws E E. B B is a thin disc of soft iron (usually one of the ferrotype plates prepared for photographers), tightly clamped, in its circumferential portion, between

[1] D'Arsonval's galvanometer is an adaptation of the principle of this instrument. The current to be measured passes through a small light coil suspended between the poles of a steel magnet.

the two parts of the wooden case H H, which are held together by screws, while its central portion is left free and nearly touches the end of the magnet. A A is the mouth-piece, through which the speaker directs his voice upon the iron disc.

Two telephones must be employed, one for transmitting, and the other for receiving, one binding-screw of each being connected with the line-wire, and the other with the earth or with a return-wire, so that their coils form parts of one and the same circuit, and every current generated in the one traverses the other. The mouth-piece A of the receiving telephone is held to the ear of the listener, and he is able to hear the words which are spoken into the transmitting telephone. There is a great falling off in loudness, and a decided nasal twang is imparted, but so much of the original character is preserved that familiar voices can be recognized. Conversations have thus been carried on through 60 or 70 miles of submarine telegraph cable, and through as much as 200 miles of wire suspended in the air on poles.

These results, which came upon the scientific world as a most startling surprise, are explained as follows. The voice of the speaker produces changes of pressure in the air in front of the iron disc, and thus causes the disc alternately to advance and recede, its movements keeping time with the sonorous vibrations, and the amplitudes of its movements being approximately proportional to those of the particles of air which convey the sound. Now a piece of soft iron, when brought near a magnet, exercises a *quasi* attraction upon the lines of force, causing them to be more closely aggregated in its own neighbourhood, and more widely separated in the other parts of the field. Hence when the disc approaches the magnet, it causes the lines of force to move in towards the axis of the disc, and when it recedes it causes them to open out again.

The lines of force thus cut the convolutions of the coil in opposite directions, according as the disc is approaching or receding, and give rise to alternate currents. These currents, passing through the coil of the receiving telephone, strengthen or weaken, according to their direction, the magnetism of its steel core, and increase or diminish the attraction of the latter for the iron disc. The disc is accordingly set in vibration, and imitates on a diminished scale the movements of the disc of the transmitter. Thus the original sonorous vibrations, having first been converted into undulating currents

of electricity, are reproduced as sonorous vibrations. The currents are excessively feeble, probably millions of times feebler than ordinary telegraphic currents; but on the other hand the ear is extremely sensitive to movements however small which recur periodically. Lord Rayleigh has made experiments from which it appears that the note of a whistle is audible at a distance at which the amplitude of the vibrating particles of air is less than a millionth of a millimetre.

When the telephone is employed for conversing through one of a number of telegraphic wires suspended on the same poles, it is found that messages sent by ordinary telegraphic instruments along the other wires are audible in the telephone as a succession of loud taps, so loud in fact as seriously to interfere with the telephonic conversation. This is an illustration of the principle, that the starting or stopping of a current in one wire gives rise to an induced current in a neighbouring wire; but the induced currents in this case, though so loudly audible in the telephone, have never been detected by any other receiving instrument. The telephone appears likely to supplant the galvanometer as a means of detecting feeble currents.

309. **Microphone.**—Fig. 211 represents one of the best forms of the microphone of Professor Hughes, the inventor of the well-known printing telegraph.

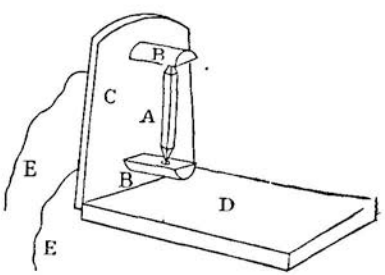


Fig. 211.—Microphone.

A is a stick of carbon about an inch long, sharpened at both ends, which rest in cavities in the two horizontal supports B B, also of carbon. The upper end of A is free to rattle about in the cavity which contains it, but not to fall away. The two wires E E are in connection respectively with the two supports B B, and are used for putting the instrument into circuit with a receiving telephone at another station. A battery, usually consisting of two or three very small cells, is also introduced into the circuit. The back C in which the supports B B are fixed, and the base D, are of wood, and, besides insulating the carbons, serve to convey to them the sonorous vibrations of the air or of surrounding bodies. These vibrations produce alternate increase and diminution of pressure at the points

of contact of the carbons with one another, and as increase of pressure gives closer contact and consequently diminished resistance, the current in the circuit undergoes corresponding changes of strength. These changes act upon the receiving telephone, and cause it to emit sounds which are often much louder than the originals. The microphone in fact acts as a relay, turning on and off the current of the battery, like the Morse relay described in § 302.

The action is improved by employing carbon which has been "metallized" by heating it white hot, and then plunging it in mercury.

The back C should be attached to the base D by a pivot which permits it to be inclined to one side. The best results for speech are usually obtained with an inclination of some 20 or 30 degrees from the vertical. When this inclination is too small there is an increase of noise in the receiving telephone, but a loss of distinctness. A microphone of the above kind transmits spoken sounds with as much distinctness as a telephone, and with much greater loudness. It has also a surprising power of transmitting very faint sounds produced by rubbing or striking the base or back with light bodies. Sounds of this kind which are quite inaudible at the place where they are produced, are easily heard by a person with his ear to the receiving telephone.

310. Telephonic Transmitters.—Though Bell's original telephone is still used as a receiving instrument, it has been almost entirely superseded as a transmitter by various forms of the microphone on the following plan.

There is a funnel for receiving the voice and converging the waves of sound upon a thin iron diaphragm, as in Bell's telephone. This diaphragm, by its vibrations to and fro, increases and diminishes the pressure at one or more points of contact in a local circuit containing a small battery and the primary coil of a miniature Ruhmkorff. The secondary coil of the Ruhmkorff is connected to the line-wire which leads to the receiving instrument.

311. Hughes' Induction Balance.—Bell's telephone, as above stated, is an extremely sensitive indicator of the currents induced in a wire by the commencement or cessation of currents in a neighbouring wire. Professor Hughes has taken advantage of this property to construct a very sensitive instrument for the instantaneous testing of metals.

A current from two or three cells is sent through two small

primary coils at a considerable distance apart. Near to them are placed two secondary coils, in circuit with a telephone, and so arranged that the induced currents in them are opposite. When the induced currents are exactly equal they destroy one another; and the adjustments are first made so as to obtain this result, that is, so as to obtain no sound in the telephone when the primary circuit is momentarily made and broken. The balance is then ready for making comparisons. Within each secondary coil is a box for containing the specimens to be compared. If precisely similar specimens are placed in the two boxes, no effect is obtained; but the slightest difference suffices to disturb the balance of the two currents and give a sound. A counterfeit coin can thus be easily distinguished from a genuine one; and even two genuine coins of the same kind will disturb the balance if one is a little more worn than the other. The best conducting metals give the most powerful effects, and a piece of wire gives a very much stronger effect when its ends touch so as to form a closed circuit than when they are apart. The effect of iron is exceptional, depending partly on currents induced in it and partly on its magnetic properties. Some successful attempts have been made, by Professor Roberts-Austen of the Mint, to apply the instrument to the testing of alloys.

312. **Electrically-controlled Clocks.**—Various schemes have been proposed for utilizing electricity in connection with the driving and government of clocks. In some of them, electricity is employed either to wind up the driving-weight, or to fulfil the office of a driving-weight by its own action, a pendulum being employed as the regulator, as in ordinary clocks. In others, electricity both drives and regulates the clock (or even a considerable number of clocks), by means of currents which keep time with the movements of a standard clock, electricity having thus to do the work both of driving and regulating the dependent clocks.

But the system which has given the best practical results is that introduced by Mr. R. L. Jones, in which the dependent clocks are complete clocks, able to go of themselves, and keep moderately good time, without the aid of electricity. The duty devolving on the electric currents is merely to supply the small amount of accelerating or retarding action necessary to prevent the dependent clocks from gaining or losing on the standard clock by whose movements the currents are timed.

The arrangements for attaining this end are shown in the annexed

figures 212, 213, which represent the pendulums of the controlling and controlled clocks respectively. These pendulums are supposed to be almost precisely of the same length, so that they would nearly synchronize if disconnected.

The controlling pendulum, in its movement to either side, comes in contact with one or the other of two weak springs S S′, which are connected with the poles of a battery P N, having one of its middle plates connected with the earth, so as to keep its poles at potentials differing from that of the earth in opposite directions. In the position represented in the figure, a current is being sent from the positive pole P into the wire W. When the pendulum swings over to the other side, a negative current will be sent.

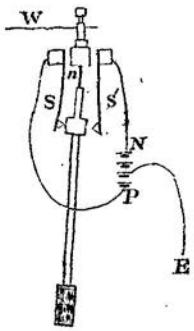


Fig. 212.—Controlling Pendulum.

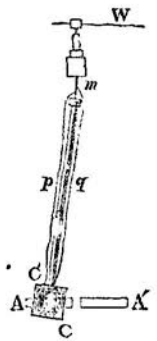


Fig. 213.—Controlled Pendulum.

The bob C C of the controlled pendulum (Fig. 213) is a hollow cylinder of soft iron encircled by a coil, whose ends are connected through two suspending springs at *m* with the wire W and the earth respectively. The consequence of this arrangement is that, whenever a current arrives by the wire W, the bob becomes an electro-magnet.

Two steel magnets A A′ are fixed, with their poles turned opposite ways, in such a position that the hollow bob of the pendulum always encircles one or both of them. Suppose, in the figure, that the poles A A′ which are turned outwards, are the two austral poles, so that the two boreal poles are facing each other. Then matters are to be so arranged that, in the position represented, the pendulum being near the left extremity of its swing, the right-hand end of the coil is a boreal pole, and magnetic force urges the pendulum to the left. When the pendulum is near the right extremity of its swing, the current is in the opposite direction, and consequently the boreal pole of the coil is its left-hand end. The pendulum will thus experience magnetic force urging it to the right. If the pendulum tends to gain upon the standard, its return from the extremities of its swing is thus opposed for a longer time than its outward movement is aided; and if it tends to lose, the assistance to its motion lasts longer than the opposition. Its tendency to deviate from the

standard clock either way is thus checked, and the correcting action is greater as the deviation from coincidence is greater. The controlling power thus obtained is so great, that even if the electrical connections are interrupted during several consecutive beats, the accumulated errors will be completely wiped off after the connections are restored.

CHAPTER XXII.

ELECTRIC LIGHT.

313. Arc Light.—When two pointed pieces of a conducting kind of carbon, such as that which is deposited in the retorts at gasworks, are connected with the poles of a powerful battery, as in Fig. 214,

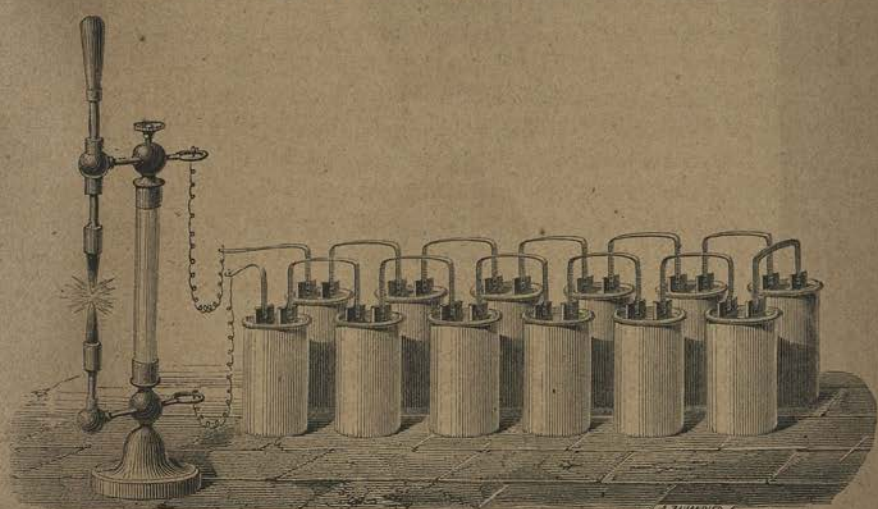


Fig. 214.—Electric Light.

a brilliant light is obtained by bringing them together so as to allow discharge to take place between them. This discharge, when once obtained, will not be interrupted by separating the points to some distance,—greater in proportion to the electromotive force of the battery; and the interval will be occupied by a luminous arch (known as the *voltaic arc*) of intense brightness and excessively high temperature. This brilliant experiment was first performed by Sir Humphry

Davy, at the commencement of the present century, with a battery of 3000 cells. The light appears to be mainly due to the incandescence of particles of carbon which traverse the space between the points.

This transport of particles can be rendered visible to a large

Fig. 215.—Image of the Carbon Points.

number of spectators by throwing an image of the heated points on a screen with the aid of a lens. Fig. 215 represents the image thus obtained, the natural size of the carbons being indicated by the sketch at the right hand. On watching the image for some time, incandescent particles will be observed traversing the length of the arc, some-

times in one direction and sometimes in the other, the prevailing direction being, however, that of the positive current. This circumstance, which appears to be connected with the higher temperature of the positive terminal, explains the difference between the forms assumed by the two carbons. The point of the positive carbon becomes concave, while the negative carbon remains pointed and wears away less rapidly. This difference is more precisely marked when the experiment is performed *in vacuo;* a kind of cone then grows up on the negative carbon, while a conical cavity is formed in the positive carbon. These phenomena are less clearly exhibited in air, on account of the combustion occasioned by the presence of oxygen.

The voltaic arc exceeds in temperature as well as in brightness all other artificial sources of heat. Despretz succeeded by its means in fusing and even volatilizing many substances which had previously proved refractory. Carbon itself was softened and bent, welded, and apparently reduced to vapour, which was condensed, in the form of black crystalline powder, on the walls of the containing vessel.

The voltaic arc must be regarded as an instance of conduction rather than of disruptive discharge, the air being rendered a conductor by the high temperature to which it is raised. Hence it is that, although discharge does not commence between the points till they have been brought close together, it is not interrupted by subsequently removing them to a considerable distance.

The voltaic arc is acted on by a magnet, according to the same laws as any other current. M. Quet, by employing a very powerful electro-magnet, with its poles at equal distances on opposite sides of the line joining the points, repelled the arc laterally to such an extent that it resembled a blowpipe flame (Fig. 216).

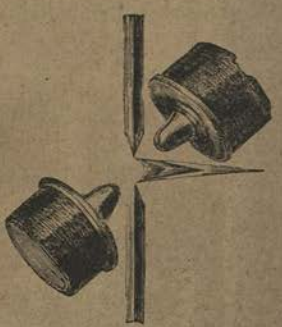


Fig. 216.—Action of Magnet on Voltaic Arc.

314. Character of the Light.—The light of the voltaic arc has a dazzling brilliancy, and attempts were long ago made to utilize it. The failures of these attempts were due not so much to its greater costliness in comparison with ordinary sources of illumination, as to the difficulty of using it effectively. Its brilliancy is painfully

and even dangerously intense, being liable to injure the eyes and produce headaches. Its small size detracts from its illuminating power—*it dazzles rather than illuminates*—and it cannot be produced on a sufficiently small scale for ordinary purposes of convenience. There is no mean between the absence of light and a light of overpowering intensity. It is much used for light-houses, for the lighting of open spaces, and for throwing images on a screen in lecture-illustrations.

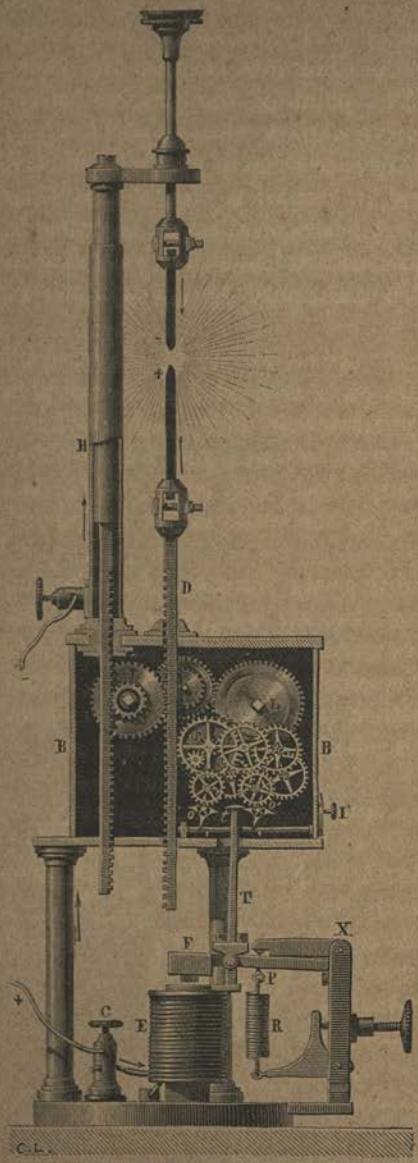


Fig. 217.—Foucault's Regulator.

As the carbons undergo waste by combustion, it is necessary to employ some means for keeping them at a nearly constant distance, so as to give a steady light. Several different regulators have been employed for this purpose, all of them depending on the principle that the strength of the current diminishes, as the distance, and consequently the resistance, increases. We will briefly describe Foucault's.

315. Foucault's Regulator.—It contains two systems of wheel-work, one for bringing the carbons nearer together, and the other for moving them further apart. Fig. 217 represents the apparatus, with the omission of a few intermediate wheels. L′ is a barrel driven by a spring inclosed within it, and driving several intermediate wheels which transmit its motion to the fly *o*.

L is the second barrel, driven by a stronger spring, and driving in like manner the fly o'. The racks which carry the carbons work with toothed wheels attached to the barrel L′, the wheel for the positive carbon having double the diameter of the other, because experience has shown that it is consumed twice as rapidly. The current enters at the binding-screw C, traverses the coil of the electro-magnet E, and passes through the wheel-work to the rack D, which carries the positive carbon. From the positive carbon it passes through the voltaic arc to the negative carbon, and thence, through the support H, to the binding-screw connected with the negative pole of the battery.

When the armature F descends towards the magnet, the other arm of the lever F P is raised, and this movement is resisted by the spiral spring R, which, however, is not attached to the lever in question, but to the end of another lever pressing on its upper side and movable about the point X. The lower side of this lever is curved, so that its point of contact with the first lever changes, giving the spring greater or less leverage according to the strength of the current. In virtue of this arrangement, which is due to Robert Houdin, the armature, instead of being placed in one or the other of two positions, as in some other regulators, has its position continuously varied according to the strength of the current. The anchor T t is rigidly connected with the lever F P, and follows its oscillations. If the current becomes too weak, the head t moves to the right, stops the fly o' and releases o, which accordingly revolves, and the carbons are moved forward. If the current becomes too strong, o is stopped, o' is released, and the carbons are drawn back. When the anchor T t is exactly vertical, both flies are arrested, and the carbons remain stationary. The curvature of the lever on which the spring acts being very slight, the oscillations of the armature and anchor are small, and very slight changes in the strength of the current and brilliancy of the light are immediately corrected.

316. Jablochkoff Candle.—The modern revival of interest in the electric light dates from the Paris Exhibition of 1878; when some of the streets of Paris were for the first time lighted by electric lamps constructed on a plan devised by M. Jablochkoff Instead of placing the two carbons end to end, and providing mechanism for keeping them at the proper distance, he dispensed with mechanism, and placed them side by side, with an insulating substance between.

them, which is gradually consumed. A A (Fig. 218) are the two carbons, separated by a stick of plaster of Paris B. The heat produced by the electric current fuses the plaster of Paris between the points of the carbons, and the fused portion acts as a conductor of high resistance, becoming brightly incandescent. To light the lamp, a piece of carbon, held by an insulator, is laid across the two carbon points until the light appears, and is then removed. The lower ends of the carbons are inserted in copper or brass tubes C C, separated from each other by asbestos; and these tubes are connected by binding-screws with the two wires which convey the current.

Fig. 218. Jablochkoff Candle.

When the current employed flows always in the same direction, the positive carbon is made twice as large in section as the negative, because it is consumed about twice as fast. When the current is alternating, which is the preferable arrangement, they are made equal.

The light was surrounded by a globe of opal glass, which served to diffuse its intensity and prevent dazzling.

317. **Incandescent or Glow Lamps.**—Another form of electric light, invented about the same time by Swan in England and Edison in America, is more suitable for the lighting of rooms and other purposes not requiring excessive brilliancy, but it gives much less light for the same expenditure of power, as compared with arc lamps. A filament of carbon prepared from bamboo, paper, or some other fibrous material, and about as thick as sewing thread, is inclosed within a vacuous glass vessel, its two ends being attached to wires which pass through the base of the lamp and serve as electrodes. A current of proper strength heats the carbon filament to whiteness, causing it to emit a soft and brilliant light, and the carbon is not consumed, as there is no oxygen to produce combustion. The vacuum must be the most perfect that can be obtained with the best Sprengel pump. An early form of these lamps is represented in Fig. 219. The thin black line in the interior represents the carbon filament, which is highly elastic and takes two turns at its upper end.

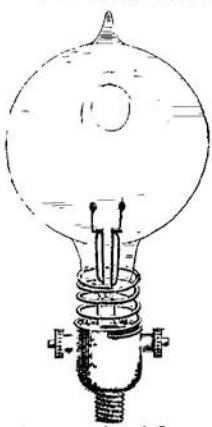


Fig. 219.—Swan's Incandescent Lamp.

CHAPTER XXIII.

ELECTROMOTORS.

318. Electromagnetic Engines.—Electromagnetic engines, more briefly styled *electromotors*, are machines driven by currents; their action depending on mechanical forces called out either between magnets and magnets, or between magnets and currents, or between currents and currents. Since their first construction in 1834 by Jacobi of St. Petersburg, who propelled a boat on the Neva by means of one, they have remained in the position of scientific toys till the recent electrical revival; but they are now becoming important in connection with the electrical transmission of driving power to a distance. A waterfall or a fixed steam-engine can be employed to generate a current of electricity by means of a dynamo or other magneto-electric machine, and this current can be made to propel a carriage or drive machinery at a distance by means of an electromotor, which receives the current and reconverts it into mechanical effect. There is always some waste of energy in this double process of conversion; but the waste is less than by any other mode of transmitting power, if the distance be considerable.

The attempts of early inventors to bring such machines, with galvanic batteries supplying their currents, into competition with steam-engines, necessarily resulted in failure, on the score of expense. A current can be more cheaply produced by a steam-engine driving a dynamo machine than by a galvanic battery; and less work would be obtainable from an electromotor driven by this dynamo than from the steam-engine direct.

319.—The most successful electromotors hitherto employed have been continuous-current dynamo machines, such as that of Siemens (Fig. 194), or of Gramme (Fig. 198), used not as dynamo machines but in the converse manner. To explain how the motion is pro-

duced, we may begin with a somewhat simpler case—that of the original Siemens' armature depicted in Figs. 190, 191. If a current from without be sent through the coil of this armature so as to make it an electro-magnet with a and b (Fig. 191) as poles, there will be a dead point in the position represented in the figure, where the fixed pole A of the field magnet is repelling a and the other fixed pole B is repelling b; but if the armature be slightly displaced from this position these two repulsions will concur in rotating the armature through about 180°. By the action of the commutator (Fig. 192) the current is interrupted for an instant at each of the two dead points (which are 180° asunder), and is then again supplied in such a direction that the pole which has just passed a pole of the field magnet is repelled by it, the repulsion lasting till the next interruption of the current. This armature is actually employed in the manner here described for some kinds of light work such as the driving of sewing-machines; but the occurrence of the dead points is a disadvantage. This difficulty could be surmounted by employing two such armatures in positions differing by 90° from each other; but still greater steadiness is obtained by employing the arrangement of Fig. 194, which resembles a combination of a number of such armatures with their poles ranged at equal distances round the circumference of the circle described.

Again in the case of the Gramme ring (Fig. 196) it is clear that if a current were sent into the coil at C from an external source and drawn off at E, this current would divide itself into two parts, one going through CDE and the other through CFE. If we suppose C to be the top and E the bottom, the current both at D and at F will circulate in the direction of watch-hands; hence both halves of the coil combine to give one pole at C and the other at E. The attractions and repulsions between these poles and the poles P P′ of the field magnet will constantly tend to turn the ring in one direction; for instance, if C is similar to P, C will be urged to the right and E to the left.

The rotations in all these cases might also have been deduced from the tendency of wires conveying a current to move across tubes of magnetic force. The two explanations in fact are fundamentally identical.

320. Quantitative Relations.—The following investigation shows the relations which exist between the work employed in driving the generator and the work given out by the motor.

Let E denote the e.m.f. of the *generator* (that is of the dynamo employed to generate the electricity), and e the reverse e.m.f. of the *motor* (which is itself a dynamo worked backwards). The whole e.m.f. in the circuit is $E-e$, and if R denote the whole resistance of the circuit, the expression for the current will be

$$C=\frac{E-e}{R}.$$

In each unit of time the quantity CE of mechanical energy is converted into electrical energy in the generator, and the quantity Ce of electrical is converted into mechanical energy in the motor.

The efficiency, as measured by the ratio of the mechanical energy given out to that put in, is accordingly $\frac{e}{E}$, and the work wasted is $C(E-e)$, which, on reference to the value above obtained for C, will be seen to be equal to C^2R, the well-known expression for the heat generated.

The expression for the efficiency shows that for economical working, the reverse e.m.f. should be a large fraction of the direct.

On the other hand, to obtain the greatest amount of work from the motor *in a given time*, when E and R are given, the product $e(E-e)$ must be made a maximum, that is, E must be divided into two parts whose product is the greatest possible; hence the two parts must be equal, e will be $\frac{1}{2}E$, and the efficiency will be $\frac{1}{2}$.

321. Economy in Transmission.—When large quantities of electrical energy are to be transmitted to great distances, whether for driving motors or for supplying electric lamps, it is important that the work spent in heating the intervening conductor should be as small as possible. The expression for this work is C^2r, where r denotes the resistance of the conductor. This latter factor can be diminished by increasing the size of the conductor, but a stout rod of copper is very expensive when the distance is several miles. The other factor C^2 can be diminished without change in the amount of energy transmitted, if we at the same time increase the electromotive force e, so as to keep the product Ce unchanged in value. If electricity is ever to be transmitted with commercial success over such distances as 50 or 100 miles, it must be by the employment of excessively high electromotive forces.

The electromotive force of a circuit supplied by a direct-current dynamo is usually called the *tension* (or the *pressure*) *of the current*.

In the case of an alternating current, the actual e.m.f. varies from

instant to instant according to a periodic law, and its *tension* or *pressure* is understood to mean the e.m.f. of a direct current which, flowing through the same resistance, would have the same energy per unit of time. It is the square root of the mean square of the actual e.m.f.

The objections to high-tension currents are the increased tendency to leakage, and the more dangerous character of the shock which will be received by a person inadvertently touching the conductor. From 500 to 600 volts is the highest tension which the Board of Trade has hitherto allowed for direct currents in circuits liable to be touched by the public.

322. **Transformation of Currents.**—In order to obtain economy in transmission combined with safety to the user, means are employed for transforming currents into others of higher or lower tension without much loss of energy.

In the case of direct currents, two methods are available. The first consists in employing the original current to charge storage cells arranged in series of a certain length, and rearranging these cells (usually in a shorter series) to produce the new current.

The second consists in employing the original current to supply an electromotor which drives by mechanical means a second dynamo, whose armature, if the tension is to be lowered, will have thicker wire than that of the original dynamo. The simplest mode of making the connection is to mount the armature of the motor and the armature of the second dynamo on the same axle. They may even be wound upon the same core and turn in the same magnetic field.

For alternate currents an apparatus specially called a *transformer* is employed, consisting of a primary and a secondary coil, both of copper wire, surrounding the same iron core. It has very various forms, one form resembling a Ruhmkorff coil with the iron wires of the core prolonged at both ends and opened out widely. Another form may be described as a composite iron ring, with the two coils wound one upon each half of it.

The weights of the two coils should be nearly equal, one of them being of thicker wire, and the other having a larger number of turns. The tensions of the two currents will be nearly in the ratio of the number of turns, and the currents themselves will be nearly in the inverse ratio. A transformer can thus be used either to increase the tension and diminish the current or to increase the current and diminish the tension.

It is now a common thing for electricity, like gas, to be supplied to a number of houses through "mains" proceeding from a central station; the "mains" consisting, in this case, of stout copper wires well insulated and laid in pairs, one wire for the direct and the other for the return current. In connection with such systems of electrical supply, it is usual to employ one of the above modes of transformation, the transformer being interposed between the mains and the houses.

In the great electric installation which has its head-quarters at Deptford, two of the dynamos at Deptford give alternate currents at a tension of 2400 volts, which undergo three transformations before reaching the consumer. They are first raised to a tension of 10,000 volts in the circuit connecting Deptford with the London centres. At these centres they are lowered again to 2400 volts in the street mains; and thirdly, there is at each consumer's house a transformer by which they are lowered to 100 volts.

323. Electric Welding.—Transformation of currents has been applied with great success by Professor Elihu Thomson to the welding of metals. The current of a powerful alternating dynamo is transformed into a current of very much lower tension and very much greater quantity.

The pieces which are to be welded together are held in massive metallic clamps which can be made to approach each other by turning a screw; and the clamps are connected with the terminals of the secondary coil. The high initial resistance of the joint compared with that of the continuous metal causes great concentration of heat at the place where heat is wanted; and the high temperature immediately after union is a further cause of high resistance and consequent concentration of heat. The weld is accordingly effected without much heating of other parts.

324. Series and Shunt Motors.—When a direct-current dynamo is giving a current, this current calls out an opposing mechanical force —in other words a force tending to make the armature rotate backwards. Hence if the field magnets retain their polarity unchanged, a reverse current supplied from without will drive the armature forwards. Whenever a dynamo is driven by a current, the current due to the rotation of its armature is opposite to the driving current.

When the brushes of a direct-current dynamo are in position, it is mechanically convenient that the armature should turn in such a

direction that the friction pulls and does not push them. It being understood that the armature is always to turn in this one direction, the following rules will be found to hold for the two kinds of machine.

In the case of a shunt dynamo, an external current in either direction drives the dynamo forwards. This appears from a glance at the three sketches, Fig. 220, in all of which *a* represents the

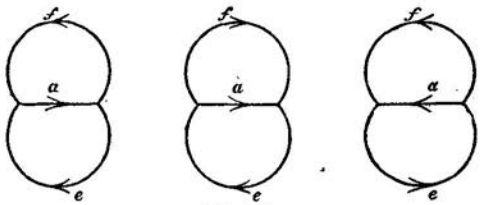


Fig. 220.

current in the armature, *f* that in the field magnets, and *e* the external current, their directions being indicated by the arrow-heads.

In the first sketch the current *a* is produced by the dynamo, and divides into *e* and *f*. In the second and third the current *e* in the external circuit divides into *a* and *f*. If the driving current *e* is in the original direction, the second sketch shows that the polarity of the field magnets is reversed, while the current in the armature is in its original direction, and therefore the armature will be driven forward. If the driving current is reversed, the third sketch shows that the field magnets retain their original polarity, but that the current through the armature is reversed, and hence the armature will still be driven forward.

In the case of a series dynamo, an external current in either direction drives the armature backwards; for the polarity of the field and the current in the armature either retain their original directions or are both reversed. The brushes must therefore be attached in a manner adapted to backward rotation.

When the field of a dynamo *is produced by a separate exciter*, or by permanent steel magnets, *the driving current must be reverse to make the armature rotate forwards.*

325. Rotating Field.—An alternating current can be used to drive a series dynamo, since, as we have seen above, the direction in which it is driven is independent of the direction of the current.

It has also been found that, under suitable conditions, one alter-

nating dynamo can drive another, and make it do mechanical work.

But a more promising application of alternating currents to motors is furnished by the discovery, made independently by Ferraris and Tesla, that two such currents, differing by a quarter-period in phase, can be made to give a rotating magnetic field, which will produce rotation of an armature (or of a magnet) placed in it.

In fact, if we denote by X the magnetic force due to the first coil at its centre, and by Y that due to the second, we shall have approximately—

$$X = a \cos \omega t \quad , \quad Y = a \sin \omega t$$

ω being a quantity such that when t is put equal to the periodic time, ωt becomes 2π, and a being the maximum value of either component.

If the coils have a common centre and have their axes at right angles, the resultant of X and Y (which act along their axes), is evidently a, and if θ denote its inclination to X we have $\tan \theta = Y/X = \tan \omega t$, $\theta = \omega t$, showing that the direction of the resultant revolves with uniform angular velocity ω.

A bar magnet, or a bar of soft iron, mounted in the field thus produced, will tend to place its length in the direction of the resultant force, and will, under proper conditions, rotate with the velocity of the field.

CHAPTER XXIV.

FURTHER DISCUSSION OF ELECTROMAGNETIC THEORY.

326. Shell.—An important conception in connection with the relations between magnetism and electricity is that of a *magnetic shell.* By this name is denoted a thin lamina, either plane or curved, magnetized in directions everywhere normal to its surfaces. It is supposed to be so thin that the areas of its two faces may be regarded as equal; and if $d\,S$ denote the area of either face of one of its elements, taken small enough to be regarded as plane, the quotient obtained by dividing the magnetic moment of the element by $d\,S$ is called the *strength of the shell.* This strength is supposed to be the same for all elements of the shell unless the contrary is stated.

It is convenient to suppose the thickness of the shell to be uniform and the magnetization to be solenoidal. Then, denoting the thickness by l, the magnetization by I, and the strength of the shell by Φ, the volume of an element is $l\,d\,S$, and its moment $I\,l\,d\,S$; hence $\Phi = I\,l$. The *strength* Φ of the shell is therefore the *product of its thickness by the intensity of its magnetization;* or by its *surface-density* of free magnetism.

327.—We shall now investigate the potential of an element of a magnetic shell.

First consider an indefinitely thin small magnet, having an amount m of free magnetism at one end A, and an amount $-m$ at the other end O. The potential at a distant point P will be

$$m/AP - m/OP = m(OP - AP)/OP.AP.$$

But, if θ denote the angle A O P, we have $OP - AP = OA\cos\theta$.

Hence the potential is $m.OA\cos\theta/QP.AP$.

But $m.OA$ is the moment of the small magnet, and $OP.AP$ is the square of its distance from P, say r^2.

If we have any number of small magnets close together, with their axes parallel, θ will be the same for them all, and their joint potential at P will be their joint moment multiplied by $\cos\theta$, and divided by r^2. Applying this result to an element of a magnetic shell, the moment of the element is $\Phi\, dS$, and the potential due to it at P is $\Phi\, dS \cos\theta / r^2$.

Imagine a small cone having dS as its base, and P as its vertex; θ is the angle between r and the normal to the element, hence $dS\cos\theta$ is the cross section of the cone, at distance r from vertex, and $dS\cos\theta/r^2$ is the solid angle of the cone. Denoting this solid angle by $d\omega$, the potential of dS at P is $\Phi\, d\omega$, and the potential of the whole shell at P is $\Phi\omega$, ω denoting the solid angle which the edge of the shell subtends at P.

The potential due to an element will be positive or negative according as the positive or the negative face of the element is turned towards P; and if a line from P cuts the shell at two points on the same side of P, elements at these points taken so as to subtend the same angle at P, will cancel each other's effects. The elementary solid angles that remain after this cancelling will constitute a solid angle bounded by the edge of the shell.

Since a closed curve drawn on the surface of a sphere of unit radius divides the surface into two parts, whose sum is the whole surface 4π, the edge of a shell subtends at any point P two solid angles, whose arithmetical sum is 4π. It is easy in each case to select the right one by attending to the elements which compose the solid angle.

328. Change $= 4\pi\Phi$ in crossing Shell.—If the shell is closed, its potential at external points is zero, and its potential at points in the inclosed hollow is $4\pi\Phi$.

If the shell is plane, its potential vanishes at external points in its plane, and has opposite signs in the two regions separated by this plane. At points close to the shell the potential is $2\pi\Phi$ on the positive side, and $-2\pi\Phi$ on the negative side.

For a shell of any form, if we take two points separated from each other by only the thickness of the shell, it will readily be seen that the angles ω subtended at these two points make up 4π when considered without regard to sign. Attending to sign, one of them is positive and the other negative; hence their algebraic difference is 4π, and the difference of their potentials is $4\pi\Phi$, which is accordingly the work that would be done by the shell on a posi-

tive unit pole, travelling round from the point on the positive face to the opposite point on the negative face. This result we shall find to be of great importance.

329. Shell equivalent to Current.—The importance of magnetic shells in electromagnetic theory arises from the fact that the effect of a shell at external points is in most respects the same as the effect of a current in a circuit coinciding with the edge of the shell.

The first step towards the establishment of this identity is furnished by the experimental fact, that the action of a current, flowing round a circle or other plane figure, is the same at distant points as the effect of a short magnet, placed within the figure, and having its magnetic axis perpendicular to the plane of the figure.

The next step is theoretical. Let A B C D be any closed curve, and A C any line joining two of its points. The effect of a current of given strength flowing round A B C D will be the same as the combined effect of two currents of the same strength flowing round A C D and C A B, the direction of flow being in each case indicated by the order of the letters. For the currents in A C and C A being equal and opposite will annul each other's effects. By successive applications of this principle, it can be shown that currents of equal strength round the meshes of any network are together equivalent to a single current of the same strength round the edge of the net.

330.—By dividing the area of a plane circuit into small equal square meshes, we can show that the moment of the equivalent magnet is proportional to the area, and can be represented by k C S, S denoting the area of the circuit, C the strength of the current in it, and k a constant depending only on the units employed. Hence a plane shell bounded by the circuit and of strength $\Phi = k$C will be equivalent to the current.

To determine the value of k, we shall take the simplest case—that of the force produced by a circular current at its centre. This force is proportional directly to C and to the length $2\pi r$ of the circuit, and inversely to r^2. Hence its value is k' C $2\pi r / r^2$ or $k' 2\pi$ C$/r$, k' being a constant depending on the units employed. The electro-magnetic system of units makes $k' = 1$; hence in this system the force at the centre is 2π C$/r$.

We have to compare this with the force at the same point due to a shell of strength k C, having the circle for its edge; we know that

all shells which fulfil these conditions have equal potentials, and therefore exert the same forces. We may conveniently regard the shell as hemispherical. The solid angle subtended at the centre is 2π; hence the potential at the centre is $2\pi k\,\mathrm{C}$. To compute the solid angle subtended at a point on the axis at a small distance x from the centre, we must add or subtract the angle subtended by an equatorial zone of breadth x and area $2\pi r x$; this angle is $2\pi x/r$. The change of potential is therefore $2\pi x k\,\mathrm{C}/r$. This is the work done by the force in moving over the distance x; hence the force is $2\pi k\,\mathrm{C}/r$. This is to be identical with the force $2\pi\,\mathrm{C}/r$ due to the current. Therefore the constant k is unity.

A shell of strength C of any form can be divided into elements each of which may be regarded as plane. Each element is equivalent to a current of strength C round it; and the aggregate of these currents reduces to a single current of strength C round the edge of the shell. We have accordingly the proposition that *a magnetic shell is equivalent to a current round its edge, the strength of the shell being equal to the strength of the current as expressed in electro-magnetic measure.*

331. Line-integral for Closed Path.—A closed curve which cuts a shell once is said to be *singly linked* with the curve which forms the edge of the shell. In travelling along such a curve from the positive face round to the negative face of a shell of strength C, the work done, or the fall of potential, is $4\pi\,\mathrm{C}$, and in completing the curve by cutting through the shell the potential rises again to its original value. When the equivalent current is substituted for the shell, the work in describing the whole of the closed curve is $4\pi\,\mathrm{C}$, for the force at all points of the path is the same as before, with the exception of the indefinitely small part representing the thickness of the shell, and the work in this is negligible.

A closed curve which cuts a shell n times in the same direction (so that in travelling along it in one direction we cut through the shell n times from the negative to the positive face) is said to be *linked n times* with the curve which forms the edge of the shell, and the work in it due to the current is $4\pi n\,\mathrm{C}$.

For a closed path not linked with the circuit, the work due to the current will be the same as that due to a shell which is not cut by the path, and will therefore be zero.

The magnetic potential due to a shell at a given point has a definite value; but the magnetic potential due to the equivalent

current at the same point has an unlimited number of values differing each from the next by $4\pi C$.

It is to be noted that though we can speak definitely of *the* current equivalent to a given shell, we cannot speak definitely of *the* shell equivalent to a given current, for all shells which have the same edge and strength and agree in sign are equivalent to one another.

It is to be understood throughout the above discussion that the work spoken of is the work done on a unit pole of positive sign.

This work is called the *line-integral of magnetic* force along the path in question. We have thus the proposition that the line-integral of magnetic force due to a current C, taken along a closed curve, is zero if the closed curve is not linked with the circuit in which the current flows, and is $4n\pi C$ if the linking is n-fold.

We may remark that the linking of two curves is mutual. Arrange two strings so that the second makes n turns round a straight portion of the first, and then straighten the second. It will be seen that the first now makes n turns round the second.

332. Force within Circular Helix.—When the circuit has the form of a uniform circular helix (like a single layer of wire wound on a Gramme ring), the line integral taken along any closed curve which runs once through its interior will from above be $4\pi n C$, if C denote the current, and n the number of turns. Let this curve be a circle of radius r coaxal with the helix. Its length will be $2\pi r$, which divided into the work $4\pi n C$ gives $2nC/r$ as the average working force. If the turns are close together, the force will be uniform, and will have this value at all points on the circle. It is to be noted that the radius of the convolutions does not enter the expressions at all; the wire carrying the current may therefore be wound in any number of layers, and the above expressions for the force and for the line-integral of force remain true. When the value of C is stated in amperes, the product nC is called the number of *ampere-turns*. This number, it will be observed, has a constant ratio to the line-integral.

333. Force within Straight Helix.—When the circuit has the form of a uniform straight[1] helix, if a pole moves from the centre of one convolution to the centre of the next, the work done is the same as if the pole were stationary and the helix moved an equal distance (which we will call one step) in the opposite direction. In this

[1] Strictly, a helix whose axis is straight.

latter displacement each convolution moves into the position previously occupied by the next, and the total motion and total work are the same as if the extreme convolution moved in succession to the positions occupied by all the other convolutions, and one step further.

This reasoning applies not only to a straight helix of any length, but also to a circular helix, whether forming a portion of a circle or a complete circle.

In the case of the straight helix, if its length is very great in comparison with its diameter, the work done on a unit pole near the middle in moving from the centre of one convolution to the centre of the next will be the same as if there were only one convolution, and the particle moved along its axis from a great distance on one side to a great distance on the other. Since this path could be converted into a closed path embracing the helix, by an external movement in which no work is done, the work in it is $4\pi C$. Let n turns of the helix occupy a length l measured along the axis of the helix, then l/n is the distance between two convolutions, and in moving through this distance the work done is $4\pi C$; hence the force is $4\pi C n/l$.

334. Flux of Force.—In a foot-note to § 52 we have pointed out an analogy between lines of force and lines of flow. It will be convenient now to expound this analogy more fully.

In the "*steady flow*" of a liquid (that is, when the state of things does not alter, but the velocity remains always the same at the same point), if we consider an imaginary tube formed by lines of flow, the quantity of liquid contained between two given sections of it remains always the same; hence the quantity which enters the included space at one of these sections must be equal to the quantity which leaves it at the other. This is true not only when the sections are perpendicular to the flow, but when they are oblique. The quantity that crosses any one section of the tube, whether perpendicular or oblique, plane or curved, must be equal to the quantity that crosses any other section in the same time. The volume of liquid that crosses a section of the tube in unit time is called the flux through the tube; and the volume that crosses any surface in unit time is called the flux through or across the surface. If the lines of flow are normal to the surface this flux is obviously equal to the surface multiplied by the velocity of the liquid. If they are inclined to the normal at an angle ϕ, the flux will be the

product of the surface, the velocity, and $\cos\phi$, or will be equal to the surface multiplied by the normal component of velocity. Denoting an element of any surface by dS, and the component velocity normal to this element by N, the flux across the surface will be the sum of such terms as $N\,dS$ taken over the whole surface. This sum $\Sigma N\,dS$ is the general expression for a flux. In the steady flow of heat, the flux of heat across a surface is $\Sigma N\,dS$, where $N\,dS$ denotes the quantity of heat that crosses dS in unit time, and N is consequently the product of the thermal conductivity by the gradient of temperature along the normal.

In the steady flow of electric currents, N will denote the electric conductivity multiplied by the gradient of electric potential along the normal to dS.

In like manner, if N denote the normal component of the intensity of force at a point, $N\,dS$ is called the flux of force across or over the element dS, and the total value of $\Sigma N\,dS$ taken over any surface is called the *flux of force* over the surface.

The intensity of force at a point is analogous to the velocity of a particle of the liquid, and the lines of force are analogous to the lines of flow of the liquid. Lines of force are always supposed to be drawn at such a distance apart that the number of lines per unit area of perpendicular section is equal to the intensity of the force; whence it will follow that the number of lines which cross any section is equal to the value of $\Sigma N\,dS$ taken over the section.

Accordingly, the three expressions,

"flux of force across a surface,"

"integral of normal force over a surface,"

"number of lines of force that cross a surface"

denote substantially the same thing, namely, $\Sigma N\,dS$.

We shall have frequent occasion to speak of the flux of (magnetic) induction B over a surface, that is to say $\Sigma N\,dS$, where N denotes the component of B normal to dS.

335. Analogy of μ to Thermal Conductivity.—We shall now show that the flux of magnetic induction B from one medium to another of different permeability follows the same laws as the steady flux of heat from one substance to another of different conductivity.

We shall use accented letters for the second medium, and unaccented for the first; shall treat the surface of junction as plane, and shall regard H, I, and B as positive when they tend from

the first medium to the second. We shall employ the subscripts n and t to denote normal and tangential components.

Since B is $H + 4\pi I$, we have, by resolving along the normal,

$$\left.\begin{aligned} B_n &= H_n + 4\pi I_n \\ B'_n &= H'_n + 4\pi I'_n \end{aligned}\right\} \qquad (1)$$

But the surface density at the junction is $I_n - I'_n$, and the repulsion due to this on either side is $2\pi(I_n - I'_n)$. Since this repulsion changes from the negative to the positive direction in passing from the first medium to the second, we have

$$H'_n - H_n = 4\pi(I_n - I'_n); \text{ hence } B_n = B'_n. \qquad (2)$$

Since the surface layer exerts no tangential force, we have

$$H_t = H'_t. \qquad (3)$$

If the media are isotropic and free from permanent magnetism, the directions of H, I, and B will coincide, making with the normal a common angle θ in the first medium, and θ' in the second. Then our last two results may be written,

$$B\cos\theta = B'\cos\theta' \qquad H\sin\theta = H'\sin\theta' \qquad (4)$$

$$\text{whence } \frac{\tan\theta}{\tan\theta'} = \frac{B}{H}\cdot\frac{H'}{B'} = \frac{\mu}{\mu'} \qquad (5)$$

$$\frac{B\sin\theta}{B'\sin\theta'} = \frac{\cos\theta'\sin\theta}{\cos\theta\sin\theta'} = \frac{\tan\theta}{\tan\theta'} = \frac{\mu}{\mu'}. \qquad (6)$$

that is, the tangential components B_t and B'_t of the flux of induction are directly as the permeabilities μ and μ', whereas the normal components B_n and B'_n are equal. The fluxes B and B′ themselves are as the secants of their inclinations to the normal.

336.—In the steady flow of heat across the surface of junction of two conductors, the normal components of flow must be equal, since each of them is the quantity of heat which in unit time crosses unit area of the junction.

The tangential flow in either medium is the product of the conductivity and the tangential gradient of temperature. This tangential gradient is the same for the two media, since the temperatures at the junction are common to both. Hence the tangential components of flow are directly as the conductivities. The tangents of the inclinations of the flows to the normal are the quotients of the tangential by the normal flows, and are therefore directly as the conductivities. Also, since the normal components are equal, the flows themselves are as the secants of their inclinations to the normal.

All these results are identical with those just obtained in the analogous problem of magnetic induction, the permeability μ taking the place of thermal conductivity.

Another analogy (from which the name *permeability* is derived) is that of a liquid forced through porous material with such small velocity that its inertia does not influence the direction of its motion. At the junction of two such materials the normal flow will be the same in both, and the tangential flows will be directly as the permeabilities. In working out the analogy pressure will take the place of temperature.

337. **Analogy to Ohm's Law.**—The analogy between magnetic induction and flow of heat is thus completely established; and it is often convenient to speak of the total "flux of induction" across a surface when we wish to denote that which is analogous to the total flow of heat (per unit time) across a surface, not necessarily a surface of junction, but any surface that we choose to imagine. This total flux is the sum of such terms as $N\,dS$, dS denoting an element of the surface, and N the component of B normal to the element. Its numerical value is often called the "number of lines of induction" that pass through the surface. A more exact designation is "number of unit tubes of induction."

In one point the analogy fails. Lines of induction are closed curves, and if we travel along one of these curves in the direction of the induction we come round to the point from which we started, whereas a line of flow of heat must have two distinct ends at different temperatures.

The analogy with flow of electricity is closer. All the reasoning of § 336 on thermal currents is equally applicable to electrical currents, and the analogy can now be extended to closed circuits.

The electrical resistance of a uniform circuit is $\frac{1}{k}\frac{l}{S}$, l denoting the length of the circuit, S its sectional area, and k the specific conductivity, or the reciprocal of the specific resistance. The analogue of this is $\frac{1}{\mu}\frac{l}{S}$, which is called the "magnetic resistance" of the circuit. For a non-uniform circuit it is $\Sigma\frac{dl}{\mu S}$.

The current C in a circuit has for its analogue the total flux BS.

Again, since the electromotive force E which forces the current C through the resistance R is CR, the magnetic analogue of E is BS multiplied by $\frac{1}{\mu}\frac{l}{S}$, that is $\frac{B}{\mu}l$, that is Hl.

If the circuit is not uniform, but the values of k and S vary from point to point, the e.m.f. between two consecutive sections at distance dl will be $C\frac{dl}{kS}$, and the magnetic analogue of this is $BS\frac{dl}{\mu S}$, that is $\frac{B}{\mu}dl$, or $H\,dl$. The analogue of the total e.m.f. in the circuit is therefore $\Sigma H\,dl$, or the line-integral of H taken round the circuit, which has accordingly received the name of *magneto-motive force*. The "sections" here considered must be surfaces to which B and H are everywhere normal. If the distance dl between two consecutive sections is sensibly different at different points of the sections, we may confine our attention to a single narrow tube of induction, and the above reasoning shows that the analogue of e.m.f. is the line-integral of H along any such tube.

If the magnetizing force is due to a current C in a wire of n convolutions linked with the magnetic circuit (for instance n turns of wire wound upon an iron ring), the value of this line-integral is $4\pi nC$ (§ 331).

If there are interruptions in the iron of the circuit, so that the circuit consists partly of iron, partly of air, and partly of copper, gutta-percha, or other non-magnetic material, the case is comparable with that of an electric circuit consisting partly of copper and partly of materials whose conductivity is only about one-thousandth of that of copper, the whole being immersed in a medium of the same small conductivity. There would be leakage through this surrounding medium; and in like manner in a dynamo there is leakage of magnetic induction through the air and other bodies which surround the armature.

In one respect the analogy with Ohm's law fails. The magnetic "resistance" of the circuit is not independent of the magnetomotive force, since it involves μ, which varies with H. On this account some writers prefer the name *magnetic reluctance*.

338. Work Dissipated in Reversals of Magnetization.—The work done by the magnetizing force H in a unit cube of iron when the magnetization I is increased by dI can be computed by multiplying the small quantity of magnetism removed from the negative to the positive end by H, and by the distance unity. It is therefore $H\,dI$. If the iron were perfectly "soft" in the magnetic sense, the total work $\Sigma H\,dI$ in a complete cycle would be zero; but owing to retentiveness, now usually called *hysteresis*, a portion of the work given is dissipated in an internal operation resembling friction, and

produces its equivalent of heat. The value of the integral $\Sigma H\,dI$ for a complete cycle, after the steady condition of recurrence has set in, is accordingly the amount of work wasted per unit volume of the iron in each complete cycle in mere magnetizing and demagnetizing.

It is desirable to use the softest possible iron for the armature-cores of dynamos, not only because it can be the most strongly magnetized, but also because it has the least hysteresis, and therefore involves the smallest wasteful generation of heat in magnetizing and demagnetizing.

Fig. 221 exhibits in graphic form the changes in B produced by a cyclic change of H in a very soft iron ring. H is represented by horizontal and B by vertical distance, the scale for H being 1000 times as large as that for B. The numbers marked are the C. G. S. values of the two variables, H ranging from $+7$ to -7, and B from $+11{,}000$ to $-11{,}000$. The area of the curve is the value of $\Sigma H\,dB$ in going once round. But B is $H + 4\pi I$, hence the area is $\Sigma H\,dH + 4\pi \Sigma H\,dI$. The first term $\Sigma H\,dH$, being the total increase of $\frac{1}{2}H^2$, vanishes for a complete cycle. Hence the area is $4\pi \Sigma H\,dI$ and is proportional to the energy wasted in overcoming hysteresis.

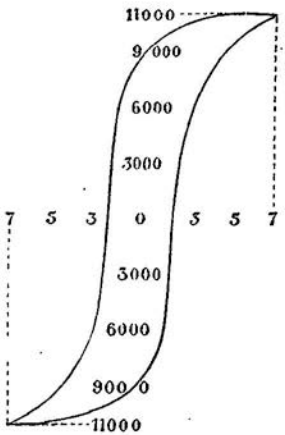


Fig. 221.—Hysteresis Curve.

339. Foucault Currents.—Another and quite distinct source of heating of iron by rapid reversals of magnetization is the production of electric currents in it by the movement of the lines of force across it. These are known by the name of *Foucault currents*, from an experiment of Foucault's in which a copper disc is heated by rapid rotation between the poles of a magnet. To guard against them in armature-cores, the iron, while continuous in the direction of magnetization, has interruptions at short intervals in the direction of the e.m.f. induced by its movement in the field. It is composed either of thin plates or of wires, and these are separated by a small thickness of some insulating substance, such as mica, paper, or varnish.

340. Potential Energy of Shell and Magnet.—We have seen that the potential at a point P, due to a magnetic shell of strength Φ, is

$\omega\,\Phi$, ω denoting the solid angle which the shell subtends at P. As regards sign, it is convenient to define magnetic potential as the work done by the magnetic forces in going *from* P *to* infinite distance. The potential of the shell will then be positive if the face which it turns towards P (or, when it turns both faces towards P, the face which preponderates by the amount ω,) is positive.

Let a quantity m of magnetism be collected at P, then the work which would be done in repelling m to infinity, which is called the *mutual potential energy* of m and the shell, is $m\,\omega\,\Phi$ Now the intensity of the force due to m at distance r is $\frac{m}{r^2}$; and the surface of a sphere of radius r described round P as centre is $4\,\pi\,r^2$. The product of these two, that is $4\,\pi\,m$, is the flux of force from m through this spherical boundary, and is the same at all distances. $4\,\pi\,m$ is, in fact, the whole flux of force from m, or the whole number of lines of force emitted by m; or the surface-integral of the force due to m taken over any closed surface which surrounds m. The flux from m within a solid angle ω is, therefore, $m\,\omega$, being the same fraction of the whole flux that ω is of the whole solid angle $4\,\pi$. Thus the expression $m\,\omega\,\Phi$, for the potential energy of m due to the shell, is equal to the strength of the shell multiplied by the flux of force from m through the shell, the flux being regarded as positive when it is incident on the positive face.

If we have any number of quantities of magnetism m_1, m_2, &c., at various points not in the substance of the shell, the total flux due to them over any surface is the algebraical sum of their separate fluxes; also their total potential energy relative to the shell is the sum of their separate energies. Therefore the mutual potential energy of a shell and a magnet is equal to the strength of the shell, multiplied by the total flux of force due to the magnet which is incident on the positive face of the shell.

341. Work in Displacement.—The work done by the mutual forces in any change of the relative position of the shell and magnet is the loss of potential energy; it is, therefore, equal to the strength of the shell multiplied by the diminution in the flux incident on its positive face, or by the increment of the flux incident on its negative face. The lines of force due to the shell itself proceed from its positive face through the air round to its negative face; they are accordingly incident on its negative face. Thus the work done is positive when the additional lines,

due to the magnet, are in the same direction as the lines due to the shell itself. Since forces tend to do positive work, the system tends to move in such a way as to produce increased flux in the direction of the flux due to the shell.

342. Coefficient of Mutual Induction.—These conclusions are applicable to a current flowing along the edge of the shell. The work done by the mutual mechanical forces of a current and a magnet in any slow movement is equal to the strength of the current multiplied by the increase in the number of lines of force (in the same direction as those due to the current) which are girdled by the current. If the movement is quick it will produce a temporary variation of the strength of the current, but the proposition will still remain true for each instant.

The magnet may be a second shell, and for this we may substitute a second current. This leads to the result that the mutual potential energy of two currents is the product of three factors, namely, the strength of the first, the strength of the second, and the number of lines of force due to unit current in one circuit which are embraced by the other circuit.

Since the potential energy cannot have two unequal values, the number of lines due to unit current in one circuit which go through the other circuit is the same whichever of the two we regard as carrying the unit current. This number is called the *coefficient of mutual induction* of the two circuits.

343. Electro-motive Force due to Motion.—Denoting by N the number of lines of force from a shell or magnet which pass through a given circuit in the direction regarded as positive, the work done *against* the mutual forces of the current and magnet in a small movement of the magnet is $-\mathrm{C}\,d\,\mathrm{N}$, C denoting the current in the circuit. The quantity of electricity that passes in the positive direction is $\mathrm{C}\,dt$, dt denoting the time occupied in the small motion. The electromotive force produced in the circuit by the motion can be calculated by dividing the work by this quantity, and is, therefore, $-d\mathrm{N}/dt$. Thus we are led to the result that the e.m.f. induced in a circuit by the motion of a magnet is equal to the rate of decrease of the flux of induction through the circuit.

This rule continues to hold when the flux of induction is due to a current, either in a second circuit or in the first circuit itself, and whether the change of flux is due to motion of one of the circuits, or to change of strength of current. *Change in the total amount*

of flux of induction through a circuit, from whatever cause it arises, calls out in the circuit an e.m.f. equal to the rate of decrease of the flux.

As regards sign, either direction having been selected as the positive direction for C, if we suppose a corkscrew, surrounded by the current, to be turned in this direction, the direction in which the corkscrew advances is to be regarded as the positive direction for flux, since this is the direction of the flux due to a positive current in the circuit.

344. Self-induction.—Let L denote the *coefficient of self-induction* of a circuit, that is to say, the flux of induction through it due to unit current in it. The flux through the circuit, due to any current in it, is proportional to the current (assuming that no iron is present), and is therefore LC; and when the current changes, the rate of decrease of this is $-L\frac{dC}{dt}$, which is accordingly the e.m.f. due to the change of current. The minus sign indicates that this e.m.f. opposes change of current (just as the inertia of a fly-wheel opposes change of angular velocity). The work done against this internal e.m.f. in the elementary time dt is the product of $L\frac{dC}{dt}$ by the quantity of electricity $C\,dt$, that is, it is $LC\,dC$. This work is stored up in the circuit, and its total amount during the increase of the current from zero to any assigned value C_1 is $\frac{1}{2}LC_1^2$, as we see at once by integrating the above expression. If the external source of current ceases, and the circuit is left to itself, this stored-up energy runs down into heat while the current is diminishing to zero. The current thus maintained after the external source is cut off is well known under the name of the "extra current."

345. Energy stored up in Two Circuits.—Next consider two neighbouring circuits, still supposing that no iron is present. Let L_1 L_2 be their coefficients of self-induction, M their coefficient of mutual induction, and C_1 C_2 the currents at time t in the two circuits.

The flux through the first circuit due to the current in the second is MC_2, and the rate of decrease of this is $-M\frac{dC_2}{dt}$, which is accordingly the e.m.f. in the first due to change of current in the second. Also, as we have seen, $-L_1\frac{dC_1}{dt}$ is the e.m.f. in the first due to change of its own current. The work done in forcing a

quantity $C_1 dt$ through the first circuit in opposition to these opposing e.m.f.s is $C_1 dt \left(L_1 \frac{dC_1}{dt} + M \frac{dC_2}{dt}\right)$, that is,

$$L_1 C_1 dC_1 + M C_1 dC_2;$$

and in like manner the work done in forcing the quantity $C_2 dt$ through the second circuit is

$$L_2 C_2 dC_2 + M C_2 dC_1.$$

The sum of these two is the energy stored up in the two circuits jointly. It can be put in the form

$$d(\tfrac{1}{2} L_1 C_1^2 + M C_1 C_2 + \tfrac{1}{2} L_2 C_2^2).$$

Accordingly the whole energy stored up during the increase of the currents from zero to their actual values is

$$\tfrac{1}{2} L_1 C_1^2 + M C_1 C_2 + \tfrac{1}{2} L_2 C_2^2.$$

346. Formula for Coefficient of Induction.—It can be shown that, when no iron or other magnetic substance is present, the coefficient of mutual induction of two circuits is the sum of such term as [1]

$$\frac{ds_1 ds_2 \cos\theta}{r}$$

ds_1 denoting any element of the first circuit, ds_2 any element of the second, θ the mutual inclination of these two elements, and r their mutual distance. Each element of the first is to be combined with each element of the second; so that if we divide each circuit into n elements, the number of terms in the summation will be n^2.

The same formula gives the coefficient of self-induction of a circuit when no magnetic substance is present, each element being taken as ds_1 and also as ds_2. This is true whether other circuits are present or not; for they do not affect the permanent flux due to the given circuit.

347. Influence of Iron on Coefficients of Induction.—The total flux of induction due to a given current flowing in a given circuit is greater (sometimes hundreds of times greater) when iron is present than when there is no iron. If the magnetization at a given point of the iron were exactly proportional to the current, L_1, L_2 and M would still be constants, though their values would not be the same as if there were no iron. This supposition, however, is not fulfilled, and L_1, L_2, and M are functions of C_1 and C_2. The most convenient meanings to be attached to the coefficients when iron is present are such that

$$L_1 \frac{dC_1}{dt} + M \frac{dC_2}{dt}$$

[1] For a very neat proof of this formula see Emtage's *Electricity and Magnetism*, p 172.

is the e.m.f. in opposition to C_1 in the first circuit, due to the changes of the two currents; in other words that $L_1 C_1 d C_1 + M C_1 d C_2 + M C_2 d C_1 + L_2 C_2 d C_2$ is the energy stored up in the two circuits in the time dt. This expression cannot now be integrated as before, since the coefficients are no longer constants.

348. Effect of Self-induction on Current of Alternating Dynamo.—In consequence of self-induction, the maxima of current-strength in the circuit of an alternating dynamo are not contemporaneous with the maxima of electromotive force, but lag behind them, and the strength of the maximum current is somewhat less than the quotient of the maximum e.m.f. by the resistance.

In applying calculation to this subject it is usual to assume that the e.m.f. is a simple harmonic function of the time. This gives the differential equation

$$C R + L \frac{dC}{dt} = E \sin \omega t; \qquad (1)$$

C denoting the current at time t, E the maximum applied e.m.f., ω the quotient of 2π by the periodic time, R the resistance of the entire circuit, and L its self-induction, which is regarded as constant.

The only variables in the equation are C and t; and we require to find C in terms of t.

The complete solution of (1) takes account of the transitory stage as well as of the permanent state. We shall confine our attention to the latter, and shall assume, from the general analogy of forced vibrations, that the current is simple harmonic with the same period as the e.m.f. This can be expressed by writing

$$C = A \cos \omega t + B \sin \omega t; \qquad (2)$$

and we have to find the values of A and B. (2) gives $dC/dt = \omega(-A \sin \omega t + B \cos \omega t)$, and when this is substituted in (1) we have

$$A R \cos \omega t + B R \sin \omega t + L \omega (-A \sin \omega t + B \cos \omega t) = E \sin \omega t. \qquad (3)$$

As this is to be true for all values of t, we may equate coefficients of $\cos \omega t$ and also of $\sin \omega t$, thus obtaining

$$A R + B L \omega = 0, \qquad B R - A L \omega = E. \qquad (4)$$

These give

$$A = -\frac{L \omega E}{R^2 + L^2 \omega^2}, \quad B = \frac{R E}{R^2 + L^2 \omega^2};$$

and when (2) is transformed into $C = a \sin(\omega t - b)$ we shall have

$$a = \frac{E}{\sqrt{(R^2 + L^2 \omega^2)}} \qquad \tan b = \frac{L \omega}{R}.$$

a here denotes the maximum value of C, and is obviously less than E/R. The quantity $\sqrt{(R^2+L^2\omega^2)}$, which takes the place of R in the denominator, is called the *impedance* of the circuit. It has sometimes been called the "apparent resistance."

Since $R/\sqrt{R^2+L^2\omega^2}$ is equal to $\cos b$, the result can be put in the form

$$C = \frac{E \cos b}{R} \sin(\omega t - b), \qquad (5)$$

which brings out the fact that self-induction affects e.m.f., not resistance, and shows the precise relation between the reduction of e.m.f. and the retardation of phase.

When the alternations are excessively rapid, another effect of self-induction becomes important—the current is more impeded in the central than in the circumferential parts of a wire, so that it is practically confined to the circumferential portions, and the central portion is useless. For this reason copper tubes are employed instead of solid conductors for conveying the alternate currents from Deptford to London.

349. Energy in Surrounding Medium.—According to Maxwell's theory, which is a development of Faraday's, electrical and magnetic effects are propagated by means of the elastic properties of the ether and of the substances which it pervades. The magnetic or magnetizing force H is to be regarded as stress, and the induction B as strain (that is, distortion). By a general law in the theory of elasticity, the energy in a unit of volume of the medium is $\frac{1}{2}$ H B, which may also be written, $\frac{1}{2}\mu H^2$, or $\frac{1}{2} B^2/\mu$. Thus, for a given value of H the energy is much greater in iron than in air, but for a given value of B it is much less. Attractions and repulsions are to be regarded as differences of pressure of the medium against the two sides of a body. The potential energy of a system of magnets is the energy of distortion of the medium in the whole of the magnetic field, and the same remark applies to what we have called the energy stored up in a circuit or in a pair of circuits. It is not really stored up in the wires which compose the circuits, but in the surrounding medium to as great a distance as the influence of the currents extends.

350. Effects of Iron on Line-integral of H and on Flux of B.—The following distinctions must be carefully noted:—

We pointed out in § 331 that when the surrounding medium is air, the line-integral of H or B (which are the same in air) along any

closed curve singly linked with a circuit carrying a current C is $4\pi C$. When iron is introduced into the field, it increases the total flux of B, but does not affect the line-integral of H, which is still equal to $4\pi C$, both for paths which pass through the iron and for paths which do not. This conclusion, which is supported by the reasoning of § 337, is believed to be rigorously true. (See Maxwell, § 499, 2nd edition.)

On the other hand, Faraday's experiments, described in Series XXIX. of his "Researches," led him to the conclusion that, in a field produced by hard-steel magnets, the introduction of iron merely serves to attract lines of induction to the iron from other parts of the field, without altering the whole flux. His experiments (see § 3221 of "Researches") indicate that there is a slight increase, and for magnets of ordinary steel not specially hardened a very considerable increase, but quite trifling in comparison with the enormous increase which occurs in fields due to currents.

351.—Faraday measured the whole flux of induction due to a magnet, by employing a loop of insulated wire just large enough to be passed over the magnet and slipped along it, the two ends of the wire being connected with a ballistic galvanometer. When the loop surrounds the magnet in the middle all the lines of induction are to be regarded as passing through it, and when the loop is at a great distance from the magnet none of them pass through it.

By the principles laid down in § 343, if R denote the joint resistance of the wire and galvanometer, and n the number of lines girdled by the loop at time t, the e.m.f. at any instant[1] is $-dn/dt$, and this must be equal to CR; hence we find (neglecting sign) that the whole quantity of electricity which passes, and which is measured by the swing of the needle, is equal to n/R.

Faraday found the swing of the needle produced by moving the loop from any one position to any other to be independent of the path taken; hence he inferred that every line of induction is continuous—in other words, that the total flux of induction is the same across every complete section of the flux.

352. Measurement of μ.—The same principle has been employed in modern observations on the magnetization of iron by currents. In one method, a wire, through which a current can be sent in either direction by means of a battery and commutator, takes several turns

[1] If the circuit has self-induction, an additional term is introduced which vanishes in the integration, and the conclusion stated in the text remains true.

round a ring of the material to be tested; and another wire, with its ends connected with a ballistic galvanometer, takes a few turns round another part of the same ring. When the primary current is passing, the flux of induction which it produces in the ring is girdled by the secondary wire; and when the primary current is reversed this flux is changed into an equal flux in the opposite direction. The quantity of electricity which passes through the galvanometer at each reversal, multiplied by the resistance R of the secondary circuit, is therefore double of the flux in the ring, if the secondary circuit is singly linked with the ring, and is $2n$ times the flux in the ring if the secondary circuit makes n turns. The value of B in the ring is obtained by dividing this flux by the sectional area. Again, if the primary wire makes N turns round the ring, and C denote the strength of the current in it, the line-integral of magnetizing force due to the current is 4πNC, and this, divided by the length of the ring measured at the middle of its thickness, gives H. Then B divided by H is the value of the permeability μ.

353. Lifting Power.—A rough determination of μ can be made by observing the mutual attraction of two electromagnets.

If an iron ring with a coil of wire wound upon it is cut across a diameter without severing the wire, and is thus converted into two half-rings round which one and the same current can be sent, the force with which they attract each other can be computed by regarding the cuts as transverse crevasses with their sides in contact. If A denote the area of one of the sections, the quantity of free magnetism on one face of it is AI, and this is to be multiplied by the intensity of force B in a transverse crevasse, minus that part of it, namely, 2πI or $\frac{1}{2}$(B−H), which is due to the face we are considering. Hence $\frac{1}{2}$(B+H)AI is the force with which each side is urged towards the other. Since I is $\frac{1}{4\pi}$(B−H) we thus obtain

$$\frac{1}{8\pi}(B^2 - H^2)$$

as the expression for the lifting power per unit of area. The lifting power when a known current is passing can be determined by direct observation, and H can be determined from the known value of the current and length of the ring, as explained in the preceding section. B can then be computed from the above formula. If C.G.S. units are employed, the lifting power must be expressed in dynes per square centimetre. When B is not uniform over the section,

the formula gives the mean value of B^2, which is greater than the square of the mean value of B; hence there is danger of over-estimation of B by this method.

The lifting powers of any two magnets similar in shape and similarly magnetized are directly as their areas, that is, as the squares of their lengths or as the squares of the cube roots of their weights.

354. Hertz's Experiments.—Maxwell maintains that light, electricity, and magnetism are all affections of one and the same medium, and that light is an electromagnetic phenomenon. This theory is supported by the experimental fact that the velocity of light is represented, in any consistent system of units, by the same number which represents the ratio of the electromagnetic to the electrostatic unit of electricity; and it has received further confirmation from the remarkable experiments of Professor Hertz, in which vibrations undoubtedly electromagnetic, and having a frequency intermediate between those of light and sound, are shown to be propagated through air and other media with a velocity which, as near as it can be measured, is that of light. To render them intelligible we must premise:

1st. That the discharge of an electrified conductor is very often of an oscillatory character. The tendency of a current, when once started, to continue, causes the conductor to be more than discharged, that is, to acquire for the moment a charge opposite in sign to that which it originally had, but smaller. The whole discharge thus consists of a rapid succession of gradually diminishing discharges in opposite directions.

2nd. That these alternate discharges, being of the nature of currents, tend to induce a rapid succession of alternate currents in any conductor in their neighbourhood.

3rd. That, according to the electromagnetic theory of light, insulators are transparent and conductors opaque. As a matter of fact, all the metals are highly opaque, and some of the best insulators are highly transparent. There is reason to believe that those insulators which are opaque to ordinary light are transparent to rays of much longer wave-length.

In Hertz's experiments the influencing system consisted of a pair of similar conductors arranged in a horizontal line, and discharging to one another by a pair of knobs, the discharges being brought about by connecting the two conductors with the two terminals of

a Ruhmkorff coil. We shall refer to this influencing system as the "primary."

Another conductor or pair of similar conductors, which we shall call the "secondary," was also employed, and was intended to show the influence of the primary. When it consisted of a pair of similar conductors these were placed in a line parallel to that occupied by the primary. When it consisted of a single conductor its form was that of a ring, with a gap at one place. In both cases the two neighbouring ends terminated in knobs very near together, their distance being regulated by a fine adjustment. The experiments consisted in getting sparks to pass between these knobs in response to the sparks of the primary.

To make the secondary more sensitive to the influence of the primary, advantage was taken of the principle known in Acoustics as Resonance, that is to say, the secondary was made of such dimensions that its natural period of electric oscillation was identical with the period of the rapid succession of alternate discharges which made up one visible discharge of the primary. This period was very much longer than the periods of ordinary luminous vibrations, but very much shorter than the periods of sonorous vibrations. The following were some of the results obtained, and they have since been verified by other observers.

The secondary, when held in a proper position, gave sparks in response to those of the primary, even at distances of several yards, and even in adjoining rooms or rooms above or below.

When a large sheet of metal was stretched across the further end of the room (a very long one), to serve as a reflector, and the secondary was held at different distances from it, a series of points alternately of maximum and minimum effect could be traced, just as nodes and antinodes can be traced when the sonorous waves produced by a tuning-fork are reflected from a wall. The inference is that the effect is propagated in waves, and that these waves can be reflected. From the wave-length as thus determined, and from the oscillation period, which could be calculated independently, the velocity of propagation of the waves was calculated, and it agreed with the velocity of light.

The fact of the reflection of the waves by a sheet of metal having been thus established, two sheets of zinc were bent into parabolic form to act as reflectors, and were placed facing each other at distant points in a room, the line of foci of each being vertical.

A suitable "primary" and "secondary" of linear form were constructed, and placed in these two lines of foci, with results which clearly showed a convergence of influence on the second line.

A gigantic prism of pitch was then placed in the path of the waves instead of the second reflector, and was found to bend the path of the waves to one side, just as a glass prism refracts light.

APPENDIX.

ELECTRICAL AND MAGNETIC UNITS.

UNITS AND DERIVED UNITS.

(1.) The numerical value of a concrete quantity is its ratio to a particular unit of the same kind; the selection of this unit being always more or less arbitrary.

(2.) One kind of quantity may, however, be so related to two or more others, as to admit of being specified in terms of units of these other kinds. For example, of the three kinds of quantity, called distance (or length), time, and velocity, any one is capable of being expressed in terms of the other two. Velocity can be specified (as regards amount) by stating the distance passed over in a specified time. Distance can be specified by stating the velocity required for describing it in a specified time, and time can be specified by stating the distance described with a specified velocity.

Force, distance, and work are in like manner three kinds of quantity, of which any two are just sufficient to specify the third.

(3.) Calculation is greatly facilitated by employing as few original or underived units as possible. These should be of kinds admitting of easy and accurate comparison; and all other units should be derived from them by the simplest modes of derivation which are available.

DIMENSIONS.

(4.) Velocity is proportional directly to distance described, and inversely to the time of its description; and is independent of all other elements. This is expressed, by saying that *the dimensions of velocity* are $\frac{\text{distance}}{\text{time}}$ or $\frac{\text{length}}{\text{time}}$.

Again, if we define the unit of velocity to be that with which unit distance would be described in unit time, the real magnitude of the unit of velocity will depend upon the units of length and time selected, being proportional directly to the real magnitude of the former, and inversely to the real magnitude of the latter. This is expressed by saying that *the dimensions of the unit of velocity* are $\frac{\text{length}}{\text{time}}$. Both forms of expression are convenient; and the ideas which they are intended to express are logically equivalent.

MECHANICAL UNITS.

(5.) All electrical and magnetic units can be derived from units of length, mass, and time. We shall denote length by l, mass by m, and time by t.

(6.) The unit of *velocity* is the velocity with which unit length is described in unit time. Its dimensions are $\frac{l}{t}$.

(7.) The unit of *acceleration* is the acceleration which gives unit increase of velocity in unit time. Its dimensions are $\frac{\text{velocity}}{\text{time}}$ or $\frac{l}{t^2}$.

(8.) The unit *force* is that which acting on unit mass produces unit acceleration. Its dimensions are mass × acceleration, or $\frac{ml}{t^2}$.

(9.) The unit of *work* is the work done by unit force working through unit length. Its dimensions are force × length, or $\frac{ml^2}{t^2}$.

(10.) The unit of *kinetic energy* is the kinetic energy of *two* units of mass moving with unit velocity (according to the formula $\frac{1}{2}mv^2$). Its dimensions are mass × (velocity)^2, or $\frac{ml^2}{t^2}$, and are the same as the dimensions of work. It might appear simpler to make it the energy of *one* unit of mass moving with unit velocity; but if this change were made, it would be necessary either to halve the unit of work, or else to make kinetic energy double of the work which produced it. Either of these alternatives would involve greater inconvenience and complexity than the selection made above.

ELECTRO-STATIC UNITS.

(11.) Let q denote *quantity* of electricity measured statically, so that the mutual repulsion of two equal quantities q at distance l,

is $\frac{q^2}{l^2}$. This being equal to a force, the dimensions of q^2 must be (length)^2 × force, or $\frac{ml^3}{t^2}$, and the dimensions of q must be $\frac{m^{\frac{1}{2}}l^{\frac{3}{2}}}{t}$.

(12.) Let V denote *difference of potential*. Then the work required to raise a quantity q through a difference of potential V, is q V. The dimensions of V are therefore $\frac{\text{work}}{q}$, or $\frac{ml^2}{t^2}\frac{t}{m^{\frac{1}{2}}l^{\frac{3}{2}}}$, or $\frac{m^{\frac{1}{2}}l^{\frac{1}{2}}}{t}$. The dimensions of potential are of course the same as those of difference of potential.

(13.) The *capacity* of a conductor is the quotient of the quantity of electricity with which it is charged, by the potential which this charge produces in the conductor. The dimensions of capacity are therefore $\frac{m^{\frac{1}{2}}l^{\frac{3}{2}}}{t} \cdot \frac{t}{m^{\frac{1}{2}}l^{\frac{1}{2}}}$, or simply l. In fact, as we have seen (§ 58), the capacity of a spherical conductor is equal to its radius.

MAGNETIC UNITS.

(14.) Let P denote the numerical value of a *pole* (or the strength of a pole). Then, since two equal poles P at distance l repel each other with the force $\frac{P^2}{l^2}$, which must be of the dimensions $\frac{ml}{t^2}$, the dimensions of P are $\frac{m^{\frac{1}{2}}l^{\frac{3}{2}}}{t}$.

(15.) Let I denote the *intensity of* a magnetic *field*. Then, a pole P in this field is acted on with a force P I. This must be of the dimensions $\frac{ml}{t^2}$. Hence, the dimensions of I are $\frac{ml}{t^2} \cdot \frac{t}{m^{\frac{1}{2}}l^{\frac{3}{2}}}$, or $\frac{m^{\frac{1}{2}}}{l^{\frac{1}{2}}t}$.

(16.) Let M denote the *moment* of a magnet. Since it is the product of the strength of a pole by the distance between two poles, its dimensions are $\frac{m^{\frac{1}{2}}l^{\frac{1}{2}}}{t}$.

(17.) Intensity of *magnetization* is the quotient of moment by volume. Its dimensions are therefore $\frac{M}{l^3}$ or $\frac{m^{\frac{1}{2}}}{l^{\frac{1}{2}}t}$. These are the same as the dimensions of intensity of field.

(18.) When a magnetic substance is placed in a magnetic field, it is magnetized by induction; and the ratio of the induced magnetization to the intensity of the field is called the *magnetic susceptibility* of the substance. The susceptibility usually increases with the intensity of the field up to a certain point at which it attains

a maximum, and then diminishes indefinitely. For diamagnetic substances, this coefficient is negative, that is to say, the induced polarity is reversed, end for end, as compared with that of a paramagnetic substance placed in the same field.

(19.) The work required to move a pole P from one point to another, is the product of P by the difference of the magnetic potentials of the two points. Hence, the dimensions of *magnetic potential* are $\frac{m l^2}{t^2} \frac{t}{m^{\frac{1}{2}} l^{\frac{1}{2}}}$ or $\frac{m^{\frac{1}{2}} l^{\frac{1}{2}}}{t}$.

ELECTROMAGNETIC UNITS.

(20.) A *current* C flowing along a circular arc, produces at the centre of the circle an intensity of field equal to C multiplied by length of arc divided by square of radius. Hence, C divided by a length is equal to a field-intensity, the dimensions of which are $\frac{m^{\frac{1}{2}}}{l^{\frac{1}{2}} t}$, and the dimensions of C are $\frac{m^{\frac{1}{2}} l^{\frac{1}{2}}}{t}$.

(21.) The *quantity* Q of electricity conveyed by a current is the product of the current by the time that it lasts. Its dimensions are therefore $m^{\frac{1}{2}} l^{\frac{1}{2}}$.

(22.) The work done in urging a quantity Q by an electromotive force E is EQ, hence the dimensions of *electromotive force* are $\frac{m l^2}{t^2} \frac{1}{m^{\frac{1}{2}} l^{\frac{1}{2}}}$ or $\frac{m^{\frac{1}{2}} l^{\frac{3}{2}}}{t^2}$; and as electromotive force is difference of potential, these are also the dimensions of *potential*.

(23.) The *capacity* of a conductor is the quotient of quantity of electricity by potential; its dimensions are therefore $\frac{t^2}{l}$.

(24.) The *resistance* R of a circuit is, by Ohm's law, equal to $\frac{E}{C}$. Its dimensions are therefore $\frac{m^{\frac{1}{2}} l^{\frac{3}{2}}}{t^2} \frac{t}{m^{\frac{1}{2}} l^{\frac{1}{2}}}$ or $\frac{l}{t}$, and are the same as the dimensions of velocity.

(25.) A coefficient of self-induction or of mutual induction is an electromotive force divided by the rate of increase of a current. Its dimensions are therefore $\frac{\text{e.m.f.}}{\text{current}}$ time = resistance × time = length.

COMPARISON OF THE TWO SETS OF UNITS.

(26.) On comparing the dimensions of the same element as measured according to the two systems, it will be observed that they are not identical. The dimensions of quantity of electricity, for

example, in the first system, are to its dimensions in the second, as l to t; and the dimensions of capacity are as l^2 to t^2.

Notwithstanding this difference of dimensions, two quantities of electricity which are equal when compared statically, are also equal when compared magnetically, or if one be double of the other when compared statically, it will also be double of the other when compared magnetically.

(27.) An illustration from a somewhat more familiar department may assist the reader in convincing himself that it is possible for one and the same kind of quantity to have different dimensions according to the line of derivation employed. It is well known that uniform spheres attract each other with a force which is directly as the product of their masses, and inversely as the square of the distance between their centres. If this law were made to furnish the unit of force, the dimensions of force would be $\frac{m^2}{l^2}$, instead of $\frac{ml}{t^2}$, as previously found.

If gravitational, electrical, and magnetic attractions are reducible to ordinary mechanical principles, all these differences of dimensions must be removable. Professor Rücker has shown that, by introducing specific inductive capacity (K) and magnetic permeability (μ) into dimensional formulæ, the electrostatic and electromagnetic dimensions can be brought into agreement by assuming that $K\mu$ is the reciprocal of the square of a velocity. (See our *C.G.S. System of Units*, 4th edition, p. 208.)

(28.) Derived units are often called *absolute* units; but it seems an abuse of language to define a unit by its *relation* to other arbitrary units, and then call it *absolute*.

(29.) By the general consent of the scientific world the Centimetre, Gramme, and Second are adopted as the basis of all derived units; and the units thence derived are distinguished by the initial letters C. G. S. prefixed.

The following units are employed for commercial purposes by the general consent of practical electricians.

Ohm = 10^9 C.G.S. units of resistance, electromagnetic system.

Volt = 10^8 C.G.S. units of e.m.f., electromagnetic system.

Ampere = current given by e.m.f. of 1 volt through resistance of 1 ohm = $\frac{1}{10}$ C.G.S.

Coulomb = quantity conveyed by 1 ampere in 1 sec. = $\frac{1}{10}$ C.G.S.

Farad = capacity of a condenser which requires 1 coulomb to charge it to 1 volt $= \left(\frac{1}{10}\right)^9$ C.G.S.

Microfarad = one-millionth of farad $= \left(\frac{1}{10}\right)^{15}$ C.G.S.

The microfarad is the unit actually employed, the farad being much too large.

Watt = rate of working of 1 ampere driven by 1 volt, or of 1 ampere flowing through 1 ohm $= 10^7$ C.G.S.

Kilowatt = 1000 watts; whereas 1 horse-power = 746 watts.

Joule = work done by 1 ampere driven by 1 volt $= 10^7$ ergs.

The self-induction (or coefficient of self-induction) of a circuit in which increase of current at the rate of 1 ampere per second produces a reverse e.m.f. of 1 volt is sometimes called a Secohm, sometimes a Quadrant. It is equal to 10^9 centimetres, which is approximately a quadrant of a meridian.

It was agreed in 1892 that the ohm should be represented by 14·4521 grammes of pure mercury at 0° C., in the form of a column of uniform section 106·3 centimetres high.

That the e.m.f. of a Clark cell at 15° C. be taken as 1·434 volt.

That ·001118 be adopted as the number of grammes of silver deposited per second from a neutral solution of nitrate of silver by a current of 1 ampere.

(30.) Ratio of the Two Units of Quantity is the Velocity of Light.—Let the units of length, mass, and time in any other system be respectively equal to

l centimetres, m grammes, t seconds.

Then the new electromagnetic unit of *quantity* will be $m^{\frac{1}{2}} l^{\frac{1}{2}}$ times the C.G.S. electromagnetic unit; and the new electrostatic unit of quantity will be $m^{\frac{1}{2}} l^{\frac{3}{2}} t^{-1}$ times the C.G.S. electrostatic unit. If the two new units of quantity are *equal*, we shall have the following relation between the two C.G.S. units, namely—

$$m^{\frac{1}{2}} l^{\frac{1}{2}} \text{ electromagnetic units} = m^{\frac{1}{2}} l^{\frac{3}{2}} t^{-1} \text{ electrostatic units};$$

that is,

$$\frac{\text{C.G.S. electromagnetic unit}}{\text{C.G.S. electrostatic unit}} = \frac{l}{t}.$$

But $\frac{l}{t}$ is clearly the value, in centimetres per second, of that velocity which would be called unity in the new system. This is a definite concrete velocity; and its numerical value will always be equal to the ratio of the electromagnetic to the electrostatic unit of quantity, whatever units of mass, length, and time are employed. Denoting

the numerical value of this velocity by v, it can be shown, by reasoning similar to the above, that the electrostatic units of potential and resistance are respectively v times and v^2 times the corresponding electromagnetic units, and that the electromagnetic unit of capacity is v^2 times the electrostatic unit.

From numerous experiments in which the same physical magnitude (quantity of electricity, difference of potential, or capacity), was measured both statically and magnetically, it appears that this velocity is (within the limits of experimental error) identical with the velocity of light.

EXAMPLES.

1. Sum to infinity the two geometric series of § 67, which express the quantities successively discharged from the two coatings.

2. Compare the energy of discharge of a single Leyden-jar with that of a battery of n such jars,

(1) When the charge of the single jar is equal to that of the battery,

(2) When both are charged to the same potential.

3. A series of n Leyden-jars are charged by cascade, the knob of the first being in connection with the conductor of the machine, the potential of which is V. If the first jar be disconnected from the machine and from the second jar, and its outer coating be connected with the earth, what will be the potential of its inner coating?

4. Show that the spark obtained by connecting the knob of the first jar in a cascade arrangement with the outer coating of the last, will be stronger than that which would be obtained by connecting the knob and outer coating of the first jar, if this jar were charged in the ordinary way with the same quantity of electricity which it has in the cascade arrangement.

5. If n similar jars are charged by cascade, and then connected so as to form an ordinary Leyden-battery, compare the energy of the discharge obtained by connecting the two coatings of the battery with that of the discharge which would have been obtained by connecting the inner coating of the first jar with the outer coating of the last in the cascade arrangement.

6. If n jars of unequal capacities $C_1, C_2, \ldots C_n$ are charged by cascade, and the potentials of their coatings are $V_0, V_1, V_2, \ldots V_n$; show that the total energy is $\frac{1}{2}(V_n - V_0)^2$ divided by $\Sigma\, 1/C$.

7. An electrified body is fixed at the distance of a few inches from the knob of an uncharged gold-leaf electroscope, and produces divergence of the leaves. How will this divergence be affected,

(1) By introducing a wire-gauze shade in connection with the earth between the charged body and the electroscope?

(2) By placing a wire-gauze shade in connection with the earth so as to inclose both the charged body and the electroscope?

8. A metallic vessel A is filled with shot, which run out, one by one, along a metal tube terminating near to an insulated positively charged conductor, but not so near that a spark can pass. The shot drop from the tube into a second metallic vessel B. What will be the final electrical condition of each of the vessels;

(1) If both are insulated?

(2) If B is insulated, but not A?

(3) If A and B are connected together by a wire, but insulated from other bodies?

Show how the principle of the conservation of energy applies in each case.

9. Show that the gain or loss of potential energy in transferring a quantity Q of electricity from one conductor to another is $Q(V_1 - V_2)$, where V_1 is the arithmetical mean of the potentials of one of the two conductors before and after the transfer, and V_2 is the similar mean for the other conductor.

In the four following examples the skeletons are supposed to be made of uniform wire, and the resistance of one side (or one edge) is called unity.

10. Find the resistance between opposite corners of a skeleton square.

11. Find the resistance between two corners of a skeleton equilateral triangle.

12. Find the resistance between two opposite corners of a skeleton cube.

13. Find the resistance between two corners of a skeleton regular tetrahedron.

14. Investigate a formula for the resistance of a wire in terms of its length, mass, density, and specific resistance.

15. The terminals of a battery of three cells are connected by a wire of resistance R; and it is found that when the terminals of a fourth cell similar to the cells of the battery are connected with the terminals of the battery, the current through R is not altered. Compare R with the resistance of one cell.

16. A battery consists of ten similar cells arranged in a series, and the circuit is completed by a wire 10 ft. long. If this wire, at a point 2 ft. from the first terminal, is allowed to touch the binding-screw which connects the second and third cells, show whether there will be any alteration in the current in either part of the wire.

17. The terminals of a battery are connected by a wire of ten thousand units' resistance. Compare the indications of an electrometer when its electrodes are joined (1) to the terminals of the battery, (2) to points in the wire separated by eight thousand units' resistance, (3) to points separated by three thousand units' resistance.

18. A galvanic battery of ten cells has its ends joined by a wire 100 ft. long with a resistance five times that of the battery. Also the junction of the third and fourth cells is in communication with the earth. Find the potentials of the other junctions, and of the terminals of the battery.

19. Investigate the effect on the current of a battery of five cells, with an external resistance equal to that of one cell,

(1) When half the liquid in one of the cells is removed,

(2) When, for the zinc of one cell another metal is substituted, such that this cell by itself would produce a current only half that of one of the other cells by itself.

20. A battery is to be constructed with plates of a given aggregate area, and at a given distance apart, for the purpose of heating a wire whose resistance is given. Find the number of cells which will heat the wire most.

21. A and B are two batteries constructed of the same materials. The resistance of A is to that of B as $a : b$, and the current in A is to that in B as $i : j$, the external resistances being negligible. Find the ratio of the amount of zinc consumed in A to that consumed in B in the same time.

22. The current from a battery of ten equal elements, passes through a voltameter, and evolves 50 cc. of hydrogen per minute. Fifty metres of wire are now introduced, in addition, into the circuit, and the volume of hydrogen now evolved in the voltameter per minute is 30 cc. If the resistance of 2·5 metres of the introduced wire be the unit of resistance, and the unit of current be that which evolves 1 cc. of hydrogen per minute, what is the electromotive force of one element of the battery?

23. Show that a given battery generates a maximum quantity of heat in the connecting wire when this wire has the same resistance as the battery.

24. A straight horizontal copper bar, in electric communication with the earth by chains hanging vertically from its two ends, is carried in a horizontal plane in the four following ways,

(1) The bar is in the magnetic meridian, and is carried in the direction of its own length.

(2) It is perpendicular to the meridian, and is carried in the direction of its own length.

(3) It is in the meridian, and is carried perpendicular to its own length.

(4) It is perpendicular to the meridian, and is carried perpendicular to its own length.

Compare the currents (if any) generated by the motion in the four cases.

25. A metal ring rotates uniformly round a horizontal diameter, which is perpendicular to the magnetic meridian. In what parts of the revolution is the induced current strongest, and in what parts does it vanish?

26. Compare the currents in the preceding question with those induced by rotating the ring with the same velocity round a vertical diameter.

27. Compare the electromotive forces generated, in two rings of different radii, by rotating with equal angular velocities round parallel diameters.

28. Prove that the distance of a fault in a submarine cable is to the whole length of the cable as

$$S - \sqrt{(T-S)(R-S)} \text{ to } R,$$

S denoting the resistance of the faulty cable when connected with earth at the further end,

T its resistance when insulated at the further end,

R the resistance of the cable before it was faulty. The fault is supposed to consist in a loss of insulation, and all parts of the cable except the fault are supposed to be perfectly insulated.

29. A copper disc revolves in a field produced by a current in a fixed coil surrounding the circumference of the disc. In a wire in circuit with the coil two points are found which have a difference of potential equal to that between the centre and circumference of the disc. Show that the resistance of the wire between these points is M/T, M denoting the coefficient of mutual induction of the coil and the circumference of the disc, and T the time of rotation. [This is Lorenz's method of absolute measurement of resistance.]

ANSWERS.

Ex. 1. Q, m Q. Ex. 2. Energy of discharge of single jar is n times that of battery in case (1), and $\frac{1}{n}$ in case (2). Ex. 3. $\frac{V}{n}$.

Ex. 4. Since Q is to equal Q', the charges and potentials in the cascade arrangement are n times as great as in the case supposed in § 77, Hence the energy of the spark is n^2 times as great. It is therefore n times the energy of the spark of the single jar. Ex. 5. The same.

Ex. 6. $Q = C_1(V_1 - V_0) = (V_1 - V_0) \div 1/C_1 = (V_2 - V_1) \div 1/C_2 = \&c. = (V_n - V_0) \div \Sigma\, 1/C$. Energy $= \frac{1}{2} Q(V_1 - V_0) + \frac{1}{2} Q(V_2 - V_1) + \&c. = \frac{1}{2} Q(V_n - V_0)$.

Ex. 7. In case (1) the electroscope will be completely screened from the influence of the electrified body, and the leaves will collapse. In case (2) electricity of the opposite sign to that of the charged body will be induced on the shade, and the divergence of the leaves will be diminished.

Ex. 8. (1) A will become positively and B negatively charged, the shots acting as carriers of negative electricity from A to B. (2) B will become negatively charged, more quickly than in case (1). (3) The negative carried from A to B by the shots returns from B to A by the wire as fast as it is supplied.

In (1) and (2) the descent of the shots is opposed by electrical forces. Hence they fall more gently, and generate less heat by concussion, than they would in the absence of electricity.

Ex. 10. 1. Ex. 11. $\frac{2}{3}$. Ex. 12. $\frac{5}{6}$. Ex. 13. $\frac{1}{2}$. Ex. 14. $\frac{\rho l^2 d}{m}$.

Ex. 15. R is $1\frac{1}{2}$ times the resistance of one cell.

Ex. 16. None; for the two points which touch had the same potential before contact.

Ex. 17. As 10, 8, and 3.

Ex. 18. First terminal $\frac{15}{6}e$, last terminal $-\frac{35}{6}e$, junctions $\frac{10}{6}e, \frac{5}{6}e, 0, -\frac{5}{6}e, -\frac{10}{6}e$, &c., e being electromotive force of one cell.

Ex. 19. The current will be diminished (1) as 7 to 6, (2) as 10 to 9.

Ex. 20. Let R denote the resistance of the wire, and r the resistance of the battery when consisting of a single cell. The number of cells n must be that which gives the smallest difference between nr and $\frac{R}{n}$.

Ex. 21. As $a i^2$ to $b j^2$.

Ex. 22. $150+\frac{1}{10}e'$, where e' denotes the reverse electromotive force of the voltameter.

Ex. 23. The heat per unit time can be shown to be E^2 divided by $R+2B+B^2/R$, and is greatest when $R+B^2/R$ is least, that is, when $R=B^2/R$.

Ex. 24. No current in cases (1), (2), and equal currents in (3), (4). The currents are due solely to the vertical magnetic force, for though the lines of horizontal force are cut by the chains in (2) and (3), the electromotive forces are *up* both chains, or *down* both, and so destroy each other.

Ex. 25. Strongest when the plane of the ring is parallel to the dipping-needle. Vanishes when plane of ring is perpendicular to dipping-needle.

Ex. 26. As total intensity to horizontal intensity, or as 1 to cosine of dip.

Ex. 27. As the squares of the radii, or directly as the areas inclosed.

Ex. 29. M C is the number of lines that go through the disc. These are all cut once in time T. Therefore e.m.f. = MC/T. But it also equals C R.

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